3. PASSIVE FILTERS.

3.1 From Equation (3.2) we get $r_L \approx 126$ mΩ.

3.2 $H(s) = \dfrac{s^2 + r_z^2}{s^2 + cs + r_p^2}$

3.3 The power that the sinusoidal signal source delivers is absorbed in R_s and R_L, since the LC network is lossless. The maximally available power from the source and that is dissipated in R_L is $P_{imax} = \dfrac{|V_{in}(\omega)|^2}{4R_s}$ where $V_{in}(\omega)$ is the effective value of the sinusoidal. For a frequency where the maximally power is dissipated in R_L is the same power dissipated in R_s. The transfer function and frequency response for an LC filter is determined by

$$\frac{P_{out}(\omega)}{P_{imax}} = \frac{\dfrac{|V_{out}(\omega)|^2}{R_L}}{\dfrac{|V_{in}(\omega)|^2}{4R_s}} = \frac{4R_s}{R_L}|H(j\omega)|^2 \le 1$$

. Usually the frequency response is normalized in order to simplify the calculations so that $|H_{norm}(j\omega)|_{max} = 1$. This means that for a doubly resistively terminated LC ladder with $R_s = R_L$ we have $|H_{denorm}(j\omega)|_{max} = 0.5$.

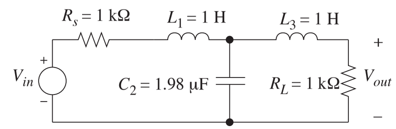
3.4 Consider Figure 3.2. The current is $I = \dfrac{V_1}{Z_1 + Z_2}$. The power dissipated in the load is $P_L = |I|^2 R_L$

$$= \left(\frac{V_1}{Z_1 + Z_2}\right)^2 R_L = \frac{|V_1|^2 R_L}{(R_s + R_L)^2 + (X_s + X_L)^2}$$

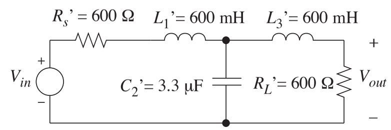
. Maximum power is dissipated when the denominator is minimum, which occur when $X_L = -X_s$ and when $\dfrac{|V_1|^2 R_L}{(R_s + R_L)^2}$ is maximum. That is when $R_L = R_s$. Hence, the maximal power transfer occur if $Z_L = Z_s^*$.

3.5 Since $R_s = R_L$, the output voltage will not be larger than half of the input voltage, i.e., the attenuation is –20 log(0.5) = 6.02 dB. At $\omega = 0$ we get $V_{outmax} = 1$ V. At the passband edge there will be some additional attenuation due to the ripple in the passband, A_{max}, where $A_{max} = -10\log(1 - \rho^2)$. With ρ = 50% we have $A_{max} = -10\log(1 - 0.5^2) = 1.249$ dB. The total attenuation at the passband edge is: 6.02 + 1.249 = 7.269 dB which correspond to $10^{-(7.269/20)} = 0.4331$. The output signal is therefore $V_{out}(t) = 0.8661\sin(\omega_c t)$.

3.6 a The impedance scaling involve multiplying all impedance with the same factor, i.e., ksL, k/sC and which yields $L \to kL$, $C \to C/k$ and $R \to kR$. Since impedance scaling is done with the same factor k, the transfer function and frequency response are not affected. in our case, $k = R_L'/R_L = 1000/600$. We get $L_1 = L_3 = k\,L'_3 = k$ 600 mH = 1 H, $C_2 = (3.3/k)$ μF = 1.98 μF and $R_s = R_L = 600/k = 1$ kΩ



b) The new frequency response shall have the passband edge at 6π Mrad/s, i.e., we increase the band edge with the factor 6000π. All terms in the expression for the frequency response contain frequency dependent impedances (L and C) as products of $\omega^2 LC$. These products will be unaffected after the frequency scaling. If we increase ω with a factor k and divide L and C with the same factor, then the products remain the same, i.e., $(k\,\omega)^2(L/k)(C/k) = \omega^2 LC$. The resistors, which are frequency independent and not affected. In this case, we get $R_s = R_L = 1$ kΩ and $L_1 = L_3 = 1/k$ H = 1/6000π = 53.0516 μH.



3.7 An RC section attenuate at the cutoff frequency with 3.0102 dB. Hence, $\omega_c = 1/RC$. For f_c = 1,2 kHz and R = 10 kΩ we get $2\pi 1200 = 1/(C\,10^4)$ => C = 13.249 nF

3.8 % LP specification: $A_{max} = 0.01$; $A_{min} = 50$; wc = 100e6; ws = 350e6; $R_s = 50$; $R_L = 50$.

a) Butterworth

```
Wc = 100*10^6; Ws = 350*10^6; Amax = 0.01; Amin = 50; Rs = 50; RL = 50;
N = BW_ORDER(Wc, Ws, Amax, Amin)
N = 8; Ladder = 0;          % pi ladder
[L, C, K] = BW_LADDER(Wc, Ws, Amax, Amin, N, Rs, RL, Ladder)
```

N = 7.0186 We select N = 8

L [μF]	C [n F]
0	0.05339068015275
0.38010948348175	0
0	0.22754961736904
0.67103268710839	0
0	0.26841307484336
0.56887404342260	0
0	0.15204379339270
0.13347670038186	0

b) Chebyshev I

```
Wc = 100*10^6; Ws = 350*10^6; Amax = 0.01; Amin = 50; Rs = 50; RL = 50;
N = CH_ORDER(Wc, Ws, Amax, Amin)
N = 5; Ladder = 0;
[L, C, K] = CH_I_LADDER(Wc, Ws, Amax, Amin, N, Rs, RL, Ladder)
```

N = 4.9281 We select N = 5

L [μF]	C [n F]
0	0.15126633060317
0.65245987094190	0
0	0.31546102537261
0.65245987094190	0
0	0.15126633060317

c) Chebyshev II

```
N = CH_ORDER(Wc, Ws, Amax, Amin)
N = 5; Ladder = 0;
[L, C, Rs, RL, W0, K] = CH_II_LADDER(Wc, Ws, Amax, Amin, N, Rs, RL, Ladder)
```

