6. FIRST- AND SECOND-ORDER SECTIONS

6.1 $H(j\omega) = \frac{r_p^2}{s^2 - 2\sigma_p s + r_p^2} = \frac{r_p^2}{s^2 + \frac{r_p}{Q}s + r_p^2}$ and $|H(j\omega)| = \frac{r_p^2}{\sqrt{(r_p^2 - \omega^2)^2 + \left(\frac{r_p}{Q}\omega\right)^2}}$

Taking the derivative with respect to ω yields

$$\frac{\partial}{\partial\omega}|H(j\omega)| = \frac{\partial}{\partial\omega}\frac{r_p^2}{\sqrt{(r_p^2 - \omega^2)^2 + \left(\frac{r_p}{Q}\omega\right)^2}} = -\frac{r_p^2\left(-4\omega(r_p^2 - \omega^2) + \frac{2r_p^2\omega}{Q^2}\right)}{2D^{3/2}}$$ where

$D = (r_p^2 - \omega^2)^2 + \left(\frac{r_p}{Q}\omega\right)^2$. The derivative is zero if the numerator is zero, i.e., if, $-4(r_p^2 - \omega^2) + \frac{2r_p^2}{Q^2} = 0$ which yields $\omega_0 = r_p\sqrt{1 - \frac{1}{2Q^2}}$. This is a maximum (the second-order derivative is a large and have a complicated expression). If $Q \le 1/\sqrt{2}$ we have a maximum. We get $|H(j\omega_0)|_{max} = 2\frac{Q^2}{\sqrt{4Q^2-1}} \approx Q$ for large Q values and $\arg\{H(j\omega_0)\} = \frac{2Q^2}{1 + j\sqrt{4Q^2 - 2}} \approx -jQ$ for large Q values, i.e., the phase is $\approx -\pi$.

6.2 It is a BP section with $H(s) = \frac{Gs}{(s - \sigma_p + j\omega_p)(s - \sigma_p - j\omega_p)} = \frac{Gs}{s^2 - 2\sigma_p s + r_p^2} = \frac{Gs}{s^2 + \frac{r_p}{Q}s + r_p^2}$.

We get $|H(j\omega)| = \left|\frac{Gj\omega}{(j\omega)^2 - 2\sigma_p j\omega + r_p^2}\right| = \left|\frac{Gj\omega}{r_p^2 - \omega^2 - 2\sigma_p j\omega}\right| = \frac{|G|\omega}{\sqrt{(r_p^2 - \omega^2)^2 + (2\sigma_p\omega)^2}} = \frac{|G|\omega}{\sqrt{D}}$

The derivative with respect to ω is $\frac{\partial}{\partial\omega}|H(j\omega)| = \frac{2|G|D - |G|\omega D'}{(2D)^{3/2}} =$

$$= \frac{2|G|((r_p^2 - \omega^2)^2 + (2\sigma_p\omega)^2) - |G|\omega[(-4\omega(r_p^2 - \omega^2) + (8\sigma_p^2\omega))]}{2D^{3/2}} =$$

We get after simplifications $\frac{\partial}{\partial\omega}|H(j\omega)| = \frac{2|G|(r_p^4 - \omega^4)}{((r_p^2 - \omega^2)^2 + (2\sigma_p\omega)^2)^{3/2}} = 0$ for $\omega = r_p$.

The second-order derivative and for $\omega = r_p$, we get after long simplifications $\frac{\partial^2}{\partial\omega^2}|H(j\omega)| = \frac{|G|}{2\sigma_p^3} < 0$.

Hence a maximum, which is $|H(jr_p)|_{max} = \left|\frac{G}{2\sigma_p}\right| = \frac{Q|G|}{r_p}$. With $\sigma_p = -1$ rad/s, we estimate from the figure a maximum of about 1.6, i.e., $G \approx 0.8$ rad/s.

6.3 $\omega_0 = \frac{r_p}{\sqrt{1 - \frac{1}{2Q^2}}}$ and $|H(j\omega_0)|_{max} = 2\frac{Q^2 r_p^2}{\sqrt{4Q^2-1}}$

6.4

6.5 The poles for a Butterworth filter are: $s_{pk} = r_{p0}(\cos(\theta_k) + j\sin(\theta_k))$ for $k = 1, \dots N$

where $\theta_k = \frac{(N - 1 + 2k)\pi}{2N}$. The Q values are: $Q_{BW} = \frac{-r_{p0}}{2\sigma_p} = \frac{-1}{2\cos(\theta_k)}$ and for a Chebyshev I filter: $s_{pk} = -\omega_c a\sin(\theta_k) + j\omega_c b\cos(\theta_k)$ where $\theta_k = \frac{(2k-1)\pi}{2N}$.

The Q values are: $Q_{CI} = \frac{-|s_k|}{2\sigma_p} = \frac{\sqrt{(a^2\sin(\theta_k)^2 + b^2\cos(\theta_k)^2)}}{2a\sin(\theta_k)}$

6.6 a) The currents into the nodes are.

$$\begin{cases} R_1^{-1}(V_1 - V_-) + R_1^{-1}(V_2 - V_-) = 0 \\ A(V_+ - V_-) = V_2 \\ sC(V_1 - V_+) + R^{-1}(0 - V_+) = 0 \end{cases}$$ which yields $$\begin{cases} V_- = \frac{V_2 + V_1}{2} \\ A(V_+ - V_-) = V_2 \\ V_+ = \frac{sRCV_1}{sRC + 1} \end{cases}$$ and

$\frac{V_2}{V_1} = \frac{sRC - 1}{1 + sRC + \frac{(sRC+1)2}{A}} \to \frac{sRC - 1}{sRC + 1}$ and $H(s) = \frac{sRC - 1}{sRC + 1}$, i.e., a first-order allpass filter.

b) The transfer function yields at DC: $H(0) = 1$ and it has a pole at $s_p = -1/RC$ and a zero at $s_z = 1/RC$ (in the right hand side of the s-plane).

c) The frequency response is

$$H(j\omega) = \frac{j\omega RC - 1}{j\omega RC + 1} = \frac{(j\omega RC - 1)(-j\omega RC + 1)}{(j\omega RC + 1)(-j\omega RC + 1)} = \frac{(\omega RC)^2 - 1 + j2\omega RC}{((\omega RC)^2 + 1)}$$

d) The phase response is $\Phi(\omega) = \mathrm{atan}\left(\frac{2\omega RC}{(\omega RC)^2 - 1}\right)$ and the group delay is

$$\tau_g(\omega) = -\frac{\partial}{\partial\omega}\Phi(\omega) = \left(-\frac{1}{1 + \left(\frac{2\omega RC}{(\omega RC)^2 - 1}\right)^2}\right)\frac{2RC[(\omega RC)^2 - 1] - (2\omega RC)2\omega(RC)^2}{((\omega RC)^2 - 1)^2} =$$

After simplifications we get $\tau_g(\omega) = \frac{2RC}{(\omega RC)^2 + 1}$, $\tau_g(0) = 200$ μs and $2RC = 200$ μs, i.e., $RC = 100$ μs. Select for example $C = 1$ nF and $R = 10$ kΩ.

