9. WAVE ACTIVE FILTERS

9.1 Determine first the $\mathbb{K}$ matrix for a shunt admittance. We get

$V_1 = V_2$

$V_1 = Y(I_1 - I_2)$

i.e., $K = \begin{pmatrix} 1, 0 \\ Y, 1 \end{pmatrix}$. Inserting $A = 1$, $B = 0$, $C = Y$ and $D = 1$ into Eq. (9.19) yields

$s_{11} = (1 - YR - 1)/\Delta$

$s_{12} = 2R_1G_2/\Delta$

$s_{21} = 2/\Delta$

$s_{22} = -(1 + YR_1 - R_1G_2)/\Delta$

$\Delta = 1 + YR_1 + R_1G_2$

After simplification we get

$s_{11} = (G_1 - G_2 - Y)/(G_1 + G_2 + Y)$ $\quad s_{11} = -Y/(2G+Y)$

$s_{12} = 2G_2/(G_1 + G_2 + Y)$ $\quad s_{12} = 2G/(2G+Y)$

$s_{21} = 2G_1/(G_1 + G_2 + Y)$ $\quad s_{21} = 2G/(2G+Y)$

$s_{22} = (G_2 - G_1 - Y)/(G_1 + G_2 + Y)$ $\quad s_{22} = -Y/(2G+Y)$

Alternatively, the scattering matrix for a series impedance Z is according to Eq. (9.20)

$S = \frac{1}{Z + 2R}\begin{pmatrix} Z, 2R \\ 2R, Z \end{pmatrix}$ using the theorem with a series impedance embedded between two gyrators. We get a shunt admittance $Y = Z/R^2$. Hence, a series impedance with $Z = YR^2$ yields

$$S = \frac{1}{Y + 2G}\begin{bmatrix} Y & 2G \\ 2G & Y \end{bmatrix}.$$

The gyrators correspond, according to Figure 9.9, to sign-inversion of the reflected waves. Hence we change the signs of the factor that is multiplied with A_2. The scattering matrix for a gyrator is $\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} A_1 \\ A_2 \end{bmatrix}$. We get the scattering matrix for an embedded two-port

$$\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} -s_{21} & -s_{22} \\ s_{11} & s_{12} \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} -s_{22} & s_{21} \\ s_{12} & -s_{11} \end{bmatrix}\begin{bmatrix} A_1 \\ A_2 \end{bmatrix}$$

and with the series impedance $Z = YR^2$ we get $S_Y = \frac{1}{Y + 2G}\begin{bmatrix} -Y & 2G \\ 2G & -Y \end{bmatrix}$

9.2 We have the incident and reflected waves to a port $a = v + Ri$ and $b = v - Ri$ where all variables are sinusoidals. This yields $V = (a + b)$ and $I = (a - b)/R$ where V and I are r.m.s. values.
The power into a port is: $P = \text{real}\{V I^*\} = (a + b)(a - b)/R = (|a|^2 - |b|^2)/R = G(|a|^2 - |b|^2)$

9.3 We get

$L_1' = 2R_0\tau_5$ $\quad C_1' = \tau_6/(2R_0)$

$L_2' = R_0\tau_3/2$ $\quad C_2' = 2\tau_4/R_0$

$L_3' = 2R_0\tau_2$ $\quad C_3' = \tau_1/(2R_0)$

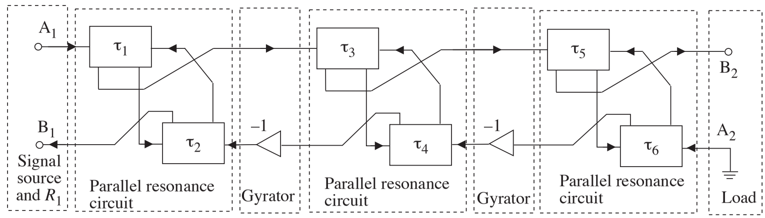
and

$\tau_1 = 2\Omega_2/\omega_I^2 L_3$ $\quad \tau_4 = C_2/2\Omega_2$

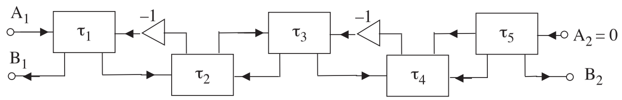
$\tau_2 = L_3/2\Omega_2$ $\quad \tau_5 = L_1/2\Omega_2$

$\tau_3 = 2\Omega_2/\omega_I^2 C_2$ $\quad \tau_6 = 2\Omega_2/\omega_I^2 L_3$

The constant $R_0 > 0$ is arbitrary and affects only the impedance level in the wave two-ports.



9.4 A crossover network has not very high stopband attenuation in the two stopbands. Hence, we may realize a lowpass filter and use to complementary output for the highpass part. We get



where the inductances and yield the t-factors $\tau_L = L/2R$ and $\tau_C = RC/2$, respectively.

9.5

9.6 According to Feldtkellers Eq. (9.37) we have: $|s_{11}|^2 + |s_{21}|^2 = 1$

In this case, we estimate that $|s_{11}| \leq 20$ dB in the passband, i.e., the maximum of the magnitude of reflection function is less than $|s_{11}| = 10^{-A_{max}\, s11/20} = 10^{-(20/20)} = 0.1$ and

$|s_{21}| = \sqrt{1 - |s_{11}|^2} = 0.994987 \Rightarrow A_{max} = 0.043648$ dB