$N = 4.9281$ We select $N = 5$

L [μF]	C [n F]	ω_0
0	0.15303549765980	
0.16897449368419	0.01669091433255	5.95455565846428 10^8
0	0.52370812504808	
0.15154721380251	0.04872239250670	3.68011778483394 10^8
0	0.12456064600872	

d) Cauer

```
Wc = 100*10^6; Ws = 350*10^6; Amax = 0.01; Amin = 50; Rs = 50; RL = 50;
N = CA_ORDER(Wc, Ws, Amax, Amin)
N = 5;
[G, Z, R_ZEROS, P, Wsnew] = CA_POLES(Wc, Ws, Amax, Amin, N);
W = [0:Ws/200:5*Ws];
H = PZ_2_FREQ_S(G, Z, P, W); Att = MAG_2_ATT(H);
plot(W, Att, 'linewidth', 2, 'color', 'r'); hold on
Ladder = 0;                    % pi ladder
[L, C, Rs, RL, W0, K] = CA_LADDER(Wc, Ws, Amax, Amin, N, Rs, RL, Ladder)
Z0 = []; T = 1;
H = LADDER_2_H(N, Z0, L, C, Rs, RL, K, W, T);
Att = MAG_2_ATT(H*2);
PLOT_ATTENUATION_S(W, Att)
```

$N = 3.888$ We can not realize a lowpass Cauer ladder of even order. There must be at least one zero at infinity for a realizable lowpass filter. We may use a modified even-order Cauer filer type C, however, here we select $N = 5$

L [μF]	C [n F]	ω_0
0	0.13999718453357	
0.61223598723062	0.01469869387294	3.33350568849650 10^8
0	0.28631519830734	
0.52231096665713	0.04187643147380	2.13821117393035 10^8
0	0.11730363270447	

3.9 From a nomogram with $\omega_s/\omega_c = 6 \Rightarrow N = 5$.

From a table with R_s and R_L, i.e., $r = 1$ we select the filter $P0516$. We get for the normalized T ladder (a T ladder begins with an inductor) and after denormalization with and $R_0 = 50\ \Omega$ and $r_{p0} = \omega_c\ \varepsilon^{-1/N} = 10^5\ \varepsilon^{-1/6} = 1.379\ 10^5$ where $\varepsilon = \sqrt{10^{0.1A_{max}} - 1}$

$r = 1$ — $R_s = R_L = R_0 r = 50\ \Omega$

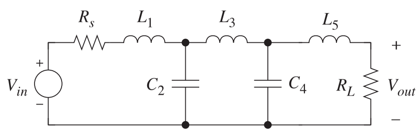
$L_1' = 0.6180$ — $L_1 = L_1' R_0/r_{p0} = 0.2589$ mH

$C_2' = 1.6180$ — $C_2 = C_2'/(R_0 r_{p0}) = 236.56$ nF

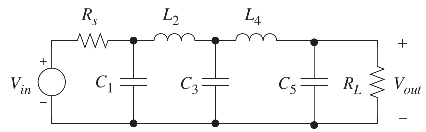
$L_3' = 2.0000$ — $L_3 = L_3' R_0/r_{p0} = 0.73103$ mH

$C_4' = 1.6180$ — $C_4 = C_4'/(R_0 r_{p0}) = 236.56$ nF

$L_5' = 0.6180$ — $L_5 = L_5' R_0/r_{p0} = 0.2589$ mH



Denormalization of the element values for Butterworth filters is done with r_{p0}. Notice the symmetry in the ladder and element values which is an indication of lower element sensitivity.



f) We get for the element values for the π ladder

$C_1' = 0.6180$ — $C_1 = C_1'/(R_0 r_{p0}) = 90.355$ nF

$L_2' = 1.6180$ — $L_2 = L_2' R_0/r_{p0} = 0.5914$ mH

$C_3' = 2.0000$ — $C_3 = C_3'/(R_0 r_{p0}) = 292.41$ nF

$L_4' = 1.6180$ — $L_4 = L_4' R_0/r_{p0} = 0.5914$ mH

$C_5' = 0.6180$ — $C_5 = C_5'/(R_0 r_{p0}) = 90.355$ nF

Also here we have symmetry and ladder has fewer inductor and is therefore cheaper to manufacture.

g) In this case, the terminating resistors are unequal. We have $r = R_L/R_s = 300/600 = 0.5$. From a table, or **BW_LADDER**, we get the element values for the π ladder. Notice when the terminating resistors are unequal we get different cases. In this case, we have N = odd and the signal source is a voltage source. We denormalize so that one of the terminating resistor becomes 300 Ω and the other becomes 600 Ω. We select $R_0 = 300\ \Omega$.

$r = 0.5$

$R_s' = 1/r = 2$ — $R_s = 600\ \Omega$

$R_L' = 1$ — $R_L = 300\ \Omega$

$C_1' = 3.1331$ — $C_1 = C_1'/(R_0 r_{p0}) = 76.346$ nF

$L_2' = 0.9237$ — $L_2 = L_2' R_0/r_{p0} = 2$ mH

$C_3' = 3.0510$ — $C_3 = C_3'/(R_0 r_{p0}) = 74.346$ nF

$L_4' = 0.4955$ — $L_4 = L_4' R_0/r_{p0} = 1.1$ mH

$C_5' = 0.6857$ — $C_5 = C_5'/(R_0 r_{p0}) = 16.709$ nF

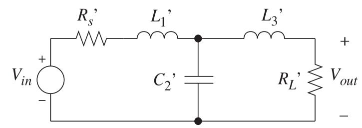
The ladder has no symmetry and the sensitivity is probably somewhat larger.

Notice that for a reciprocal ladder it is possible to place the signal source (voltage source) either in series with R_L or R_s and use the voltage of the other resistor as output signal (signal carrier). The frequency response will be the same in both cases, except for a constant factor. In this case, according to Problem 3.5 the maximal output voltage is $R_L V_{in}/(R_s + R_L) = 300\ V_{in}/(600 + 300) = V_{in}/3$ compared to $V_{in}/2$ in case a) and b). Usually we have no reason to not select equal termination when the ladder is used as reference filter for synthesis of active, SC or digital filters.