6.7 a) The currents into node V_8 and V_- and the equation for the amplifier are

$$\begin{cases} Y_8V_8 - Y_3(V_1 - V_8) - Y_7(V_- - V_8) - Y_9(V_2 - V_8) = 0 \\ Y_7(V_- - V_8) + Y_6(V_- - V_2) = 0 \\ V_2 = A(V_+ - V_-) \\ V_+ = 0 \end{cases}$$ We get $$\begin{cases} V_8 = \frac{Y_9V_2 + Y_3V_1 + Y_7V_-}{Y_9 + Y_8 + Y_7 + Y_3} \\ V_- = \frac{Y_7V_8 + Y_6V_2}{Y_6 + Y_7} \\ V_2 = -AV_- \end{cases}$$

and $$\begin{cases} V_- = \frac{(Y_9Y_7 + N_1Y_6)V_2 + Y_7Y_3V_1}{(Y_6 + Y_7)N_1 - Y_7^2} \\ V_2 = -AV_- \end{cases}$$ where $N_1 = Y_9 + Y_8 + Y_7 + Y_3$

Finally we get $H(s) = \frac{-Y_7Y_3}{Y_9Y_7 + N_1Y_6 + \frac{N_1Y_7 + N_1Y - Y_7^2}{A}}$

With $Y_3 = sC_3$, $Y_7 = sC_7$, $Y_9 = sC_9$, $Y_6 = 1/R_6$, and $Y_8 = 1/R_8$ and $C_3 = C_7 = C_9 = C$, $R_6 = R$, and $R_8 = R/(9Q^2)$ we get $N_1 = \frac{9Q^2}{R} + 3sC$ and $r_p = 3\frac{Q}{RC}$

$$H(s) = -\frac{s^2}{s^2 + \frac{3s}{RC} + \frac{9Q^2}{(RC)^2} + \frac{2R^2C^2s^2 + 3CR(3Q^2+1)s + 9Q^2}{AR^2C^2}}$$ **A HP section!**

$$H(s) = -\frac{s^2}{\left(s^2 + \frac{r_p}{Q}s + r_p^2\right) + \frac{\left(2s^2 + (3Q^2+1)\frac{r_p}{Q}s + r_p^2\right)}{A}}$$

b) Select $C = 100$ pF which yields $R = \frac{3Q}{r_p C} = 30$ kΩ, i.e., $R_6 = 30$ kΩ, $R_8 = 30/900$ kΩ $= 33.33$ Ω. The error function is $E(s) = 2s^2 + (3Q^2+1)\frac{r_p}{Q}s + r_p^2$

6.8 We get by simplifying Equations (6.35) and (6.41) with respect to the circuit elements that are present in the LP section $H(s) = \frac{-Y_7Y_3}{Y_9Y_7 + N_1Y_6 + \frac{N_1Y_7 + N_1Y - Y_7^2}{A}}$ where $N_1 = Y_9 + Y_8 + Y_7 + Y_3$.

With the following elements we get: $Y_3 = 1/R_3$, $Y_7 = 1/R_7$, $Y_9 = 1/R_9$, $Y_6 = sC_6$, $Y_8 = sC_8$ and $C_8 = C$, $R_3 = R_7 = R_9 = R$, and $C_6 = \frac{C}{9Q^2}$ we get $r_p = 3\frac{Q}{RC}$. Insertion above yields $N_1 = sC + \frac{3}{R}$ and finally we get

$$H(s) = \frac{-r_p^2}{\left(s^2 + \frac{r_p}{Q}s + r_p^2\right) + \frac{\left(s^2 + (3Q^2+1)\frac{r_p}{Q}s + 2r_p^2\right)}{A}} \rightarrow \frac{-r_p^2}{s^2 + \frac{r_p}{Q}s + r_p^2}$$ when $A \rightarrow \infty$.

6.9 We get $\frac{\partial Y}{\partial x} = \frac{\partial}{\partial x}x^k = kx^{(k-1)}$ and $S_x^Y = \frac{x\partial Y}{Y\partial x} = \frac{x}{Y}kx^{(k-1)} = \frac{x}{x^k}kx^{(k-1)} = k$.

The expression for the pole radius is: $r_p = 1/\sqrt{R_7R_9C_6C_8}$, i.e., the sensitivities with respect to these elements are $S_x^{r_p} = -\frac{1}{2}$ since the exponent for all elements are –1/2. Generally, the possible exponent for the passive elements, R and C, are ±1 or ±1/2.

6.10 a) We get with the following selection of the element: $C_7 = C_9 = C$, $R_6 = R = r$ and $R_3 = \frac{R}{4Q^2}$

$H(s) = \frac{-2r_pQs}{s^2 + \left(\frac{r_p}{Q}\right)s + r_p^2}$ where $r_p = \frac{1}{\sqrt{R_3R_6C_7C_9}} = \frac{2Q}{RC}$. We select $C = 100$ pF and get

$$R = \frac{2Q}{Cr_p} = \frac{2\cdot 5}{100\cdot 10^{-12}\cdot 4\pi\cdot 10^6} = 7.9577\ \text{k}\Omega.$$

b) The gain constant is $G = -2r_pQ = -2\cdot 5\cdot 4\pi\cdot 10^6 = -40\pi\cdot 10^6$. The largest gain, which according to Problem 6.2 is at $\omega = r_p$, is $|H(jr_p)|_{max} = \frac{Q|G|}{r_p} = \frac{5\cdot 40\pi\cdot 10^6}{4\pi\cdot 10^6} = 50$

c) We have selected $R_3 = \frac{R}{4Q^2}$, i.e., the spread in the resistor values is $\frac{R}{R_3} = 4Q^2 = 100$ and $R_3 = 79.58$ Ω and the spread in the capacitor capacitance values is = 1.

d) For NF sections we have, according to Eq. (6.39) – (6.40). We get with $GS_{A_0}^Q = 2Q^2 = 50$

$\delta = -\frac{1}{2Q}\frac{r_p}{\omega_t}GS_{A_0}^Q = -\frac{1}{2\cdot 5}\frac{4\pi\cdot 10^6}{200\pi\cdot 10^6}50 = -0.1$ which is a too large error in the pole radius. The ratio $\frac{r_p}{\omega_t} = 0.02$ is, hence, too large and an operational amplifier with larger GB must be selected. If an operational amplifier with twice as large bandwidth is selected, i.e., with $\frac{r_p}{\omega_t} = 0.01$ we get $\delta = -0.05$ which is a relatively small error. The error in the Q value is, i.e., $Q = \eta\, Q_{nominal}$ where

$\eta = \frac{1}{1 + \left(\frac{r_p}{\omega_t}\right)\left(\frac{r_p}{\omega_t} - \frac{1}{2Q}\right)GS_{A_0}^Q} = 1.087$ which also is a relatively large error (8.7%).

