Extended Kalman Filter Applied to Industrial Manipulators

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Abstract

This paper summarizes previous work on tool position estimation on industrial manipulators, and emphasize the problems that must be taken care of in order to get a satisfied result. The acceleration of the robot tool, measured by an accelerometer, togheter with measurements of motor angles are used. The states are estimated with an extended kalman filter. A method for tuning the covariance matrices for the noise, used in the observer, is suggested. The work has been focused on a robot with two degrees of freedom.

1 Introduction

Current industrial robot development is focused on increasing the robot performance, reducing the robot cost, improving safety, and introducing new functionalities as described in [1]. The need for cost reduction results in the use of cost optimized robot components with increased elasticity and larger individual variation, such as variation of gearbox stiffness or in the parameters describing the mechanical arm. Cost reduction also implies weight-optimized robots and thus lower mechanical stiffness and more complicated vibration modes. To maintain or improve the robot performance, the motion control must be improved for this new generation of robots. For robots with traditional measurement systems, where only the motor angular position is measured, this can be obtained by improving the model-based control as described in [2]. Another option is to use inertial sensors to improve the estimation of the robot tool position. This paper investigates how the tool position can be estimated by the use of observers.

One early contribution is [3], which describes how the nonlinear dynamics of elastic robots can be handled. The problems of gravity compensation for elastic robots is studied in [4]. One commonly used observer is the linear Kalman filter (KF) or the extended Kalman filter (EKF), used for nonlinear systems. A KF is used for a single-axis robot arm in [5] and [6]. EKFs are used in [7], and also in [8], where a two-axis robot, with tool position and joint speed measurements, are used. Estimation using motor measurements only is studied in [9]. In [10], accelerometers are used, and estimation is performed by particle filters as well as EKFs.

The estimated tool position can be used for on-line feedback control as a mean of increasing both the robust and the nominal performance of the robot. Another possible use of tool estimation is iterative learning control (ILC) [11]. In [12] it is shown that motor side learning is insufficient if the mechanical resonances are exited by the robot trajectory. Other applications in need of tool position estimation are, e.g., model identification, supervision, diagnostics, and automatic controller tuning.

The estimation problem can be divided into one static (low frequencies) and one dynamic (mid to high frequencies) estimation problem. A large industrial robot typically has a static volumetric accuracy of 2–15 mm due to gravity deflection, component tolerance, and variations in the assembly procedure. One common solution to the static problem is to perform an off-line identification of an extended kinematic model and an elasto-static model, i.e., to solve the problem by model-based control. In this way a static accuracy of 0.5 mm can be obtained. Thus, tool estimation by observers is most interesting to investigate for the dynamic problem, i.e., for frequencies larger than, e.g., 1 Hz[1]. The static and dynamic estimates can then be fused, for example, in the frequency domain.

This paper investigates methods for estimation of the robot tool position. It is assumed that the motor angular position and the tool acceleration

[1] The frequency should be well below the lowest mechanical resonance of the robot.

are measured. The observer considered is an EKF. A method for tuning the observers is suggested and the robustness of the methods is investigated. The observer is evaluated on simulated data from a two axes model of an industrial robot. However, it is straight forward to adapt the methods to a six degrees-of-freedom (DOF) industrial robot.

2 Problem Formulation

To achieve a satisfactory result of the estimation in terms of the dynamic accuracy it is instrumental to have a good model of the plant and to find the noise covariances in the filter that minimizes the estimation error. The challange lies in the high accuracy that is required by the robot user. Typically, the dynamic path error can be up to serveral millimeters, in some cases more than 10 mm. The high accuracy is obtained using a combination of feedback and feedforward control using complex nonlinear dynamic models of the robot structure. One problem with this approach is that by relying only on more and more complex models there is an obvious risk that the control system becomes less robust to model uncertainties. One solution to gain robustness can be to use estimations of the acceleration, the velocity and the position for the joints and the tool by fusing information from additional sensors. Several fundamental problems related to the estimation procedure have emerged in previous work by the authors, see Section 3. In Section 4 the open issues are further discussed and some suggestions for possible improvements are presented.

3 Previous Work

This section gives a summary of the results in [13], which is a continuation of [14]. The robot model and sensor model are the same for these works. The differences are that [14] only uses experimental data and different types of EKFs based on reduced models while [13] only uses simulated data and the full EKF. In [13] the tuning and the model errors are considered in a more detailed study developing the results from [14] even more.