3.10 We have $\omega_c = 2\pi\ 500$ krad/s and $\omega_s = 2\pi\ 2500$ krad/s. A nomogram yields with $\omega_s/\omega_c = 5 \Rightarrow N = 3$. We select the filter $T0316$, with $R_s = R_L = 1$, the normalized T ladder have the following normalized element values

$R_s' = R_L' = 1$, $L_1' = L_3' = 1.5963$ and $C_2' = 1.0967$

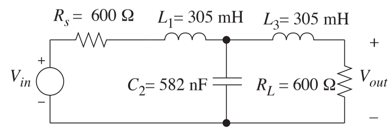
Denormalizing with $R_0 = 600\ \Omega$ and $\omega_c = 1000\ \pi$ rad/s gives $R_s = R_L = R_0 R_s' = 600\ \Omega$



$$L_1 = L_3 = L_3' \frac{R_0}{\omega_c} = 1.5936 \cdot \frac{600}{1000\pi} = 0.3049 \text{ H}$$

$$C_2 = C_2' \cdot \frac{1}{R_0 \omega_c} = \frac{1.0967}{600 \cdot 1000\pi} = 581.8 \text{ nF}$$

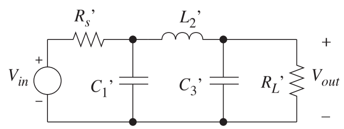
The denormalized T ladder is shown to the right. For a π ladder we get the normalized element values: $R_s' = R_L' = 1$, $C_1' = C_3' = 1.0967$ and $L_2' = 1.5963$ and after denormalization



$R_s = R_L = R_0 R_s' = 600\ \Omega$

$$L_2 = L_2' \frac{R_0}{\omega_c} = 1.5936 \cdot \frac{600}{1000\pi} = 0.3049\ \text{H}$$

$$C_1 = C_3 = C_3' \cdot \frac{1}{R_0 \omega_c} = \frac{1.0967}{600 \cdot 1000\pi} = 581.8\ \text{nF}$$



Hence the same numerical values in the two ladders, but the π ladder have one inductor less.

3.11 Chebyshev I filter, π ladder, $R_s = R_L = 50\ \Omega$

$\omega_c = 2\pi \cdot 22 \cdot 10^6$ rad/s

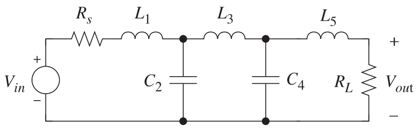
$\omega_s = 2\pi \cdot 33 \cdot 10^6$ rad/s

$R_0 = 50\ \Omega$

From the nomogram we get $N = 5$ and since $R_s = R_L$ => we select the table with $r = 1$

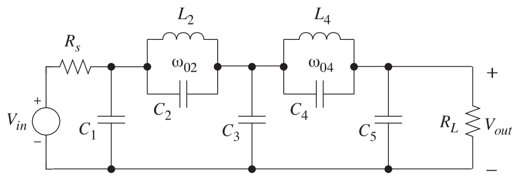
Normalized values	Denormalized values
$r = 1$	$R_s = R_L = R_0\, r = 50r = 50\ \Omega$
$C_1' = 1.7058$	$C_1 = C_1'/(R_0\omega_c) = 246.8$ pF
$L_2' = 1.2296$	$L_2 = L_2' R_0/\omega_c = 0.445\ \mu$H
$C_3' = 2.5408$	$C_3 = 367.6$ pF
$L_4' = 1.2296$	$L_4 = 0.445\ \mu$H
$C_5' = 1.7058$	$C_5 = 246.8$ pF

Denormalized ladder.



3.12 We select the filter C051529 and get the normalized element values shown below with $r^2 = 1$, i.e., equal terminating resistors. We should also have selected the filter C051528 that also meet the requirement. We get after denormalization with ω_c and $R_0 = 50\ \Omega$.

$C_1' = 1.08877$	$C_1 = C_1'/(R_0\omega_c) = 1.08877/(50 \cdot 10^8) = 217.75$ pF
$L_2' = 1.29869$	$L_2 = L_2' R_0/\omega_c = 1.29869 \cdot 50/10^8 = 649.35$ nH
$C_2' = 0.06809$	$C_2 = C_2'/(R_0\omega_c) = 0.06809/(50 \cdot 10^8) = 13.618$ pF
$\omega'_{02} = 3.3629$	$\omega_{02} = \omega'_{02}\omega_c = 336.29$ Mrad/s
$C_3' = 1.80288$	$C_3 = C_3'/(R_0\omega_c) = 1.80288/(50 \cdot 10^8) = 360.576$ pF
$L_4' = 1.15805$	$L_4 = L_4' R_0/\omega_c = 1.15805 \cdot 50/10^8 = 579.03$ nH
$C_4' = 0.18583$	$C_4 = C_4'/(R_0\omega_c) = 0.18583/(50 \cdot 10^8) = 37.166$ pF
$\omega'_{04} = 2.1556$	$\omega_{04} = \omega'_{04}\omega_c = 215.56$ Mrad/s
$C_5' = 0.98556$	$C_5 = C_5'/(R_0\omega_c) = 0.98556/(50 \cdot 10^8) = 197.11$ pF



The π ladder have fewer inductors and is often preferred for the realizing LP filters. The T ladder have series resonance circuit in the shunt arms and the voltage over the capacitors in the shunt arms may become too large if the losses in the inductors is small.