6.11 a) According to Eq. (6.43): $H(s) = -\frac{1}{R_3R_7C_6C_8}\frac{1}{\left(s^2 + \left(\frac{1}{R_3} + \frac{1}{R_7} + \frac{1}{R_9}\right)\frac{s}{C_8} + \frac{1}{R_7R_9C_6C_8}\right)}$ and

$r_p^2 = \frac{1}{R_7R_9C_6C_8}$. Taking the derivative with respect to R_7 yields $2r_p\frac{\partial r_p}{\partial R_7} = \frac{-1}{R_7^2R_9C_6C_8} = \frac{-r_p^2}{R_7}$

which yields the sensitivity $S_{R_7}^{r_p} = \frac{R_7\partial r_p}{r_p\partial R_7} = -\frac{1}{2}$. In the same way we get $S_{R_9}^{r_p} = -\frac{1}{2}$ and $S_{R_3}^{r_p} = 0$

since r_p is independent of R_3. Check that we according to Eq. (6.25) have $\sum_i S_{R_i}^{r_p} = -1$. In the same way we get $S_{C_6}^{r_p} = -\frac{1}{2}$ and $S_{C_8}^{r_p} = -\frac{1}{2}$. Check that we according to Eq. (6.26) have $\sum_i S_{C_i}^{r_p} = -1$.

b) From $H(s)$ we get $\frac{r_p}{Q} = \left(\frac{1}{R_3} + \frac{1}{R_7} + \frac{1}{R_9}\right)\frac{1}{C_8}$ => $Q = \frac{r_p}{\left(\frac{1}{R_3} + \frac{1}{R_7} + \frac{1}{R_9}\right)\frac{1}{C_8}}$. Taking the derivative of both sides with respect to R_7 yields

$$\frac{\partial Q}{\partial R_7} = \frac{\left(\frac{1}{R_3} + \frac{1}{R_7} + \frac{1}{R_9}\right)\frac{1}{C_8}\frac{\partial}{\partial R_7}(r_p) - r_p\left(\frac{-1}{R_7^2}\right)\frac{1}{C_8}}{\left(\left(\frac{1}{R_3} + \frac{1}{R_7} + \frac{1}{R_9}\right)\frac{1}{C_8}\right)^2} = \frac{\left(\frac{1}{R_3} + \frac{1}{R_7} + \frac{1}{R_9}\right)\frac{1}{C_8}\frac{\partial}{\partial R_7}(r_p) + \left(\frac{r_p}{R_7^2}\right)\frac{1}{C_8}}{\left(\frac{r_p}{Q}\right)^2}$$

$$S_{R_7}^Q = \frac{R_7\partial Q}{Q\,\partial R_7} = \frac{R_7}{Q}\frac{\left(\left(\frac{r_p}{Q}\right)\left(\frac{-r_p}{2R_7}\right) + \left(\frac{r_p}{R_7^2}\right)\frac{1}{C_8}\right)}{\left(\frac{r_p}{Q}\right)^2} = Q\frac{\left(-\frac{r_p}{2Q} + \frac{1}{R_7C_8}\right)}{r_p} = \frac{Q}{R_7C_8r_p} - \frac{1}{2}$$

With $R_7 = R_9 = R_3 = R$, $C_6 = \frac{C_8}{9Q^2}$, $C_8 = C$ and $r_p = 3Q/RC$ we get $S_{R_7}^Q = \frac{R_7\partial Q}{Q\,\partial R_7} = -\frac{1}{6}$, $S_{R_9}^Q = \frac{R_9\partial Q}{Q\,\partial R_9} = \frac{-1}{6}$ and $S_{R_3}^Q = \frac{R_3\partial Q}{Q\,\partial R_3} = \frac{1}{3}$ which agrees with Eq. (6.27), i.e., the sum of these three sensitivities is zero. In the same way we get $S_{C_6}^Q = \frac{C_6\partial Q}{Q\,\partial C_6} = -\frac{1}{2}$ and $S_{C_8}^Q = \frac{C_8\partial Q}{Q\,\partial C_8} = \frac{1}{2}$ which agrees with Eq. (6.28).

6.14 In order to determine the sensitivities we must first determine the error function $E(s)$. This requires in general very long and complicated expression, that most conveniently are performed by using a symbolic algebra program, e.g., Mathematica™ or Maple™. In Problem 6.8 we derived the following expression

$$H(s) = \frac{-r_p^2}{\left(s^2 + \frac{r_p}{Q}s + r_p^2\right) + \frac{\left(s^2 + (3Q^2+1)\frac{r_p}{Q}s + 2r_p^2\right)}{A}} \quad \text{and}$$

$$H(s) = \frac{-r_p^2 \frac{A}{A+1}}{s^2 + \left(\frac{(A+3Q^2+1)}{A+1}\right)\frac{r_p}{Q}s + \left(\frac{A+2}{A+1}\right)r_p^2} \quad \text{where } r_p = 3\frac{Q}{RC}$$

. Note: The coefficients in the error function, $E(s)$, depends on the selection of circuit elements and differ for LP and HP filters.

We get the new pole radius and the Q value: $r_{px}^2 = \left(\frac{A+2}{A+1}\right)r_p^2$ and $\frac{r_p}{Q_x} = \left(\frac{(A+3Q^2+1)}{A+1}\right)\frac{r_p}{Q}$, i.e.,

$$Q_x = \frac{(A+1)Q}{A+3Q^2+1}$$

Taking the derivative of r_p with respect to A yields $2r_{px}\frac{\partial r_{px}}{\partial A} = \frac{-r_p^2}{(A+1)^2}$ and we get

$$GS_A^{r_p} = A\left(\frac{A}{r_p}\frac{\partial r_p}{\partial A}\right) = \frac{A^2}{r_p}\frac{\partial r_p}{\partial A} = \frac{A^2}{r_p}\frac{-r_p^2}{2(A+1)^2} = -\frac{A^2}{2(A+1)^2} \to -\frac{1}{2} \text{ when } A \to \infty.$$

The gain-sensitivity product for the pole radius is small, which is necessary for a useful section since A varies significantly.

For the Q value we have $Q_x = \frac{(A+1)Q}{A+3Q^2+1}$. Taking the derivative with respect to A yields

$$\frac{\partial Q_x}{\partial A} = \frac{Q(A+3Q^2+1)-(A+1)Q}{(A+3Q^2+1)^2} = \frac{3Q^3}{(A+3Q^2+1)^2} \quad \text{which yields}$$

$$GS_A^Q = A\left(\frac{A}{Q}\frac{\partial Q_x}{\partial A}\right) = \frac{A^2}{Q}\frac{\partial Q_x}{\partial A} = \frac{3A^2Q^2}{(A+3Q^2+1)^2} \to 3Q^2 \text{ when } A \to \infty.$$

The gain-sensitivity product for the Q value is large for large Q values!

