9.8 a) The only values that need to be computed explicitly are the values to be stored in the delay elements and the outputs. Eliminating all intermediate node values in the recurrence equations, which were derived in Problem 6.7, we get the difference equations in computable order.

 $\begin{array}{l} v_2(n+1) \coloneqq v_1(n) \\ v_1(n+1) \coloneqq (\alpha_1 - 1) \; x(n) - \alpha_1 \; v_2(n) \\ v_4(n+1) \coloneqq v_3(n) \\ v_3(n+1) \coloneqq (\alpha_3 + 1) \; v_0(n) - \alpha_3 \; v_4(n) \\ v_6(n+1) \coloneqq v_5(n) \\ v_5(n+1) \coloneqq v_5(n) \\ v_5(n+1) \coloneqq \alpha_1(1 + \alpha_5) \; x(n) + \left[\alpha_5 - \alpha_1(1 + \alpha_5)\right] \; v_2(n) - \alpha_5 \; v_6(n) \\ v_0(n+1) \coloneqq x(n) \\ y_1(n) \coloneqq \alpha_1 \alpha_5 \; x(n) + \alpha_3 \; v_0(n) - \alpha_5(1 - \alpha_1) \; v_2(n) + (1 - \alpha_3) \; v_4(n) + (1 + \alpha_5) \; v_6(n) \\ y_2(n) \coloneqq \alpha_1 \alpha_5 \; x(n) - \alpha_3 \; v_0(n) - \alpha_5(1 - \alpha_1) \; v_2(n) - (1 - \alpha_3) \; v_4(n) + (1 + \alpha_5) \; v_6(n) \end{array}$

b) Inserting the quantized adaptor coefficient values we get

 $v_1(n+1) := [1123 \ x(n) + 99 \ v_2(n)] \ 2^{-10}$ $v_3(n+1) := [1405 \ v_0(n) + 381 \ v_4(n)] 2^{-10}$ $v_5(n+1) := [-53361 \ x(n) + 1598577 \ v_2(n) + 1545216 \ v_6(n)] \ 2^{-21}$ $v_0(n+1) := x(n)$ $v_1(n) := [149391 \ x(n) + 780288 \ v_0(n) + 1694607 \ v_0(n) + 287744$

 $y_1(n) := \begin{bmatrix} 149391 \ x(n) + 780288 \ v_0(n) + 1694607 \ v_2(n) + 2877440 \ v_4(n) + \\ + 551936 \ v_6(n) \end{bmatrix} 2^{-21}$

 $\begin{array}{l} y_2(n) := \left[149391 \, x(n) - 780288 \, v_0(n) + 1694607 \, v_2(n) - 2877440 \, v_4(n) + \right. \\ \left. + \, 551936 \, v_6(n) \right] 2^{-21} \end{array}$

In practice, this new set of equations should be scaled in order to optimize the dynamic range. The scaled coefficients tend to be of the same magnitude. The word length of these new coefficients is rather long, since they have not been optimized for this application. Instead, the adaptor coefficients have been optimized to have a favorable representation with few nonzero digits.

Five vector-multipliers are needed for the fully parallel implementation showed in Fig. P9.8.



Fig. P9.8. Vector-multiplier based realization of lattice wave digital filter.