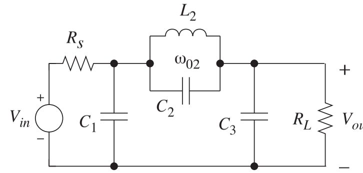
3.13 The notation C031534 correspond to $N = 3$, $\rho = 15\%$, $\Theta = 34^o$, passband ripple $A_{max} = 0.09883$ dB and minimum stopband attenuation $A_{min} = 20.53$ dB, $\omega_c = 800\pi$ Mrad/s, and $\omega_s = 1.7883 \cdot 800\pi$ Mrad/s = 4.494488 Grad/s. From a table or the MATLAB program `CA_LADDER` we get a normalized π ladder with the element values $C_1' = C_3' = 0.8544$, $C_2' = 0.2792$, $L_2' = 0.8780$, and $R_s' = R_L' = r = 1$

The normalized zero is at $s = \pm j\omega_Z = \pm j2.0199$. The denormalized element values using $R_0 = 50\ \Omega$ and $\omega_0 = \omega_c = 800\pi$ Mrad/s are

$$C_1 = C_3 = \frac{C_1'}{R_0\omega_0} = 6.799\ \text{pF},\ C_2 = 2.2218\ \text{pF}$$

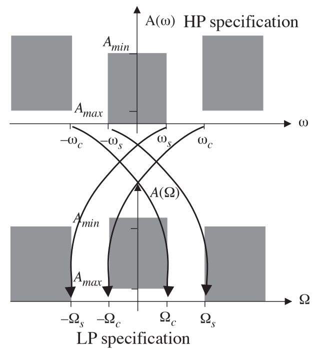
$$L_2 = \frac{R_0}{\omega_0} \cdot L_2' = 17.467\ \text{nH}$$

where $\omega_{02} = \dfrac{1}{\sqrt{L_2 C_2}} = 5.0762$ Grad/s



3.14 For the LP-HP transformation we have $S = \dfrac{\omega_I^2}{s}$

and $\Omega = -\dfrac{\omega_I^2}{\omega}$. The band edges are mapped as shown.



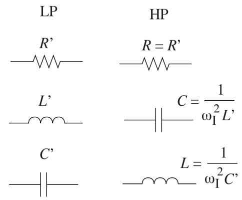
The passband and stopband attenuation requirements are the same in the two specifications.

We get the impedances in the HP filter by changing S to s in the expression for the corresponding impedance in the LP filter. Resistors are not affected since they are frequency independent. An inductor has the impedance

$$SL' \rightarrow \frac{\omega_I^2}{s} L' = \frac{1}{sC_{HP}} \quad \text{i.e.,} \quad C_{HP} = \frac{1}{\omega_I^2 L'}$$

Hence an inductor, L', in the LP filter is replaced with a capacitor, C_{HP}, in the HP filter. In the same way, a capacitor in the LP filter is replaces with an inductor

$$\frac{1}{SC'} \rightarrow \frac{s}{\omega_I^2 C'} = sL_{HP} \Rightarrow L_{HP} = \frac{1}{\omega_I^2 C'}$$



3.15 $A_{max} = 0.5$ dB, $A_{min} = 25$ dB, $f_c = 15$ kHz, $f_s = 6$ kHz

$\omega_c = 2\pi f_c = 2\pi \cdot 15$ krad/s and $\omega_s = 2\pi f_s = 2\pi \cdot 6$ krad/s

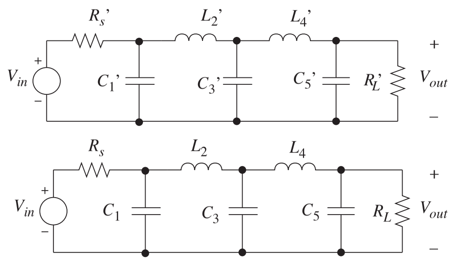
Transform to a corresponding specification for the LP filter. $\Omega_c = \omega_I^2/\omega_c$ and $\Omega_s = \omega_I^2/\omega_s$

We select ω_I^2 so that $\Omega_c = 1$, which allow us to directly use the normalized values in a table, i.e., => $\omega_I^2 = \omega_c$ and $\Omega_s = \omega_I^2/\omega_s = \omega_c/\omega_s = 2\pi f_c/(2\pi f_s) = 15/6 = 2.5$

From a nomogram we get: $N = 5$ and a π ladder with the normalized element values $C_1' = C_5' = 0.6180$, $L_2' = L_4' = 1.6180$ and $C_3' = 2.0000$

Compute ε and r_{p0}.

$\varepsilon = \sqrt{10^{0.1A_{max}} - 1} = 0.3493114$ and $r_{p0} = \Omega_c\ \varepsilon^{-1/N} = 1.23412016$ (Only for Butterworth filters)



Denormalize with $R_0 = 50\ \Omega$ and r_{p0}, i.e., multiply all capacitances with $1/(R_0 r_{p0})$ and all inductances with R_0/r_{p0}.

$R_s = R_L = 50\ \Omega$

$C_1 = C_5 = C_1'/(R_0 r_{p0}) = 1.001523 \cdot 10^{-2}$ F

$L_2 = L_4 = L_2' R_0/r_{p0} = 65.55277$ H

$C_3 = C_3'/(R_0 r_{p0}) = 3.241118 \cdot 10^{-2}$ F

Transform from LP to a HP filter yields:

$L_1 = L_5 = 1/(\omega_I^2 C_1) = 1/(\omega_c C_1) = 1.059$ mH

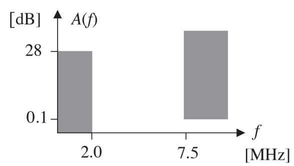
$C_2 = C_4 = 1/(\omega_I^2 L_2) = 1/(\omega_c L_2) = 161.86$ nF

$L_3 = 1/(\omega_I^2 C) = 1/(\omega_c C_1) = 0.327$ mH

3.16 In the same way as in Problem 3.15 (error in the book) we get $\Omega_S = \omega_I^2/\omega_S \Rightarrow \omega_c/\omega_S = 2\pi f_c/(2\pi f_S) = 15/6 = 2.5$ and we get $N = 4$. The *LC* ladder with equal terminating resistors does not exist since the attenuation at $\omega = 0$ is A_{max}. We may either select a modified approximation of type *c* or select unequal terminating resistors.

Incomplete solution!

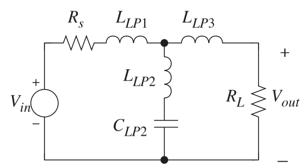
3.17 HP->LP $\omega_c = \dfrac{\omega_I^2}{\Omega_c}$ $\quad \omega_s = \dfrac{\omega_I^2}{\Omega_s}$ Select $\omega_I^2 = \omega_c$,



which yields: $\Omega_c = 1$ rad/s and $\Omega_s = 3.75$ rad/s. From the toolbox or nomogram we get $N = 3$.

In the LP->HP transformation, an inductors is replaced with a capacitor and vice versa. Select therefore the LP filter with the fewest number of capacitors in order to get the fewest number of inductors in the HP filter.

