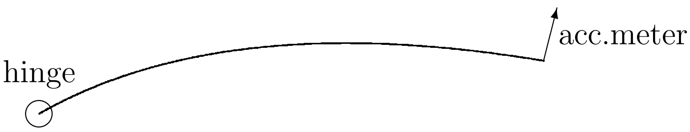
1 A Flexible Beam



Consider a flexible beam hinged at one end with a moment motor. The angle of the beam at the motor hinge is measured and an accelerometer is placed at the other end. The flexible modes are at $\omega_i = k_i^2$ rad/s and with a relative dampings of $\zeta_i = 0.01$.

The eigenmodes of the flexible beam $x \in [0, 1]$ can be described by

$$f_i(x) = a_0 \sin(k_i x) + a_1 \cos(k_i x) + a_2 \sinh(k_i x) + a_3 \cosh(k_i x)$$

with boundary conditions given by $f_i(0) = f_i''(0) = 0$, and $f_i''(1) = f_i'''(1) = 0$. This yields $a_1 = a_3 = 0$ and $\tan(k_i) = \tanh(k_i)$ such that

$$f_0(x) = \sqrt{3}x, \quad k_0 = 0$$

and

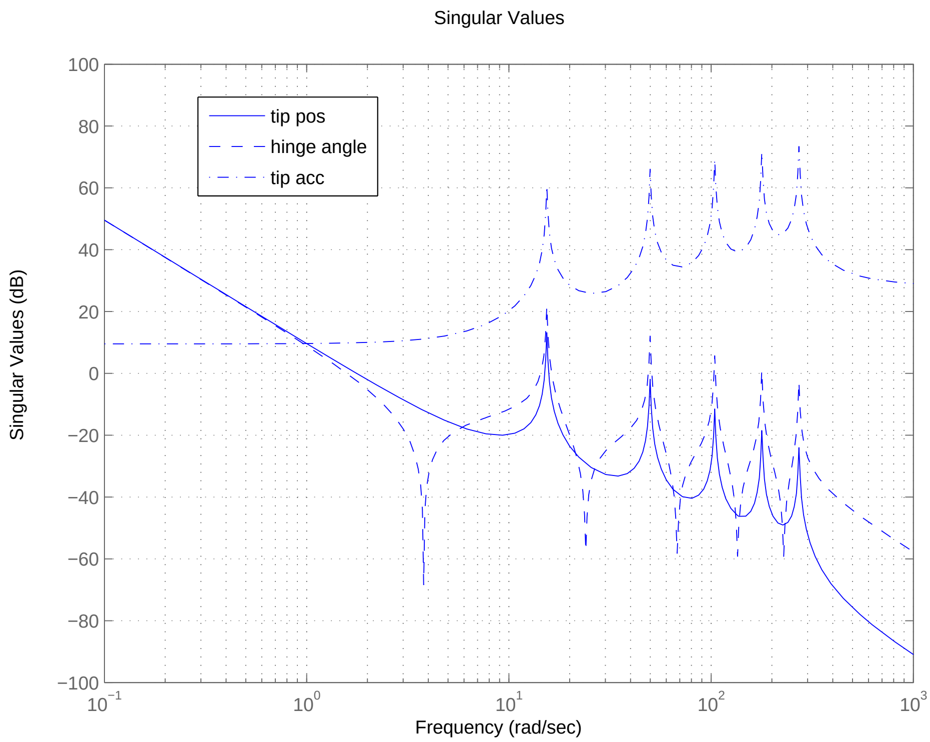
$$f_i(x) = \frac{\sin(k_i x)}{\sin k_i} + \frac{\sinh(k_i x)}{\sinh k_i}, \quad k_i = 3.9226, 7.0686, \ldots$$

Note that $\int_0^1 f_i(x) f_j(x) dx = \delta_{ij}$ where δ_{ij} is Kronecker's delta ($\delta_{ii} = 1$ and $\delta_{ij} = 0, i \neq j$) and, consequently, $\mu_i = 1$.

The dynamics, with one flexible mode, can be described by

$$G = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] = \left[\begin{array}{cccc|c} 0 & 0 & 0 & 0 & 1.732 \\ 0 & -0.1542 & 0 & -237.7 & -5.4 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 1.732 & 2 & 0 \\ 0 & 0 & 1.732 & -5.4 & 0 \\ 0 & -0.3084 & 0 & -475.4 & -7.801 \end{array}\right]$$

The input u is the commanded motor torque and the outputs y contain the true position of the tip, the angle at the motor hinge and acceleration of the tip. A Matlab function, `genbeam.m`, is available for generating this model. The figure below shows the response for a model with five flexible modes.



We would now like to design a controller with hinge angle and tip acceleration as inputs. The true tip position should be controlled to follow a reference signal.

First, make a SISO design with only hinge angle as input. Then, design a controller with two measurement inputs (hinge angle and acceleration) and one actuator output (hinge torque).

Assume that the bandwidth is limited a value in the order of 30 rad/s. The first flexible mode is at 15.4 rad/s, while the second is at 50 rad/s. This means that you can control the first flexible mode actively, but you should reduce the gain above 30 rad/s. Aim at a gain margin of at least 10 dB for the flexible modes (2nd and higher order) since the damping is uncertain.