3.1 Robot and Accelerometer Models

The robot model is a joint flexible two axes model from [15]. The model assumes rigid links and flexible joints. Each joint is described with two angles, the arm angle q_{ai}, and the motor angle q_{mi}. Now, the difference $q_{ai} - q_{mi}$ describes the deflection in joint i.

The state vector is given by

$$x = \begin{pmatrix} q_{\mathrm{a}}^T & q_{\mathrm{m}}^T & \dot{q}_{\mathrm{a}}^T & \dot{q}_{\mathrm{m}}^T \end{pmatrix}^T, \tag{3.1}$$

where $q_a = \begin{pmatrix} q_{\mathrm{a}1} & q_{\mathrm{a}2} \end{pmatrix}^T$, $q_m = \begin{pmatrix} q_{\mathrm{m}1} & q_{\mathrm{m}2} \end{pmatrix}^T$, contain the arm angles q_{a} and the motor angles q_{m} of both joints. The model accounts for flexibilities in the joints via nonlinear stiffness, nonlinear friction and linear viscous damping. A state space model of the system is given by,

$$\dot{x} = \begin{pmatrix} x_3 \\ x_4 \\ M_a^{-1}(x_1)\big(-C(x_1,x_3) - G(x_1) - A(x)\big) \\ M_m^{-1}\big(A(x) + \kappa(x_4) + u\big) \end{pmatrix}, \tag{3.2}$$

where $A(x) = D(x_3 - x_4) + \tau_s(x_1, x_2)$. $A(x)$ accounts for the flexibilities in the joints, via the linear viscous damping $D(x_3 - x_4)$ and the nonlinear stiffness $\tau_s(x_1, x_2)$. In other words, if we dispense with $A(x)$, we are back at a standard rigid robot model. Furthermore, $M_a(x_1)$ and M_m are the mass matrices for the arm and motor, $C(x_1, x_3)$ accounts for the centrifugal and centripetal torques, and $G(x_1)$ accounts for the effect of gravity on the links. The nonlinear friction is described by $\kappa(x_3)$ and u represents the motor torque applied to the robot.

3.2 Observer

Given the general nonlinear model (3.2) the estimation of the unknown states can be made in many different ways. Here, an EKF [16] is used. The EKF addresses the estimation problem for a general nonlinear discrete-time system,

$$x_{k+1} = F(x_k, u_k) + v_k, \quad v_k \sim \mathcal{N}(0, Q_k), \tag{3.3a}$$
$$z_k = h(x_k, u_k) + w_k, \quad w_k \sim \mathcal{N}(0, R_k). \tag{3.3b}$$

In order to compute estimates of the states, the system is linearized around the previous estimate and the EKF is implemented as a two-step procedure, consisting of measurement update

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k(z_k - h(\hat{x}_{k|k-1}, u_k)), \tag{3.4a}$$
$$P_{k|k} = P_{k|k-1} - K_k H_k P_{k|k-1}, \tag{3.4b}$$
$$K_k = P_{k|k-1} H_k^T (H_k P_{k|k-1} H_k^T + R_k)^{-1}. \tag{3.4c}$$

and the time update

$$\hat{x}_{k+1|k} = F(\hat{x}_{k|k}, u_k), \tag{3.5a}$$

$$P_{k+1|k} = A_k P_{k|k} A_k^T + Q_k. \tag{3.5b}$$

The following representation is used for the linearized system and output matrices,

$$A_k = \left.\frac{\partial F(x, u_k)}{\partial x}\right|_{x=\hat{x}_{k|k}}, \quad H_k = \left.\frac{\partial h(x, u_k)}{\partial x}\right|_{x=\hat{x}_{k|k-1}}.$$

The measurement equation (3.3b) includes the motor measurements as well as the accelerometer measurements,

$$h(x_k, u_k) = \begin{bmatrix} x_{2k} \\ \ddot{\rho}_s(x_k) \end{bmatrix}, \tag{3.6}$$

where

$$\ddot{\rho}_s(x_k) = R_w^s(x_{1k})(J(x_{1k})\dot{x}_{3k} + \dot{J}(x_{1k})x_{3k} + G_w). \tag{3.7}$$

Since $\dot{x}_3$ is not a state, it is replaced by the $\dot{x}_3$ equation in (3.2). $J(x_1)$ is the Jacobian of the manipulator kinematics and G_w is the gravity vector measured by the accelerometer.