6.15 a) The section has two bridge-T networks. The two grounded elements in the T networks has been lifted in order to allow the input signal to be injected. From Eqs. (6.52) and (6.53) we have

$$N(s) = KN_3 + N_1Y_{11}Y_{10} + (Y_7Y_3 + N_1Y_2)N_2$$

$$H(s) = -\frac{N(s)}{D(s) + \frac{E(s)}{A}} \quad \text{where } D(s) = N_1Y_{11}^2 + N_2Y_7^2 + N_1N_2Y_6, \quad E(s) = -N_3$$

$$N_1 = Y_9 + Y_8 + Y_7 + Y_3 = sC + 0 + sC + \frac{2}{R} = 2\frac{(sRC+1)}{R}$$

$$N_2 = Y_{13} + Y_{12} + Y_{11} + Y_{10} = \frac{1}{R} + 0 + \frac{1}{R} + s2C = N_1$$

$$N_3 = Y_{11} + Y_7 + Y_6 + Y_4 + Y_2 = \frac{1}{R} + sC + \frac{a}{R} + 0 + 0 = \frac{sRC+a+1}{R}$$

$$E = (Y_7^2 - (Y_7 + Y_6 + Y_4 + Y_2)N_1)N_2 + (Y_{11} - N_2)N_1Y_{11} \quad \text{and} \quad K = \frac{Y_1}{Y_1 + Y_5} = 0$$

We have selected $N_1 = N_2$ and a pole and a zero cancels, i.e., N_2 can cancels since the term is present in both the numerator and the denominator. Furthermore, we get

$$E = ((Y_7^2 - (Y_7 + Y_6 + Y_4 + Y_2))N_2 + (Y_{11} - N_2)Y_{11})N_2$$
$$= \left(\frac{2a+1}{R^2} + \frac{2(a+2)C}{R}s + C^2s^2\right)N_2$$

Inserting into the transfer function (BP section) yields

$$H(s) = -\frac{\frac{4s}{RC}}{s^2 + \frac{r_p}{Q}s + r_p^2 + \frac{\left(s^2 + \left(1 + \frac{a+2}{a}\right)\frac{r_p}{Q}s + r_p^2\right)}{A}} \quad \text{where} \quad \begin{cases} r_p^2 = \frac{2a+1}{(RC)^2} \\ Q = \frac{RCr_p}{2a} \end{cases}$$

b) We select $C = 100$ pF and eliminate the constant a we get

$$R = \frac{\sqrt{1+4Q^2}+1}{2CQr_p} = \frac{\sqrt{1+4\cdot 20^2}+1}{2\cdot 100\cdot 10^{-12}\cdot 20\cdot 4\pi\cdot 10^6} = 815.92\ \Omega$$

c) The spreads are proportional to a, i.e., $a = \frac{RCr_p}{2Q} = 0.025633$, i.e., $1/a = 39.012$

d) We get $\delta = -\frac{1}{2Q}\frac{r_p}{\omega_t}GS_{A_0}^Q = -\frac{1}{2\cdot 20}\frac{4\pi\cdot 10^6}{200\pi\cdot 10^6}78.025 = -0.039012$ and

$$\eta = \frac{1}{1 + \left(\frac{r_p}{\omega_t}\right)\left(\frac{r_p}{\omega_t} - \frac{1}{2Q}\right)GS_{A_0}^Q} = 1.0079$$

. Hence the deviations are small.

6.16 a) To get a transfer function of BP type we have: $f = b = 0$, Hence, the resistor R/b and the capacitor fC disappear and we get $r_p = \frac{\sqrt{1+2a}}{RC}$, $Q = \frac{r_pRC}{2a}$ and $g = 2 - e$. We select, e.g., $C = 50$ pF and $R = 4$ kΩ, i.e., $RC = 0.2$ μs which yields $a = 2.6583$. We may freely select the parameter e to, for example, $e = 0$ which yields $g = 2$ and the resistor R/e disappear. We get the elements $R/a = 1.5047$ kΩ and $R/g = 2$ kΩ.

b) The gain factor is $G = -\frac{g}{RC} = -\frac{2}{0.2\cdot 10^{-6}} = -10^7$. The largest gain occur according to Problem 6.2 at $\omega = r_p$ and is $|H(jr_p)|_{max} = \frac{Q|G|}{r_p} = \frac{5\cdot 10^7}{4\pi\cdot 10^6} = 3.9789$

c) The element spreads = a = 2.6583 which is low spread.

d) We get with $GS_{A_0}^Q = \frac{4Q}{RCr_p} = 0.7524$ =>

$\delta = -\frac{1}{2Q}\frac{r_p}{\omega_t}GS_{A_0}^Q = -\frac{1}{2\cdot 5}\frac{4\pi\cdot 10^6}{200\pi\cdot 10^6}0.7524 = -0.0015$ which is a much smaller error in the pole radius compared with the errors in the corresponding NF1 section and we get for the Q value

$$\eta = \frac{1}{1 + \left(\frac{r_p}{\omega_t}\right)\left(\frac{r_p}{\omega_t} - \frac{1}{2Q}\right)GS_{A_0}^Q} = 1.0012$$

which also is a small error.

6.17 For the NF1 BP section shown in Fig. 6.21 we have $GS_{A_0}^Q = 2Q^2$. We get from Eq. (6.39) and Eq (6.40): $\delta \approx -\frac{1}{2Q}\left(\frac{r_p}{\omega_t}\right)GS_{A_0}^Q = -\frac{1}{2\cdot 10}\left(\frac{20\pi 10^4}{2\pi 10^7}\right)2\cdot 10^2 = -1.0$ and

$$\eta \approx \frac{1}{1+\left(\frac{20\pi 10^3}{2\pi 10^7}\right)\left(\frac{20\pi 10^3}{2\pi 10^7}-\frac{1}{2(10)}\right)2(10^2)} = 1.087$$

The pole radius will thus decrease with 100% while the Q value increases insignificantly. According to Figure 6.9 is the second-order section significantly more sensitive for errors in the pole radius compared to the Q value. Hence, δ is in this case too large. Since the factor $GS^Q_{A_0} = 2Q^2$ if fixed we have only the option to select an operational amplifier with larger GB.

For the NF2 BP section shown in Figure 6.24 we have $GS^Q_{A_0} = 3Q$ and we get

$$\delta \approx -\frac{1}{2Q}\left(\frac{r_p}{\omega_t}\right)GS^Q_{A_0} = -\frac{1}{2\cdot 10}\left(\frac{20\pi 10^4}{2\pi 10^7}\right)3\cdot 10 = -0.15 \text{ and}$$

$$\eta \approx \frac{1}{1+\left(\frac{20\pi 10^3}{2\pi 10^7}\right)\left(\frac{20\pi 10^3}{2\pi 10^7}-\frac{1}{2(10)}\right)3(10)} = 1.0121$$. The pole radius will be reduced with 15% while the Q value increases insignificantly. Also in this case we must select an operational amplifier with larger GB.