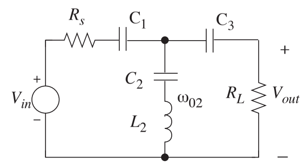
We get ($\Omega_s = 3.75$ rad/s and $A_{min} = 40$ dB) from the table. We select a filter that satisfy both the transition bandwidth and attenuation. Hence, filter C0315Θ with $\Theta = 16$ have $\Omega_s/\Omega_c = 3.6279$ and $A_{min} = 40.2$ dB. The normalized element values for this filter is ($r^2 = R_s/R_L = 1$) and they are denomalized with $C = C'/(R_0\Omega_s)$ and $L = R_0 L'/\Omega_s$. We get the element values



$L'_{LP1} = L'_{LP3} = 0.9897$ $\quad L_{LP1} = L_{LP3} = 49.485$ H

$L'_{LP2} = 0.0529$ $\quad L_{LP2} = 2.645$ H

$C'_{LP2} = 1.0869$ $\quad C_{LP2} = 21.738$ mF



LP->HP-transformation yields

$C_1 = C_3 = 1/(\omega_I^2 L_{LP1}) = 1/(\omega_c L_{LP1}) = 2.6955$ nF

$C_2 = 1/(\omega_I^2 L_{LP2}) = 1/(\omega_c L_{LP2}) = 50.41$ nF

$L_2 = 1/(\omega_I^2 C_{LP2}) = 1/(\omega_c C_{LP2}) = 6.1337\ \mu$H

3.18a) Determine the element values in a 3rd-order Butterworth filter with the passband edge 2.5 kHz. At the band edge for the LP filter and HP filter we have according to Problem 4.15, $|H_{LP}(j\omega)|^2 + |H_{HP}(j\omega)|^2 = 1$, i.e., for this frequency, half of the power are dissipated in each of

the filters and the attenuation at the passband edge is: $A_{max} = -10\log(0.5) = 3.01$ dB. We get $\varepsilon = \sqrt{10^{0.1A_{max}} - 1} = \sqrt{10^{0.301} - 1} = 1$. From the table (or MATLAB program) we get the normalized element values $L_1 = L_3 = 1$ and $C_2 = 2$. Denormalize with $R_0 = 8\ \Omega$ and $r_{p0} = \omega_c\, e^{(-1/N)} = \omega_c = 5000\,\pi$ rad/s. The denormalized element values for the singly terminated LP ladder are

$r = 0$	$R_s = 8\ \Omega$
$L_1' = 0.5000$	$L_1 = L_1' R_0/\omega_c = 0.5000 \cdot 8/5000\pi = 0.2547$ mH
$C_2' = 1.3333$	$C_2 = C_2'/(R_0\omega_c) = 1.3333/(8 \cdot 5000\pi) = 10.61\ \mu$F
$L_3' = 1.5000$	$L_3 = L_3' R_0/\omega_c = 1.5000 \cdot 8/5000\pi = 0.7639$ mH

The element values in the HP filter are with $\omega_I^2 = \omega_c^2$

$C_3 = 1/(R_0\omega_c L_1') = 15.915\ \mu$F

$L_4 = R_0/(\omega_c C_3') = 381.98\ \mu$H

$C_5 = 1/(R_0\omega_c L_3') = 5.305\ \mu$F

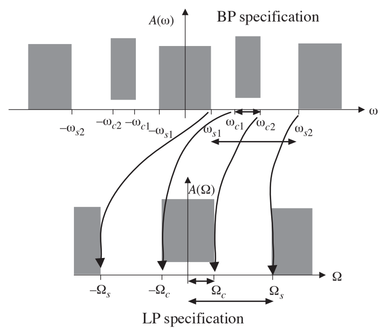
Note: If the speakers are placed relatively far from each other (one wavelength or more at the passband edge frequency) then this method is appropriate. If the speakers are placed closer, then the attenuation at the passband edge should instead be 6 dB. This requires a more complicated filter solution. Notice that the specification and design of the crossover flitters is controversial and an intensely discussed topic among audio freaks.

b) We get after long calculations $Z_{in} = 8\ \Omega$.

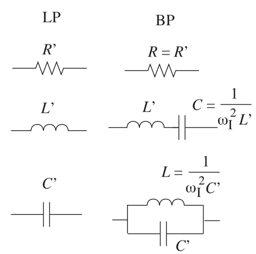
c) It is possible to replace the resistor R_s with another diplexer with the input impedance $R_s = 8\ \Omega$ and that divide the lower frequency band (0 – 2.5 kHz) in two bands.

3.19 We have for the LP-BP transformation

$S = s + \omega_I^2/s$ and $\Omega = \omega - \omega_I^2/\omega$. The band edges, see the figure. The attenuations are the same. Notice that the width of the passband in the BP filter and LP filter are the same and that the stopband edge in the LP filter equals the difference in stopband edges in the BP filter. We get the impedances in the BP filter be changing S for s in the expressions for the impedances in the LP filter. Resistors are frequency independent and are not affected.



For an inductor with the impedance SL' we get $SL' \to sL' + \omega_I^2 L'/s$. Hence an inductor, L' in the LP filter is replaced by a series resonance circuit consisting of a capacitor, $C_{BP} = 1/(\omega_I^2 L')$ and an inductor, $L_{BP} = L'$. In the same way we get for a capacitor, C', in the LP filter, i.e., for an admittance SC' we get $SC' \to sC' + \dfrac{\omega_I^2}{s}C'$. We get a parallel resonance circuit consisting of a capacitor, $C_{BP} = C'$ and an inductor, $L_{BP} = 1/(\omega_I^2 C')$. In the same way we may derive expressions for more complex impedances.



3.20 $\omega_{c1} \cdot \omega_{c2} = 40 \cdot 90 = 3600$ (Mrad/s)^{2}and $\omega_{s1} \cdot \omega_{s2} = 11.6 \cdot 311.6 = 3614.56$ (Mrad/s)^2 We select to increase ω_{s1} to ω_{s1} := 3600/311.6 = 11.5533 Mrad/s. We get $\omega_I^2 = 3600$ (Mrad/s)^2. $\Omega_c = \omega_{c2} - \omega_{c1} = 90 - 40 = 50$ Mrad/s, $\Omega_S = \omega_{S2} - \omega_{S1} = 311.6 - 11.5533 = 300.046$ Mrad/s and

$\varepsilon = \sqrt{10^{0.1A_{max}} - 1} = 0.152620 \Rightarrow N = 4$. $r_{p0} = 79.99579$ Mrad/s. We select a π ladder.