The tuning of the noise covariances in the EKF can be stated as a general system identification problem. It is here solved by minimizing the estimation error using a set of measurement data where the motor angles and the tool acceleration are available, together with measurements of the true tool position. The covariance matrices are parameterized as

$$\widetilde{Q}_\lambda = \begin{bmatrix} \lambda_1 I^{2\times2} & 0 & 0 & 0 \\ 0 & \lambda_2 I^{2\times2} & 0 & 0 \\ 0 & 0 & \lambda_3 I^{2\times2} & 0 \\ 0 & 0 & 0 & \lambda_4 I^{2\times2} \end{bmatrix} \widetilde{Q},$$

$$\widetilde{R}_\lambda = \begin{bmatrix} \lambda_5 I^{2\times2} & 0 \\ 0 & I^{2\times2} \end{bmatrix} \widetilde{R},$$

where $\widetilde{Q}$ and $\widetilde{R}$ are diagonal matrices and represent initial guesses and λ_i, $i = 1, \ldots, 5$ are free variables in the optimization. The objective function is to minimize the 2-norm of the estimation error in the two Cartesian dimensions. The resulting optimization problem was solved in [14] using ComplexRF, see [17]. The method was changed to an Active Set method (*fmincon* in MATLAB) in [13] due to the stochastic behaviour in ComplexRF.

Table 3.1: Max and mean error in mm for the EKF.

	Cov1		Cov2		Cov3	
	Max	Mean	Max	Mean	Max	Mean
Sim1	**0.078**	*0.025*	0.080	0.025	0.080	0.026
Sim2	1.681	0.550	**1.577**	*0.543*	1.910	0.661
Sim3	0.400	0.113	0.903	0.172	**0.079**	*0.027*

3.3 Results

The results here are short versions of [13] on simulated data, more results on experimental data can be found in [14]. The path in Figure 3.1 was used during the simulation. The simulation was made without errors (Sim1) with calibration errors, offset and model errors (Sim2) and with only calibration errors and offset (Sim3). Optimization of the covariance matrices were made on these three sets of simulated data, called Cov1, Cov2 and Cov3 in the rest of this text. All nine combinations of simulations and covariance matrices were used to estimate the tool position.

Figure 3.2 shows the true path for Sim1, Sim2 and Sim3 togheter with the estimated paths using the three sets of covariance matrices in each figure. Figure 3.2(a) shows that the estimated path for Cov1, Cov2 and Cov3 are very similar to the true path which is expected since Sim1 is without any errors. The estimates differ more in Figure 3.2(b) than in Figure 3.2(c) and the reason is model errors which is to be expected as well. Model errors are a big problem which in practice is inevitable. The observer must therefore be robust against model errors. Table 3.1 shows the maximum and the mean path errors for the estimations. The smallest maximum path error is indicated with bold numbers and the smallest mean path error is indicated with italic numbers for each set of simulated data. We can see that Cov1 gives minimum path errors for Sim1 and so on which is good since Cov1 is optimized on Sim1. But this is not a general result, sometimes can for instance Cov3 give a better estimation for Sim2 than Cov2. The conclusion is that the optimization gives a local minimum. The simulation in [13] was made on other types of paths as well. The estimations on these paths were made with the covariance matrices that are optimized for the path in Figure 3.1. The path errors for these paths are larger than for the path in Figure 3.1 which could imply that the covariance matrices are dependent of the states.

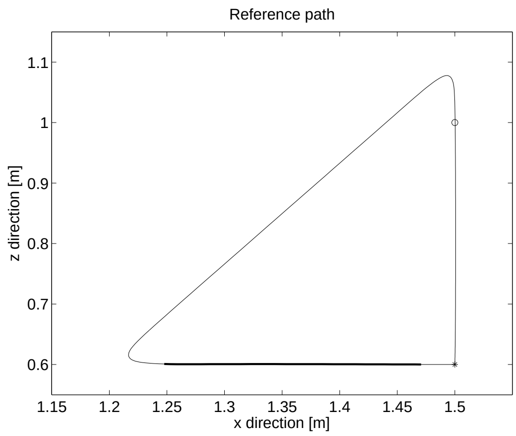


Figure 3.1: Reference for the path that has been simulated. The path starts at the star and the robot moves clockwise. The circle indicates the tool position for the zero-position, and the thicker segment of the path shows which part that is magnified in Figure 3.2, to see better.

