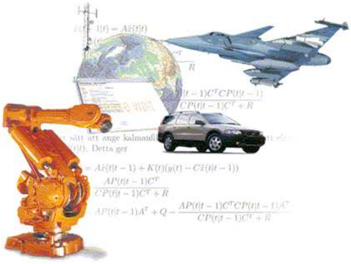
Robust Multivariable Control

Lecture 9



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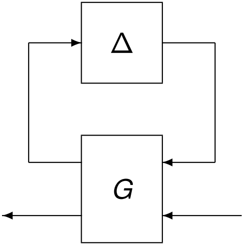
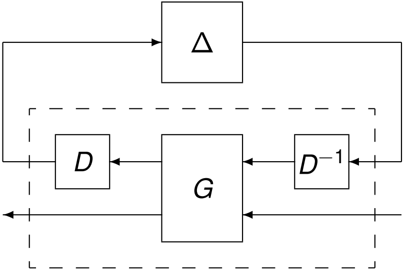
Today's topics

- D scalings
- Structured singular values μ
- The rocket example

D scalings

$$\|\Delta\|_\infty \leq 1$$





Small gain theorem

$$\|\mathscr{F}_\ell(G,K)\| < 1$$

$$D\Delta = \Delta D$$

$$\|D\mathscr{F}_\ell(G,K)D^{-1}\|_\infty < 1$$

Robustness for performance

Include a Δ-block for performance, Δ_∞, and one for parameter uncertainties, Δ_θ.

$$\Delta = \begin{bmatrix} \Delta_\theta & 0 \\ 0 & \Delta_\infty \end{bmatrix}$$

The corresponding scalings are

$$D = \begin{bmatrix} D_\theta & 0 \\ 0 & I \end{bmatrix}$$

Minimize $\|D\mathscr{F}_\ell(G,K)D^{-1}\|_\infty$ with respect to D.

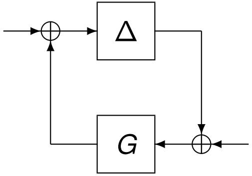
If Δ_θ is constant then we can let D_θ become frequency dependent (dynamic).

Structured singular values, μ

Definition

$$\frac{1}{\mu(M)} = \min_{\Delta \in \text{structure}} \{\bar{\sigma}(\Delta) : \det(I - \Delta M) = 0\}$$

Theorem: if $\gamma = \sup_{\omega} \mu(G(j\omega))$ then the system



is stable if and only if $\|\Delta\|_\infty < 1/\gamma$.

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Upper and lower bounds

$$\max_U \rho(UM) \underbrace{\leq}_{\text{equality}} \mu(M) \underbrace{\leq}_{\text{equality in some cases}} \inf_D \bar{\sigma}\left(DMD^{-1}\right)$$

The lower bound is an equality, but it is difficult to compute since it is not convex (difficult to find the global maximum).

The upper bound is relatively easy to find (convex), but there is often a gap to the true value (typically 15%).

The upper bound is safe; the lower bound is not valid when parameters are slowly varying, however slow they may vary.

Complex and Real uncertainties

The bound $\inf_D \bar{\sigma}\left(DMD^{-1}\right)$ can sometimes be improved if there is more structure in the uncertainties.

For *complex* uncertainties

$$\mu(M) \leq \inf_{\nu, P \succ 0} \left\{ \nu : M^* P M \preceq \nu^2 P \right\}$$

For *real* uncertainties

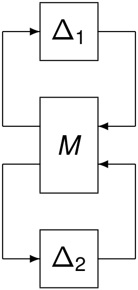
$$\mu(M) \leq \inf_{\nu, P \succ 0, G} \left\{ \nu : M^* P M + j(GM - M^* G) \preceq \nu^2 P \right\}$$

where $G = G^*$ satisfies $G\Delta = \Delta^* G$, $\forall \Delta$

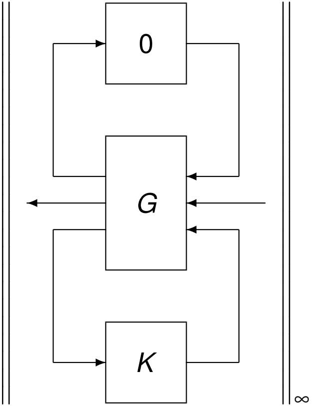
Main loop theorem

$$\mu_\Delta(M) < 1 \qquad \Leftrightarrow \qquad \begin{cases} \mu_{\Delta_2}(M_{22}) < 1 \\ \max\limits_{\|\Delta_2\| \le 1} \mu_{\Delta_1}\left(\mathscr{F}_\ell(M, \Delta_2)\right) < 1 \end{cases}$$