$C_1' = 0.7654$, $L_2' = 1.8478$, $C_3' = 1.8478$, $L_4' = 0.7654$ and denormalize by multiplying L with R_0/r_{p0}

and C with $1/R_0Rr_{p0}$. $C_1 = 95.68$ pF, $L_2 = 2.3098$ μH, $C_3 = 230.987$ pF, $L_4 = 956.80$ nH.
Transform to a BP ladder. $l_1 = 2.90$ μH, $c_1 = 95.68$ pF, $l_2 = 2.3098$ μH, $c_2 = 120.2$ pF, $l_3 = 1.2026$ μH, $c_3 = 230.987$ pF, $l_4 = 956.80$ μH, $c_4 = 290.31$ pF.
The number of zeros at s = 0 is 4 and at s = ∞ it is also 4 and the number of poles is 8.

3.21 $\omega_{c1} = 2\pi \cdot 6 \cdot 10^3$ rad/s, ω_{c2} $\omega_{c2}2\pi \cdot 9 \cdot 10^3$ rad/s => $\omega_{c1}\omega_{c2} = 4\pi^2 \cdot 54 \cdot 10^6$ (rad/s)^2

$\omega_{s1} = 2\pi \cdot 4 \cdot 10^3$ rad/s, $\omega_{s2} = 2\pi \cdot 12 \cdot 10^3$ rad/s => $\omega_{s1}\omega_{s2} = 4\pi^2 \cdot 48 \cdot 10^6$ (rad/s)^2

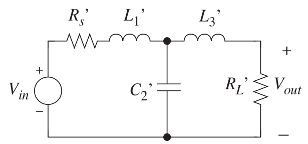
The symmetry requirement is not satisfied. We select $\omega_I^2 = 4\pi^2 \cdot 54 \cdot 10^6$ (rad/s)^2

Increase ω_{s1} => $\omega_{s1} = 4\pi^2 \cdot 54 \cdot 10^6 / 2\pi \cdot 12 \cdot 10^3 = 2\pi \cdot 4.5 \cdot 10^3$ rad/s

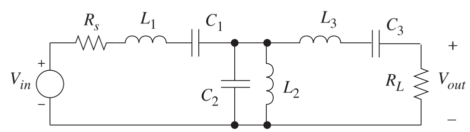
$\Omega_c = \omega_{c2} - \omega_{c1} = 2\pi\,(9-6) \cdot 10^3 = 2\pi \cdot 3 \cdot 10^3$ rad/s

$\Omega_s = \omega_{s2} - \omega_{s1} = 2\pi\,(12-4.5) \cdot 10^3 = 2\pi \cdot 7.5 \cdot 10^3$ rad/s

The filter order: $A_{max} = 0.5$ dB, $A_{min} = 20$ dB and
$\Omega_s/\Omega_c = 7.5/3 = 2.5$ => $N = 3$
Select a T ladder with the normalized element values
$L_1' = L_3' = 1.5963$
$C_2' = 1.0967$



Denormalize with $R_0 = 500$ Ω and $\Omega_c = 2\pi \cdot 12 \cdot 10^3$ rad/s.
$R_s = R_L = 500$ Ω, $L_1 = L_3 = L_1' R_0/\Omega_c = 42.343$ mH
$C_2 = C_2'/(R_0\Omega_c) = 1.164$ nF
Transforming from LP to BP.
The new BP element values are:
$C_1 = C_3 = 1/(\omega_I^2 L_1) = 1.108$ nF
$L_2 = 1/(\omega_I^2 C) = 4.0312$ mH



3.22 $\omega_{c1} = 2\pi \cdot 5 \cdot 10^3$ rad/s, $\omega_{c2} = 2\pi \cdot 10 \cdot 10^3$ rad/s => $\omega_{c1}\omega_{c2} = 4\pi^2 \cdot 50 \cdot 10^6$ (rad/s)^2

$\omega_{s1} = 2\pi \cdot 1 \cdot 10^3$ rad/s, $\omega_{s2} = 2\pi \cdot 20 \cdot 10^3$ rad/s => $\omega_{s1}\omega_{s2} = 4\pi^2 \cdot 20 \cdot 10^6$ (rad/s)^2

The symmetry requirement is not satisfied. We select $\omega_I^2 = 4\pi^2 \cdot 50 \cdot 10^6$ (rad/s)^2

Increase ω_{s1} => $\omega_{s1} = 4\pi^2 \cdot 50 \cdot 10^6 / 2\pi \cdot 20 \cdot 10^3 = 2\pi \cdot 2.5 \cdot 10^3$ rad/s

$\Omega_c = \omega_{c2} - \omega_{c1} = 2\pi\,(10-5) \cdot 10^3 = 2\pi \cdot 7 \cdot 10^3$ rad/s

$\Omega_S = \omega_{s2} - \omega_{s1} = 2\pi\,(20-1) \cdot 10^3 = 2\pi \cdot 19 \cdot 10^3$ rad/s

The filter order: $A_{max} = 0.09883$ dB, $A_{min} = 22$ dB and
$\Omega_S/\Omega_c = 19/7 = 2.71$ Using the program below we get $N = 3.085$. Hence we select $N = 4$:
Now, the case `RL = 1000 and Rs = 500` do not exist. Hence we interchange the terminating resistors and use tables with $r = 0.5$.

```
Wc = 7; Ws = 19; Amax = 0.09883; Amin = 22;
N = CH_ORDER(Wc, Ws, Amax, Amin)
Ladder = 0, Rs = 1000; RL = 500;
[L, C, K] = CH_I_LADDER(Wc, Ws, Amax, Amin, N, Rs, RL, Ladder)
```

L	C
0	0.33630616954450
0.11374902548282	0
0	0.37962163022420
0.05162282824702	0

KI = [9 2 9 2], which according to the function `LADDER_2_H`, represent a π network, i.e., from left to right:

