

- 5.17 a) The critical overflow node that shall be scaled in direct form I is the output node. The transfer function to this node is:

$$H_{Critical}(z) = H_{AP}(z) = \frac{z^2 + a z + b}{b z^2 + a z + 1}$$

The L_p -norm of the magnitude function is:

$$\begin{aligned} \left\| H_{Critical}(e^{j\omega T}) \right\|_p &= \sqrt[p]{\frac{1}{2\pi} \int_{-\pi}^{\pi} |H_{AP}(e^{j\omega T})|^p d\omega T} \\ &= \sqrt[p]{\frac{1}{2\pi} \int_{-\pi}^{\pi} 1^p d\omega T} = 1 \end{aligned}$$

Hence, all of the outputs are properly scaled and their scaling is independent of the L_p -norm. Of course, this is only valid for direct form I. For example, the direct form II structure is not automatically scaled.

- b) The ordering of the sections does not effect the signal range since the transfer function from the input of the allpass filter to each of the critical nodes are allpass functions. Also this fact is a special case that is valid only for direct form I.