6.18 The currents into the nodes V_0 and V_+ are

$$\begin{cases} V_2 = A(V_+ - V_-) \\ V_- = \dfrac{R_7V_2}{(R_7+R_8)} = \dfrac{V_2}{K} \\ Y_3(V_0 - V_+) + Y_5(V_1 - V_+) - Y_4V_+ = 0 \\ Y_1(V_1 - V_0) - Y_2V_0 - Y_3(V_0 - V_+) - Y_6(V_0 - V_2) = 0 \end{cases}$$

Elimination of V_0 and V_+ yields $\begin{cases} V_0 = \dfrac{Y_6V_2 + Y_3V_+ + V_1V_1}{N_1} \\ V_+ = -\dfrac{(N_1Y_5 + Y_3Y_1)V_1 + Y_6Y_3V_2}{Y_3^2 - N_1(Y_5+Y_4+Y_3)} \end{cases}$ and after simplification we get

$$H(s) = \frac{-(N_1Y_5+Y_3Y_1)K}{(N_2+KY_6Y_3)+\dfrac{KN_2}{A}} \text{ were } \begin{cases} N_1 = Y_6+Y_3+Y_2+Y_1 \\ N_2 = Y_3^2 - N_1(Y_5+Y_4+Y_3) \end{cases}$$

6.19 a We get from Problem 6.18 $N_1 = Y_6+Y_3+Y_2+Y_1 = sC_6 + \frac{1}{R_3} + 0 + \frac{1}{R_1}$

$$N_2 = Y_3^2 - N_1(Y_5+Y_4+Y_3) = \frac{1}{R_3^2} - \left(\left(sC_6 + \frac{1}{R_3}+\frac{1}{R_1}\right)\left(0 + sC_4 + \frac{1}{R_3}\right)\right)$$

We select $R_1 = R_3 = R$, $C_4 = C_6 = C$ and $C_6 = \frac{1}{r_pR_3}$ and we get $N_1 = sC + \frac{2}{R}$ and

$$N_2 = \frac{1}{R^2} - \left(sC+\frac{2}{R}\right)\left(sC+\frac{1}{R}\right) = \frac{C^2R^2s^2+3CRs+3}{R^2}$$

$$H(s) = \frac{-(N_1Y_5+Y_3Y_1)K}{(N_2+KY_6Y_3)+\dfrac{KN_2}{A}} = -\frac{Kr_p^2}{\left(s^2+\frac{r_p}{Q}s+r_p^2\right)+K\dfrac{\left(s^2+(1+KRCQr_p)\frac{r_p}{Q}s+r_p^2\right)}{A}}$$

where $r_p = \frac{1}{\sqrt{R_1R_3C_4C_6}} = \frac{1}{RC}$ and $\frac{r_p}{Q} = \frac{1}{R_1C_6}+\frac{1}{R_3C_6}+\frac{1-K}{R_3C_4} = \frac{3-K}{RC}$ The gain K must be selected < 3 otherwise the section will be unstable. With this selection of element values the passive sensitivities with respect to Q are proportional to Q, i.e., high

$$H(s) = -\frac{Kr_p^2}{s^2+\frac{r_p}{Q}s+r_p^2+K\dfrac{\left(s^2+(1+KQ)\frac{r_p}{Q}s+r_p^2\right)}{A}} \text{ and } K = 3-\frac{1}{Q} = 2.8$$

W. Saraga suggested the following selection of the elements. Select a suitable value for C_4 and $R_1 = \frac{1}{Qr_pC_4}$, $R_3 = \frac{1}{\sqrt{3}r_pC_4}$, $C_6 = \sqrt{3}QC_4$ which gives $K = 4/3$. We get the transfer function

$$H(s) = -\frac{4r_p^2}{3\left(s^2+\frac{r_p}{Q}s+r_p^2\right)+4\dfrac{\left(s^2+\left(1+\frac{4Q}{\sqrt{3}}\right)\frac{r_p}{Q}s+r_p^2\right)}{A}}$$

b)
c)
d)

6.20 The transfer function, according to Eq. (6.59), is $H(s) = \frac{Kr_p^2}{s^2+\left(\frac{r_p}{Q}\right)s+r_p^2}$ where

$r_p = \frac{1}{\sqrt{R_1R_3C_4C_6}}$ and $\frac{r_p}{Q} = \frac{1}{R_1C_6}+\frac{1}{R_3C_6}+\frac{1-K}{R_3C_4}$. We select $R_1 = R_3 = R$, $C_4 = C_6 = C$ and $R_3 = \frac{1}{r_pC_6}$ and get $r_p = \frac{1}{RC}$ and $G = K = 3-\frac{1}{Q}$. We select $C = 10$ nF (which in practice may/should be measured) and we get $R_3 = \frac{1}{r_pC} = \frac{1}{2\pi 10^3\cdot 10^{-8}} = 15.916$ kΩ. The DC gain $G = 3 - 1/5 = 2.8$, which is too large. We select, e.g., $R_7 = 20$ kΩ and $R_8 = 11.11$ kΩ in order to get $G = K = 2.8$ and reduce the gain to 2.0 by splitting R_1 into a resistive voltage divider connected to ground, with $R_{a1}//R_{b1} = R$ and $KR_{b1}/(R_{a1}+R_{b1}) = 2.0$. We may select $R_{a1} = 22.282$ kΩ and $R_{b1} = 55.706$ kΩ.

6.21 a

6.22 a) We have for the currents into the nodes $\begin{cases} \dfrac{(V_1-V_3)}{R_1} + (V_2-V_3)sC_6 + \dfrac{V_4-V_3}{R_3} = 0 \\ \dfrac{V_3-V_4}{R_3} - V_4sC_4 = 0 \\ V_2 = KV_4 \end{cases}$

After elimination of V_3 and V_4 we get $H(s) = \frac{Kr_p^2}{s^2+\left(\frac{r_p}{Q}\right)s+r_p^2}$ where $r_p = \frac{1}{\sqrt{R_1R_3C_4C_6}}$ and

$$\frac{r_p}{Q} = \frac{1}{R_1C_6}+\frac{1}{R_3C_6}+\frac{1-K}{R_3C_4}$$

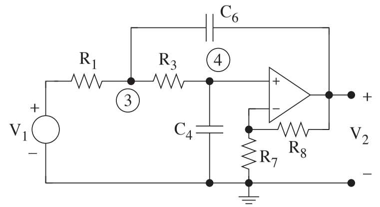
b) We get from the definition of sensitivity

$$S_{R_1}^{r_p} = \frac{R_1}{r_p}\frac{\partial r_p}{\partial R_1} = \frac{R_1}{r_p}\frac{\partial}{\partial R_1}\frac{1}{\sqrt{R_1R_3C_4C_6}} = \frac{R_1}{r_p}\left(\frac{-1}{2}\right)\frac{R_3C_4C_6}{(R_1R_3C_4C_6)^{3/2}} = -\frac{1}{2}.$$

In the same way we get for all elements that are part of the expression for r_p:

$$S_{R_1}^{r_p} = S_{R_3}^{r_p} = S_{C_4}^{r_p} = S_{C_6}^{r_p} = -\frac{1}{2}.$$