Shunt capacitor (9) – Series inductor (2) – Shunt capacitor (9)– Series inductor (2)

where $R_s = R_L = 500$ Ω

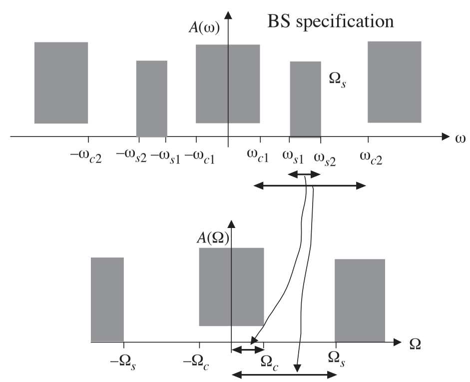
Since the ladder is reciprocal we may place the signal source in series with the "normal load resistor" and take the output across the "normal source resistor". Hence, we obtain a T ladder.
Next step: Perform the LP to BP transformation! Incomplete solution!

3.23 a) The geometric symmetry requirement yields $\omega_{c1}\omega_{c2} = \omega_{s1}\omega_{s2} = \omega_I^2$. We select to increase the lower stopband edge => $f'_{s1} = f_{c1}f_{c2}/f_{s2} = 500 \cdot 1100/2100 = 262$ kHz
$\Omega_c = \omega_{c2} - \omega_{c1} = (1100-500)\,2\pi = 1200\pi$ krad/s
$\Omega_s = \omega_{s2} - \omega_{s1} = (2100-262) \cdot 2\pi = 3676\pi$ krad/s
and $\Omega_s/\Omega_c = 3.06$. From a table we get $N = 3$
The normalized LP ladder (See for figures Problem 3.21)
$L_1' = 2.0236$
$L_3' = L_1'$
$C_2' = 0.9941$
Denormalize with $R_0 = R_L = 600$ Ω
$L_1 = L_1' R_0/\Omega_c = 322$ mH
$C_2 = C_2'/(R_0\Omega_c) = 439$ nF
$L_3 = L_1$
Transform to a BP filter.
$C_1 = C_3 = 1/(\omega_I^2 L_3) = 143$ nF
$L_2 = 1/(\omega_I^2 C_2) = 105$ mH

b) Incomplete solution!

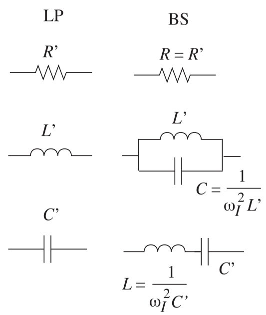
3.24 $A_{max} = 0.09883$ dB, $A_{min} = 40$ dB, $\omega_{c1} = 40$ Mrad/s, $\omega_{c2} = 90$ Mrad/s, $\omega_{s1} = 24$ Mrad/s, and $\omega_{s2} = 150$ Mrad/s. => $\omega_{c1}\omega_{c2} = 3600$ (Mrad/s)^2 and $\omega_{s1}\omega_{s2} = 3600$ (Mrad/s)^2. Hence, we have geometric symmetry and we do not need the modify any of the edges.
$\Omega_c = \omega_{c2} - \omega_{c1} = (90–40) = 50$ Mrad/s and $\Omega_s = \omega_{s2} - \omega_{s1} = (150–24) = 126$ Mrad/s => $N = 3.4726$ and we select $N = 4$ and a π ladder. A 4th-order lowpass ladder can not be realized and need to be modified to a type b approximation.
Hint use; `CA_B_POLES` and `CA_LADDER` Incomplete solution!

3.25 For the LP-BS transformation we have $S = \dfrac{\omega_I^2 s}{s^2 + \omega_I^2}$ and $\Omega = \dfrac{\omega_I^2 \omega}{\omega_I^2 - \omega^2}$. The band edges are mapped as shown below. The passband and stopband attenuations are retained.



We find the impedances in the BS filter by exchanging S with s in the expression for the corresponding impedance in the LP filter. Resistors are frequency independent and are not affected. For an inductor with the admittance $1/SL'$ we get

$$\frac{1}{SL'} \to 1/\frac{\omega_I^2 sL'}{s^2+\omega_I^2} = \frac{s^2+\omega_I^2}{\omega_I^2 sL'} = \frac{s}{\omega_I^2 L'} + \frac{1}{sL'}$$



Hence the resulting admittance in the BS filter consist of an inductor, L' in parallel with a capacitor with the capacitance $C_{BS} = 1/(\omega_I^2 L')$. In the same way we get for a capacitor, C', in the LP filter, i.e., for a impedance $1/SC'$ we get

$$\frac{1}{SC} \to 1/\frac{\omega_I^2 sC}{s^2+\omega_I^2} = \frac{s^2+\omega_I^2}{\omega_I^2 sC} = \frac{s}{\omega_I^2 C} + \frac{1}{sC}$$

We get a series resonance circuit with a capacitor, $C_{BP} = C$ and an inductor, $L_{BP} = 1/(\omega_I^2 C)$.

3.26 We have $\omega_{c2}\,\omega_{c1} = 9991\,\pi^2$ and $\omega_{s1}\omega_{s2} = 9999\,\pi^2$, i.e., we have not geometric symmetry. We select here to increase the lower band passband edge since it is more important to retain the attenuation around 50 Hz. We get $\omega_{c1} = \omega_{s1}\omega_{s2}/\omega_{c2} = 48.539 \cdot 2\pi$ rad/s.