4 Conclusions and Future Work

The prevoius work has so far introduced more problems than solutions as mentioned earlier. Most of these problems have not been solved yet and are put to future work. To begin with, the noise model for the process must be investigated in more detail. Now, the noise is an additive term independent of the states in (3.3a). A more accurate noise model could be

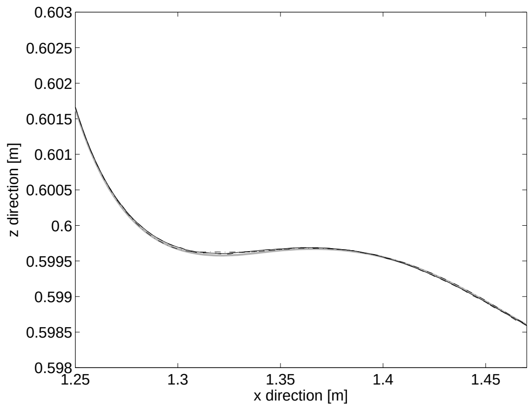
$$\dot{x} = \begin{pmatrix} x_3 \\ x_4 \\ M_a^{-1}(x_1)\big(-C(x_1,x_3) - G(x_1) - A(x) + v_a\big) \\ M_m^{-1}\big(A(x) + \kappa(x_4) + u + v_m\big) \end{pmatrix}, \tag{4.1}$$

with the discretized model in the form

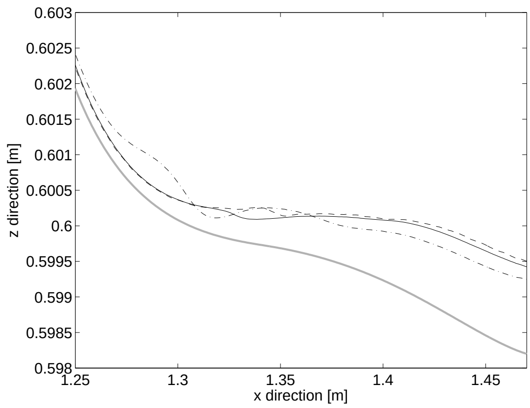
$$x_{k+1} = F(x_k, u_k) + B(x_k)v_k, \quad v_k \sim \mathcal{N}(0, Q_k). \tag{4.2}$$

This new model could reduce the state dependence in the covariance matrices as discussed in Section 3.3, since $B(x_k)$ is a matrix that depends on the states.

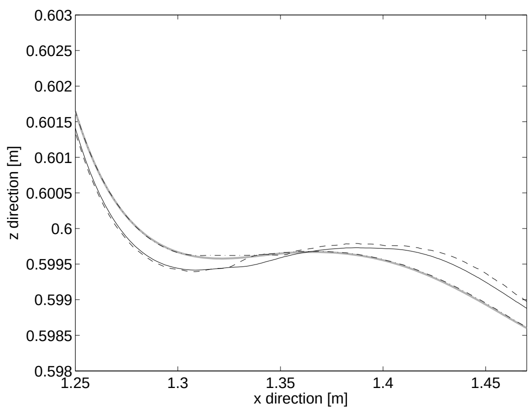
Another problem that has come up during the work is the discretization of the continious state space model. Euler forward has been used to discretize the model, but the errors that are intruduced can be significant for the EKF. One way to solve this is of course to use a more accurate discretization method which leads to more complex



(a) Sim1



(b) Sim2



(c) Sim3

Figure 3.2: Estimation of the three different sets of simulated datar with Cov1 (-), Cov2 (--) and Cov3 (-.). The grey line is the true path.

expressions. A simpler way can be to decrease the sample time during the time update (3.5), i.e., perform the time update several times during one iteration in the EKF.

One important thing to also investigate further is how the estimate is affected by the magnitude of the model errors, the calibration errors and the offset, which requires a structuralized sensitivity analysis.

How to choose the covariance matrices is crucial for the estimated result, and must be investigated more carefully. The proposed optimization method is one obvious choice but other directions should be analysed as well.

When the above problems have been solved it is natural to fully use the 6-DOF capabilities of the robot and extend the robot model to this case. The computational complexity of the EKF will increase even more and therefore other implementations based on numerical computation of the Jacobian should be tested. One such example is the Unscented Kalman Filter [18]. The sensor system could be extended as well and a first step would be to include a gyro to get a 6-DOF measurement. Another important issue is the robustness with respect to trajectory and configuration in work area, i.e., to investigate the need for observer gain scheduling. Robustness for different robot individuals and tools must also be further studied.

Finally, it is also important to do new experiments and validate the result more thorough. An important thing concerning experimental data is to validate the models that are used in the filter. This is not a problem during the simulations since it is the same models that are simulated and used in the filter with some differences in the parameter values.

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