The sensitivity with respect to an element, that is not in the expression, is, of course, zero. In the same way we get the sensitivities of σ_p with respect to the passive elements where $-2\sigma_p = \frac{r_p}{Q}$.



$$\sigma_p = -\frac{1}{2}\left(\frac{1}{R_1C_6} + \frac{1}{R_3C_6} + \frac{1-K}{R_3C_4}\right) \qquad S_{R_1}^{\sigma_p} = \frac{R_1}{\sigma_p}\frac{\partial\sigma_p}{\partial R_1} = \frac{R_1}{\sigma_p}\left(\frac{1}{2R_1^2C_6}\right) = \frac{1}{2R_1C_6\sigma_p}$$

$$S_{R_3}^{\sigma_p} = \frac{R_3}{\sigma_p}\frac{\partial\sigma_p}{\partial R_3} = \frac{R_3}{\sigma_p}\left(\frac{C_4 - C_6(1-K)}{2R_3^2C_4C_6}\right) = \frac{C_4 - C_6(1-K)}{2R_3C_4C_6\sigma_p}$$

$$S_{C_4}^{\sigma_p} = \frac{C_4}{\sigma_p}\frac{\partial\sigma_p}{\partial C_4} = \frac{C_4}{\sigma_p}\left(\frac{1-K}{2R_3C_4^2}\right) = \frac{1-K}{2R_3C_4\sigma_p}$$

$$S_{C_6}^{\sigma_p} = \frac{C_6}{\sigma_p}\frac{\partial\sigma_p}{\partial C_6} = \frac{C_6}{\sigma_p}\left(\frac{R_1+R_3}{2R_1R_3C_6^2}\right) = \frac{R_1+R_3}{2R_1R_3C_6\sigma_p}$$

c) Note that the gain is here the gain of the amplifier and NOT the gain of the operational amplifier. Instead it is $K = (R_7+R_8)/R_7$. We get

$$S_K^{\sigma_p} = \frac{K}{\sigma_p}\frac{\partial\sigma_p}{\partial K} = \frac{K}{\sigma_p}\left(\frac{1}{2R_3C_4}\right) = \frac{K}{2R_3C_4\sigma_p}$$. The sensitivities are large if the real part of the pole is small.

6.23 a) The sections are the type PF2. The HP section to the left and an LP section to the right. For the PF2 LP section we have $H(s) = \frac{r_p^2}{s^2 + \left(\frac{r_p}{Q}\right)s + r_p^2}$ where $r_p = \frac{1}{RC}$, $Q = \frac{RCr_p}{2}$ and for the HP section

$$H(s) = \frac{s^2}{s^2 + \left(\frac{r_p}{Q}\right)s + r_p^2} \quad \text{where } \frac{r_p}{Q} = \frac{1}{R_4C_1} + \frac{1}{R_4C_3} = \frac{2}{RC}.$$

b) At the crossover frequency, the power is divided equally between the two filters, i.e., $|H(j\omega_0)| = \frac{1}{\sqrt{2}}$ and the attenuation is $A(\omega_0) = -20\log(|H(j\omega_0)|) = 3.01$ dB.

Hence the 3-dB edge = r_p and we select, e.g., $C = 10$ nF which yields $R = 9362\ \Omega$. Both the LP and HP sections Q value are $Q = 0.5$.

6.24 a

6.25 The section is a UG section where the resistor n have been replaced by a resistive voltage divider, R_2 and R_1 to reduce the gain of the section, i.e., $n = R_0 = \frac{R_1R_2}{R_1+R_2}$. The transfer function is

$$H(s) = \frac{G}{s^2 + \left(\frac{r_p}{Q}\right)s + r_p^2} \quad \text{where } r_p = \frac{1}{\sqrt{R_0R_3C_4C_6}},\ Q = \frac{R_0R_3C_6r_p}{R_0+R_3},\ G = \frac{R_2}{R_1+R_2}r_p^2.$$ we have

$$\frac{\partial Q}{\partial C_6} = \frac{\partial}{\partial C_6}\frac{R_0R_3C_6r_p}{R_0+R_3} = \frac{R_0R_3r_p}{R_0+R_3}$$ and we get $S_{C_6}^{Q} = \frac{C_6}{Q}\frac{\partial Q}{\partial C_6} = \frac{C_6}{Q}\frac{R_0R_3r_p}{R_0+R_3} = 1$.

In the same way we get $\frac{\partial r_p}{\partial C_6} = \frac{\partial}{\partial C_6}\frac{1}{\sqrt{R_0R_3C_4C_6}} = \frac{-R_0R_3C_4}{2(R_0R_3C_4C_6)^{3/2}}$ and $S_{C_6}^{r_p} = \frac{C_6}{r_p}\frac{\partial r_p}{\partial C_6} =$

$$= \frac{C_6}{\frac{1}{\sqrt{R_0R_3C_4C_6}}}\frac{-R_0R_3C_4}{2(R_0R_3C_4C_6)^{3/2}} = -\frac{1}{2}.$$ Compare with Problem 6.9 where $r_p \propto (C_6)^{-1/2}$.

If the element (C_6), for which we want to determine the sensitivity appear as a rational function with a power of 1 or –1, then the sensitivity is 1 or –1, respectively. For example, the sensitivity of Q with respect to C_6 is +1 since $Q = \frac{R_0R_3C_6r_p}{R_0+R_3}$. If the element appears as a square root, then the sensitivity is either 0.5 or –0.5. For example, the sensitivity of r_p with respect to C_6 is –0.5 since $r_p = \frac{1}{\sqrt{R_0R_3C_4C_6}}$.

6.25

6.26

6.27

6.28 Select, e.g., $C_1 = C_2 = C = 5$ nF and we get $R = 1/(r_pC) = 10.0$ kΩ and $R_1 = R_2 = R_4 = R_5 = R$ and $R_3 = Q\,R = 7.072$ kΩ. The DC-gain = 2.

6.29 The gyrator has the input impedance $Z_{in}(s) = \frac{sC_2}{g^2} = 6.75 10^{-8}s$, i.e., it corresponds to an inductor with the inductance $L = 67.5$ nH. The admittance for the whole circuit is $Y = 1/R + sC_1 + 1/sL$ =>

$$Z = \frac{\frac{s}{C_1}}{s^2 + \frac{s}{C_1R} + \frac{g^2}{C_2C_1}} = \frac{10^8s}{s^2 + 10^4s + 1.4815 10^{15}} \Rightarrow r_p = \sqrt{1.4815 10^{15}} = 338.49 \text{ Mrad/s},$$

$$Q = \frac{r_p}{10^4} = 3849$$ which is a high Q value.

6.30 a) We select $C_1 = C_2 = C = 50$ pF and $R_2 = R_5 = 10$ kΩ we get $R_3 = R_6 = \frac{1}{Cr_p} = 1.5915$ kΩ.