$\omega_I^2 = 98686.1744$ (rad/s)^2 and $(\omega_{c2} - \omega_{c1}) =$

$\omega_{c2} - \omega_{c1} = \omega_I^2/\Omega_c$

$\Omega_c = \omega_I^2/(\omega_{c2} - \omega_{c1}) = 98686.1744/6\pi = 5304.12598$ rad/s

$\omega_{s2} - \omega_{s1} = \omega_I^2/\Omega_s$

$\Omega_s = 98686.1744/2\pi = 15706.3925$ rad/s

which yields $\Omega_s/\Omega_c = 2.9612$ and finally from the nomogram $N = 3$

The LP filter $T0361$ ($A_{max} = 2$ dB) meets the requirement and have the element values for a π ladder ($r = 1$): $L_1' = L_3' = 2.7107$ and $C_2' = 0.8327$

Denormalize with $\omega_0 = \Omega_c$ and $R_0 = R_L = 100\Omega$ yields $L_1 = L_3 = L_3' R_0/\Omega_c = 51.105498$ mH and $C_2 = C_2'/(R_0\Omega_c) = 1.5699099\ \mu$F

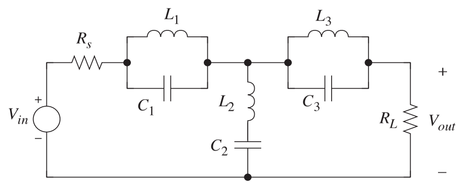
LP-BS transformation of the element values yields. Inductor => Parallel resonance circuits with the element values

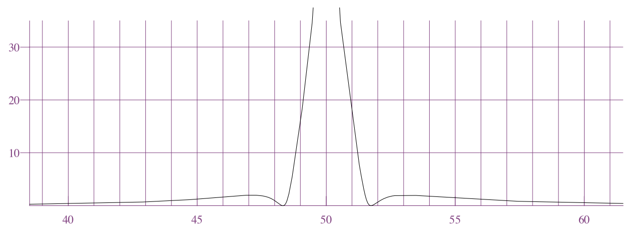
$C_1 = C_3 = 1/(\omega_I^2 L_3) = 198.278699\ \mu$F

L_1 and L_3, according to the above, i.e.,

$L_1 = L_3 = 51.105$ mH

Capacitor => Series resonance circuits with the element values $L_2 = 1/(\omega_I^2 C_2') = 6.454594$ H and according to the above $C_2 = 1.5699\ \mu$F.

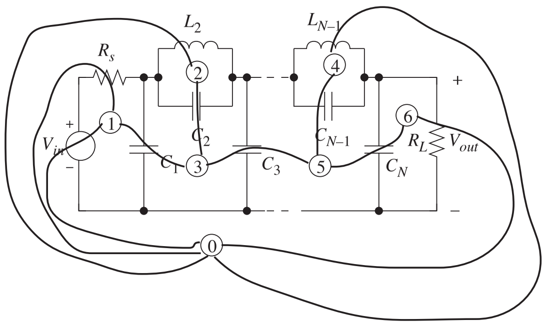




3.27 A π ladder with the element values:

Θ	ω_s/ω_c	A_{min}	C_1	C_2	L_2	C_3	C_4	L_4	C_5
23	2.559304665	61.2	0.732110	0.044113	1.261137	1.496225	0.121060	1.149490	0.662813

Place a node inside every closed loop in the π ladder and one outside the loops as shown below. Connect the nodes as shown.



Every branch that crosses an impedance is replaced with its dual element (admittance) as discussed in section 3.6.1, e.g., a capacitor is replaced with an inductor with the same numerical value. We get

23	2.559304665	61.2	0.732110	0.044113	1.261137	1.496225	0.121060	1.149490	0.662813
Θ	ω_s/ω_c	A_{min}	L_1	L_2	C_2	L_3	L_4	C_4	L_5

3.28 We get a ladder with a current source in parallel with a source conductance; a shunt branch consisting of a parallel resonance circuit; a series branch with a series resonance circuit; shunt branch consisting of a parallel resonance circuit; a series branch with a series resonance circuit; shunt branch consisting of a parallel resonance circuit, and, finally, a load conductance.

3.29 Derive the transfer function for a second-order symmetrical constant-R lattice section.

We get after very long computations $Z_a Z_b = R^2$. Incomplete solution!

3.30 According to the figure shown below, the load resistor can be replaced with one or several constant-R lattice section since their input impedance is $= R$ (in this case $= R_L$). The bridged-T network used below requires fewer element compared to a complete lattice section.

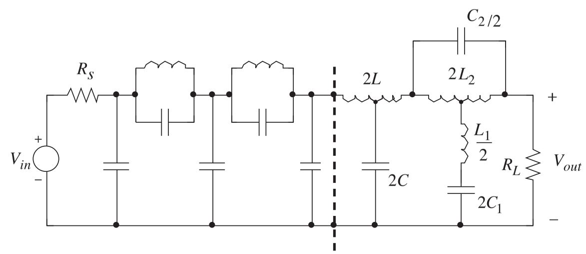
The real pole $s_{p0} = -1$ and Eq.(4.29), gives $H(s) = \frac{Z_a - R}{R + Z_a}$, with $R = 1$ we get $R + Z_a = s + 1$ and $Z_a = s$. Hence, in the first section we get $L = 1$ and $C = 1$. For the second section we get with $Z_a =$ parallel resonance circuit:

$$H(s) = \frac{Z_a - R}{R + Z_a} = \frac{s^2 - 6s + 25}{s^2 + 6s + 25} = \frac{\frac{sL}{LCs^2+1} - R}{R + \frac{sL}{LCs^2+1}} = \frac{s^2 - \frac{s}{C} + \frac{1}{LC}}{s^2 + \frac{s}{C} + \frac{1}{LC}}$$

. Identifying the term gives $L_{parallel} = 6/25$ and $C_{parallel} = 1/6$. Z_b is a series resonance circuit with the impedance

$$Z_b = \frac{R^2}{Z_a} = \frac{R^2}{\frac{sL}{LCs^2+1}} = \frac{\frac{s^2}{25} + 1}{s\frac{6}{25}} = \frac{s}{6} + \frac{25}{6s}$$

, with $L_{series} = 1/6$ and $C_{series} = 6/25$. The lattice sections are denormalized with $R_0 = R_L$ and the LP filter is denormalized with r_{p0} for a Butterworth filter and with ω_c for Chebyshev I and Cauer filters.



3.31

3.32

3.33

3.34

3.35 a) Poles in the lhp and zeros may only in the rhp and on the $j\omega$-axis.
b) Poles in the lhp and zeros may lie anywhere in the s-plane
c) Significantly lower element sensitivity.
d) Singly terminated ladders are used for example as crossover filters in audio systems and for suppressing out-of-band signals due to distortion in the power amplifier in a transmitter. A singly terminated ladder is selected since we do not want to lose power and the frequency selective requirement is low.