

3.26 Let $X(k) = A + jB$ and $W^{nk} = W_R + jW_I$. According to the suggested algorithm we start by computing the DFT of $X(k)$. Hence, we compute the DFT where we have interchanged the real and imaginary parts of discrete Fourier transform, $X(k)$

$$\begin{aligned} \sum_{n=0}^{N-1} X(n) W^{nk} &= \sum_{n=0}^{N-1} (B + jA)(W_R + jW_I) = \\ &= \sum_{n=0}^{N-1} (B W_R + jB W_I + jA W_A - A W_I) = \\ &= \sum_{n=0}^{N-1} (B W_R - A W_I) + j(B W_I + A W_A) \end{aligned}$$

Now, we interchange the real and imaginary parts of the DFT

$$\begin{aligned} \sum_{n=0}^{N-1} (B W_I + A W_A) + j(B W_R - A W_I) &= \\ &= \sum_{n=0}^{N-1} (A + jB)(W_R - jW_I) = \\ &= \sum_{n=0}^{N-1} (A + jB) W^{-nk} = \sum_{n=0}^{N-1} X(n) W^{-nk} = x(n) \end{aligned}$$

Thus, we have obtained the IFFT except of the factor $\frac{1}{N}$. However, this factor will, in practice, be included inside the butterflies in order to achieve a safe scaling of the signals.

An alternative method is as follows

$$\begin{aligned} x(n) &= \frac{1}{N} \sum_{n=0}^{N-1} X(n) W^{-nk} = \frac{1}{N} \left[\left(\sum_{n=0}^{N-1} X(n) W^{-nk} \right)^* \right]^* = \\ &= \frac{1}{N} \left[\sum_{n=0}^{N-1} X^*(n) W^{nk} \right]^* = \frac{1}{N} \left[\text{DFT}\{ X^*(n) \} \right]^* \end{aligned}$$

This alternative method to compute the IFFT by using two complex conjugate operations is more expensive than the method discussed above, since it involves two changes of the sign of the imaginary parts.