3.22 Show that $\sum_{n=0}^{N-1} W^{kn} = \begin{cases} N & k = 0 \\ 0 & \text{otherwise} \end{cases}$ where $W = e^{-i2\pi/N}$.

(i) If $k = 0$, $\sum_{n=0}^{N-1} W^{kn} = \sum_{n=0}^{N-1} W^{0} = \sum_{n=0}^{N-1} 1 = N$.

(ii) If $k \neq 0$, $W^{k} = e^{-i2\pi k/N} \neq 1$,

$$\sum_{n=0}^{N-1} W^{kn} = \frac{1 - W^{kN}}{1 - W^{k}} = \frac{1 - (e^{-i2\pi/N})^{kN}}{1 - e^{-i2\pi k/N}} = \frac{1 - e^{-i2\pi k}}{1 - e^{-i2\pi k/N}}$$

$$= \frac{1 - 1}{1 - e^{-i2\pi k/N}} = 0$$

With the results for (i) and (ii), we have shown that $\sum_{n=0}^{N-1} W^{kn} = \begin{cases} N & k = 0 \\ 0 & \text{otherwise} \end{cases}$