Furthermore, we select $R_4 = 10$ kΩ which yields $R_1 = (2Q–1)R_4 = 390$ kΩ and $G = 2 – 1/Q = 1.95$.

b) $R_1/R_2 = 245$

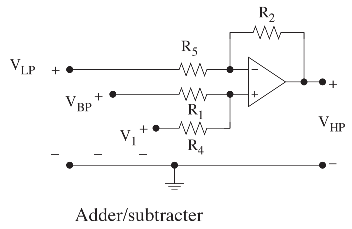
c) We get $\delta = –0.04$ and $\eta = –1.6667$ which is a too large error and the section is not useful in practice. Note that $\omega_t/r_p = 50$. Obviously, an operational amplifier with larger bandwidth is required. for example, with $\omega_t/r_p = 250$ we get $\eta = –1.0331$.

d)

6.31 We get $H(s) = -\dfrac{s^2 - \frac{1}{2RC}s + \frac{1}{2R^2C^2}}{s^2 + \frac{1}{2RC}s + \frac{1}{2R^2C^2}}$, i.e., an AP section. Generally for an AP filter we have

$H_{AP}(s) = P(-s)/P(s)$ where $P(s)$ is a Hurwitz polynomial.

6.32 We get from the figure by using the superposition principle three cases, i.e., one for each input signal when the other signals are = 0.



Adder/subtracter

For the case when V_{LP} is the input signal and $V_{BP} = V_1 = 0$ we get an inverting amplifier with the gain (the resistors R_1 and R_4 are grounded) $V_{HP} = -\dfrac{R_2}{R_5}V_{LP}$

For the case when V_{BP} is the input signal and $V_{LP} = V_1 = 0$ we get a noninverting amplifier the gain $\dfrac{R_2 + R_5}{R_5}$ and the input signal has been divided between R_1 and R_4. We get therefore the contribution

$$V_{HP} = \frac{R_4}{R_1 + R_4}\left(\frac{R_2 + R_5}{R_5}\right)V_{BP}$$

Finally for the case when V_1 is the input signal and $V_{LP} = V_{BP} = 0$ we get a noninverting amplifier and input signal has been divided between R_4 and R_1. We get therefore the contribution $V_{HP} = \dfrac{R_1}{R_1 + R_4}\left(\dfrac{R_2 + R_5}{R_5}\right)V_1$. By adding the three contributions we get

$$V_{HP} = \frac{R_4}{R_1 + R_4}\left(\frac{R_2 + R_5}{R_5}\right)V_{BP} - \frac{R_2}{R_5}V_{LP} + \frac{R_1}{R_1 + R_4}\left(\frac{R_2 + R_5}{R_5}\right)V_1$$

6.33

6.34 a) By superposition of the output signals from the three (ideal) operational amplifier

$V_{out} = \dfrac{-Z_2V_{in}}{Z_1} + \dfrac{-Z_2V_2}{R_5}$, $V_3 = \dfrac{-V_{in}}{R_7sC_2} + \dfrac{-V_{out}}{R_6sC_2}$ and $V_2 = \dfrac{-R_4V_{in}}{R_8} + \dfrac{-R_4V_4}{R_3}$ where $Z_1 = R_1//C_3 = \dfrac{R_1}{1 + sR_1C_3}$ and $Z_2 = R_2//C_1 = \dfrac{R_2}{1 + sR_2C_1}$. Elimination and simplification yields

$$H(s) = \frac{-C_3}{C_1}\frac{\left(s^2 + \left(\frac{1}{R_1} - \frac{R_4}{R_5R_8}\right)\frac{s}{C_3} + r_z^2\right)}{\left(s^2 + \frac{r_p}{Q}s + r_p^2\right)} \text{ where } r_p^2 = \frac{R_4}{R_3R_5R_6C_1C_2},\ \frac{r_p}{Q} = \frac{1}{R_2C_1}$$

$$r_z^2 = \frac{R_4}{R_3R_5R_7C_1C_3} \text{ and } \frac{r_z}{Q_z} = \frac{1}{R_1C_3} - \frac{R_4}{R_5R_8C_3}$$

b) A BP section is obtained if $C_3 = 0$ and $R_8 = \infty$ which yields $H(s) = \dfrac{-1}{R_1C_1}\dfrac{s}{\left(s^2 + \frac{r_p}{Q}s + r_p^2\right)}$ We select $C_1 = C_2 = 20$ pF and with $\dfrac{r_p}{Q} = \dfrac{1}{R_2C_1}$ yields $R_2 = 79.5775$ kΩ. With $R_3 = R_4 = 20$ kΩ and $R_5R_6 = 1/(C^2r_p^2)$ we get with equal resistors $R_5 = R_6 = 3.97887$ kΩ. We select R_1 to get the desired gain constant $G = -5\ 10^{10}/R_1$

c) The element spread is $R_2/R_5 = 20$.

d) The Tow-Thomas section has about the same sensitivity as the KHN section, i.e., much too large sensitivity for these requirements and with these operational amplifiers.

e) Suitable element for tuning is
r_p is tuned with R_5
Q_p is tuned with R_2.
r_z is tuned with R_7
Q_z is tuned with R_1 and R_8.
G is tuned with the ratio of C_1 and C_3.
The parameters of the sections can thus be tuned independently of each other.

6.35

6.36

6.37a) The pole are the same for all outputs, only the zeros differ. When the switches are open we get $|V_2|^2 + |V_3|^2 = |V_1|^2$. The attenuation at the crossover frequency is 3 dB. The crossover filter is thus power complementary and suitable when the speakers are relatively far away from each other and the listener.

b) When the switches are closed, only the Barker-output is effected. We get $|V_2| + |V_3| + |V_4| = |V_1|$. The attenuation at the crossover frequency is 6 dB and this setting is suitable when the speakers are relatively close to each other and the listener.

6.38

6.39

6.40

6.41

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6.46

6.47 $V_{out}/V_{in} = (g_{m3}g_{m2}g_{m0})/(g_{m4}g_{m3}g_{m2}+g_{m3}g_{m2}g_{m1} + s^2(C_{c2}C_{c1}g_{m3}+C_{c3}C_{c2}g_{m1}) + s(C_{c3}g_{m4}g_{m2}+C_{c1}g_{m3}g_{m2}+C_{c2}g_{m3}g_{m1}) + C_{c3}C_{c2}C_{c1}s^3$

Very long calculations! Not an interesting problem

6.48

6.49 $I_1 = g_{m1}V_{in}$
$I_2 = -g_{m2}V_{BP}$
$I_3 = g_{m3}V_{BP}$
$I_4 = -g_{m4}V_{LP}$
$I_1 + I_2 + I_4 = sC_1V_{LP}$
$I_3 = sC_2V_{LP}$
$\Rightarrow V_{BP} = -C_2\, g_{m1}\, sV_{in}/(C_1C_2s^2 + C_2g_{m2}\, s + g_{m3}\, g_{m4})$ and
$\Rightarrow V_{LP} = g_{m1}\, g_{m3}\, V_{in}/(C_1C_2s^2 + C_2g_{m2}\, s + g_{m3}\, g_{m4})$

6,50 Extra

Reealize an active filter of type Chebyshev I that fulfills the specification given below.

