7.1 a) These butterfly pairs, which computed concurrently, are determined by the relative indices. We assume that the butterflies are labeled with numbers 0 to 7 from top to down for each butterfly in Figure 7.4.

First alternative

Execute butterflies p and $p + N/4$ simultaneously for all stages, the butterfly pairs are $\{0, 4\}$, $\{1, 5\}$, $\{2, 6\}$, and $\{3, 7\}$ for all stages.

Second alternative

Execute butterflies p and $p + N_s/2$ at the first stage, p and $p + N_s$ for other stages. Note that $N_s = 2^{4-stage}$, the butterflies paies are therefore $\{0, 4\}$, $\{1, 5\}$, $\{2, 6\}$, and $\{3, 7\}$ for the first and the second stage, $\{0, 2\}$, $\{1, 3\}$, $\{4, 6\}$ and $\{5, 7\}$ for the third stage, and $\{0, 1\}$,$\{2, 3\}$,$\{4, 5\}$, and $\{6, 7\}$ for the final stage.

b) Obviously, m ranges from 0 to 3. $N_s = 2^{n-stage}$, i.e., $N_s = 8$ for the first stage, $N_s = 4$ for the second stage, $N_s = 2$ for the third stage, and $N_s = 1$ for the final stage. See a) for the range of p-values.

First alternative

$k_1 = 2N_s\lfloor m/N_s \rfloor + [m \bmod(N_s)]$

$$k_2 = \begin{cases} k_1 + N/4 & Stage = 1 \\ k_1 + N/2 & Stage \geq 2 \end{cases}$$

$Stage = 1$:

$2N_s\lfloor m/N_s \rfloor = 0$ and $m \bmod(N_s) = m$, $k_1 = m$, $k_2 = m + N/4 = m + 4$,

$k_{1N_s} = k_1 + N_s = m + 8$ and $k_{2N_s} = k_2 + N_s = m + 4 + 8 = m + 12$.

In the same manner, we can determine the k_1, k_2, k_{1N_s}, and k_{2N_s} for the other stage. The result is listed in the following table.

Stage	k_1	k_2	k_{1N_s}	k_{2N_s}
1	0,1,2,3	4,5,6,7	8,9,10,11	12,13,14,15
2	0,1,2,3	8,9,10,11	4,5,6,7	12,13,14,15
3	0,1,4,5	8,9,12,13	2,3,6,7	10,11,14,15
4	0,2,4,6	8,10,12,14	1,3,5,7	9,11,13,15

Second alternative

$k_1 = 4N_s\lfloor m/N_s \rfloor + [m \bmod(N_s)]$

$$k_2 = \begin{cases} k_1 + N_s/2 & Stage = 1 \\ k_1 + 2N_s & Stage \geq 2 \end{cases}$$

$Stage = 1$:
$4N_s\lfloor m/N_s \rfloor = 0$ and $m \bmod(N_s) = m$, $k_1 = m$, $k_2 = k_1 + N_s/2 = m + 4$,
$k_{1N_s} = k_1 + N_s = m + 8$ and $k_{2N_s} = k_2 + N_s = m + 4 + 8 = m + 12$.

In the same way, we can determine the values for k_1, k_2, k_{1N_s}, and k_{2N_s} at each stage. This results the following table.

Stage	k_1	k_2	k_{1N_s}	k_{2N_s}
1	0,1,2,3	4,5,6,7	8,9,10,11	12,13,14,15
2	0,1,2,3	4,5,6,7	8,9,10,11	12,13,14,15
3	0,1,8,9	4,5,12,13	2,3,10,11	6,7,14,15
4	0,4,8,12	2,6,10,14	1,5,8,13	3,7,11,15

c) We consider the simplification of one indice in the first alternative, the other simplifications are left to the readers. All indices is represented with binary numbers, for example, m is $m = m_1 \cdot 2^1 + m_0$, where $m_i = 0, 1$. We give only on example for the simplification here, i.e., k_2.

$Stage = 00: k_2 = (01m_1m_0)_2$

$Stage = 01: k_2 = (10m_1m_0)_2$

$Stage = 10: k_2 = (1m_10m_0)_2$

$Stage = 11: k_2 = (1m_1m_00)_2$

which means that $k_2 = s_1 + s_0, s_1 \cdot m_1 + \overline{s_1} \cdot \overline{s_0}, \overline{s_1} \cdot m_1 + s_1 \cdot m_0 \cdot s_0, (\overline{s_1} + \overline{s_0}) \cdot m_0$.
Hence the addition is not neccesilly in th ecomputation of k_2.

d) Observe that the separation of $\{k_1, k_{1N_s}\}$ and $\{k_2, k_{2N_s}\}$ does not effected by m.
Hence the butterflies operations does not changed except the orders.