where $M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$

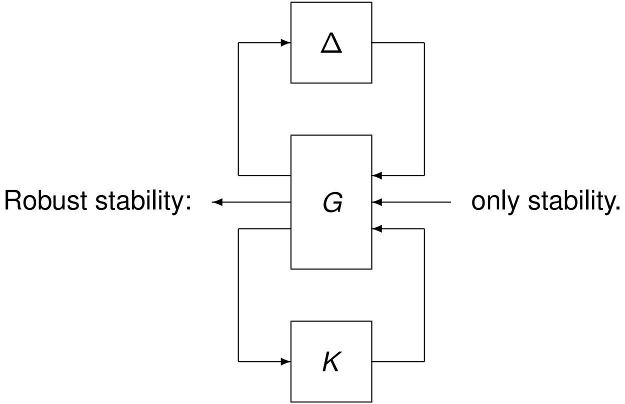


Nominal performance

Nominal performance: 

H_∞-norm.

Robust stability



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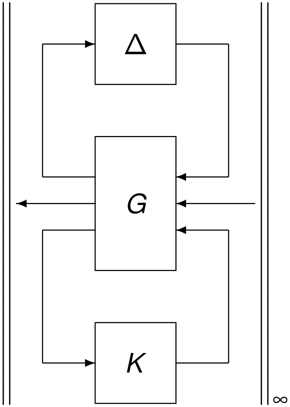
Robust performance

Robust performance:



worst case H_∞-norm.

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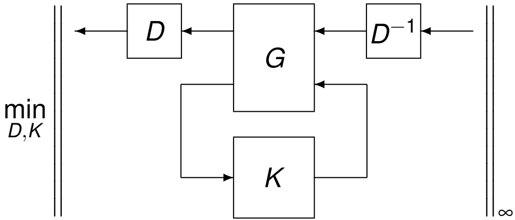
μ-design

Replace $\mu(M)$ with its upper bound: $\min_D \bar{\sigma}\left(DMD^{-1}\right)$:



Difficult to optimize with respect to both D and K – Not a convex problem!

μ-design

(i) Find a K such that $\|\mathscr{F}_\ell(G,K)\|_\infty$ is minimized (H_∞-synthesis).

(ii) Find a D (with fixed K) such that $\left\|D\mathscr{F}_\ell(G,K)D^{-1}\right\|_\infty$ is minimized (`mu` and `musynfit`).

(iii) Find a K (with fixed D) such that $\left\|D\mathscr{F}_\ell(G,K)D^{-1}\right\|_\infty$ is minimized (H_∞-synthesis).

(iv) Repeat (ii) and (iii) until convergence or until $\gamma = \left\|D\mathscr{F}_\ell(G,K)D^{-1}\right\|_\infty$ becomes sufficiently small

μ-design

Step (ii), that is to say

(ii) Find a D (with fixed K) such that $\left\| D\mathscr{F}_\ell(G,K)D^{-1} \right\|_\infty$ is minimized.

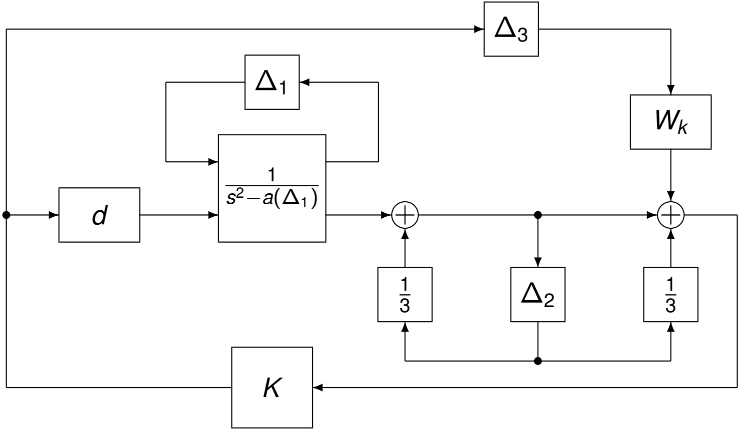
Can be divided into two substeps:

(iia) Find a D_ω for "all" ω (sufficiently dense in ω).

(iib) Fit D_ω to $D(j\omega)$ where D is stable and minimum phase.

Relatively simple for scalar D, but more difficult for repeated uncertainties.

An example



where $d = \frac{1-0.03s}{1+0.03s}$ corresponds to a delay of 0.06 s.

Requirements

- Delay of 0.06 s.
- Gain and phase margins: 6 dB and 35 deg.
- Reduce the controller gain to -6 dB above 50 rad/s.

W_k

Reduce the controller gain to -6 dB above 50 rad/s.