$A_{max} = 0.1$ dB $\omega_s = 4{,}5$ krad/s

$A_{min} = 35$ dB $\omega_c = 3$ krad/s

Use PF2 sections, 2 kohm resistors, and calculate the other component values.

Nomogram gives $N = 6.86 \Rightarrow N = 7$.

$s_{p1,7} = -251.5229 \pm j\,3125.500$ rad/s	$Q_1 = 6.233238$
$s_{p2,6} = -704.751 \pm j\,2506.456$ rad/s	$Q_2 = 1.847212$
$s_{p3,5} = -1018.395 \pm j\,1390.978$ rad/s	$Q_3 = 0.846397$
$s_{p4} = -1130.334$ rad/s	$Q_4 = 0.5$

Place the section in the order Q_4, Q_3, Q_2, Q_1. Select $R_1 = R_3 = R_2 = 2$ kohm

Q_4: $R = 2$ kohm	$C = 4.42347$ e-7 F
Q_3: $C_6 = 4.90968$ μF	$C_4 = 1.45017$ e-7 F
Q_2: $C_6 = 7.094699$ μF	$C_4 = 9.60190$ e-8 F
Q_1: $C_6 = 1.98789$ μF	$C_4 = 7.97294$ e-8 F

6.51 Extra

Calculate the component values of an active lowpass filter without finite zeros that fulfills the specification below. The order should be as low as possible.

$A_{max} = 0{,}1$dB $\omega_c = 5$ krad/s

$A_{min} = 22$ dB $\omega_s = 15$ krad/s

Use PF1 sections, 10 kΩ resistors, and draw the schematic of the filter.

Chebyshev I, $N = 3$

Find the normalized poles:

–0.191020 ± 0.927074i

–0.38203944

Denormalized poles:

–955.1 ± 4635.37i rad/s

–1910.1972 rad/s

Q values: 2.477617 and 0.5

$r_p = 4732.744557$ rad/s

Put the 1st-order section first, followed by the 2nd-order PF1 section, with a voltage follower as a buffer in between.

$R_1 = R_3 = R = 10$ kohm, $C_4 = C_6 = C$, => $C = \dfrac{1}{r_p R} = 21.29$ nF, $K = 3 - \dfrac{1}{Q}$ $K = \dfrac{R_7 + R_8}{R_7} =$ 2.596386

Select the resistor and capacitor in the 1st-order section so that $1/RC = 1910.1972 \Rightarrow R = 10$ kΩ and $C = 52.351$ nF.

6.52 Extra

Realize an active filter of type Chebyshev I that fulfills the specification given below.

$A_{max} = 0.1$ dB $\omega_s = 1$ krad/s

$A_{min} = 50$ dB $\omega_c = 9$ krad/s

Use PF2 sections, 1nF capacitors, and calculate the other component values.

Chebyshev I N = 3

Find the normalized poles:

-0.48470285451503 + 1.20615528499652i

-0.96940570903005

-0.48470285451503 - 1.20615528499652i

Denormalized poles:

1.0e+03 *

-2.42351427257513 + 6.03077642498262i

-4.84702854515027

-2.42351427257513 - 6.03077642498262i

Q values: 1.341 and 0.5

Put the 1st-order section first, followed by the 2nd-order PF1 section, with a voltage follower as a buffer in between.

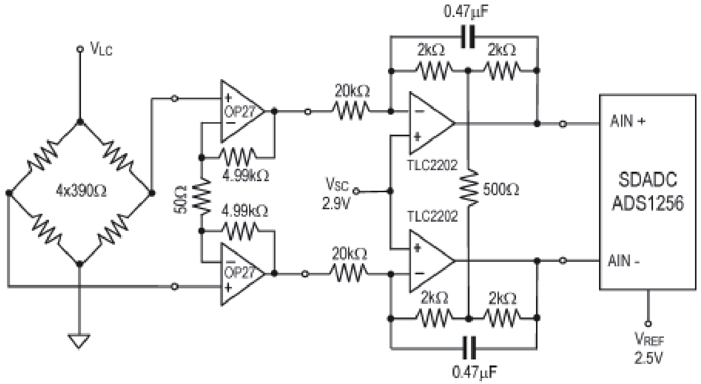
$$R_1 = R_3 = R \quad C_4 = C_6 = C \quad C = \frac{1}{r_p R} \quad K = 3 - \frac{1}{Q} \quad K = \frac{R_7 + R_8}{R_7}$$

Select the resistor and capacitor in the 1st-order section so that $\dfrac{1}{RC} = 4.848028$.

6.53 Extra

The following circuit is found in the paper:

Enrique Mario Spinelli, Pablo Andrés García, and Dardo Oscar Guaraglia: A Dual-Mode Conditioning Circuit for Differential Analog-to-Digital Converters, IEEE Trans. on Instrumentation and Measurements, Vol. 59, No. 1, pp. 195-199, Jan. 2010



a) Determine the transfer function for the interface, i.e., from the bridge terminals to the analog-to-digital converter. Hint, the 50 Ω and 500 Ω resistors can be split into two resistor in order to create a virtual ground.

b) What is the DC gain.

c) What is the advantage of using a differential circuit?

a) The gain of the first stage, i.e., the two OP27 amplifiers, is $G = (4990 + 25)/25 = 200.6$

The second stage is an NF1 section with the transfer function

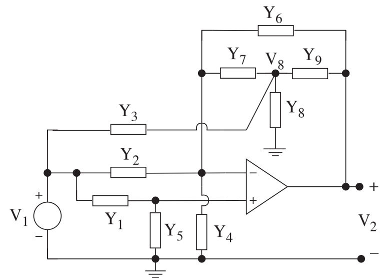
NF1

$$N(s) = \frac{Y_1}{Y_1 + Y_2} N_2 + Y_7 Y_3 + N_1 Y_2$$

$$D(s) = Y_7^2 + N_1 Y_6$$

$$N_1 = Y_9 + Y_8 + Y_7 + Y_3$$

$$N_2 = Y_7^2 - (Y_7 + Y_6 + Y_4 + Y_2) N_1$$



$N_1 = G_3 + G_7 + G_8 + G_9$

$N_2 = G_7^2 - (G_7 + s\,C_6) N_1$

$N = G_7 G_3 + N_1 G_2$

$D = G_7 + N_1 (sC_6)$

$H = N/D = (G_7 G_3 + G_2 N_1)/(G_7^2 + N_1 s C_6) =$

$= (G_7G_3+(G_9+G_8+G_7+G_3)G_2)/(G_7^2+sC_6(G_9+G_8+G_7+G_3)) = 10.53297/(s+105.3297)$

Hence a first-order anti-aliasing filter in front of the ADC. The DC gain is

$Gain_{DC} = 200.6*10.53297/(0+105.3297) = 20.06$

b) A differential circuit is used to suppress common mode noise and it reduces the effect of nonliniarities. See Chapter 9.