Single Joint Control of a Flexible Industrial Manipulator using $\mathcal{H}_\infty$ Loop Shaping

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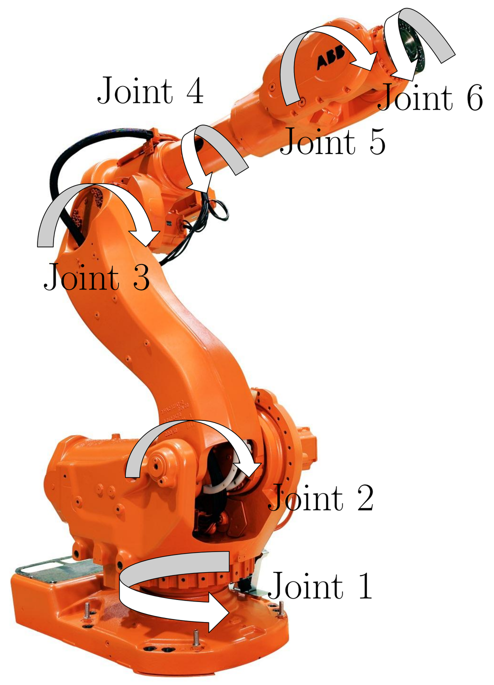
Contribution

Control of a flexible joint of an industrial manipulator using **1)** only actuator position, as well as, **2)** actuator position and acceleration of the end-effector, as measurements. The controllers are synthesized using $\mathcal{H}_\infty$ loop shaping and compared to an ordinary PID controller in simulation.

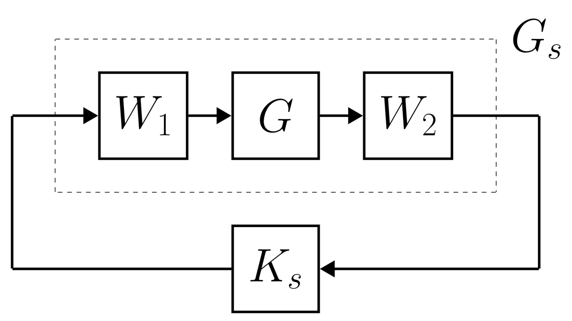
Background

1. Typical standard control configuration for industrial manipulators: actuator positions are the only measurements used.
2. As a result of the development of cost efficient manipulators the mechanical structure has become less rigid: need for new control structures have emerged.
3. To support the proposed control structures: necessary to introduce new sensors such as encoders, measuring joint position after the gearbox, and accelerometers, measuring the end-effector acceleration.



Loop Shaping using $\mathcal{H}_\infty$ Synthesis

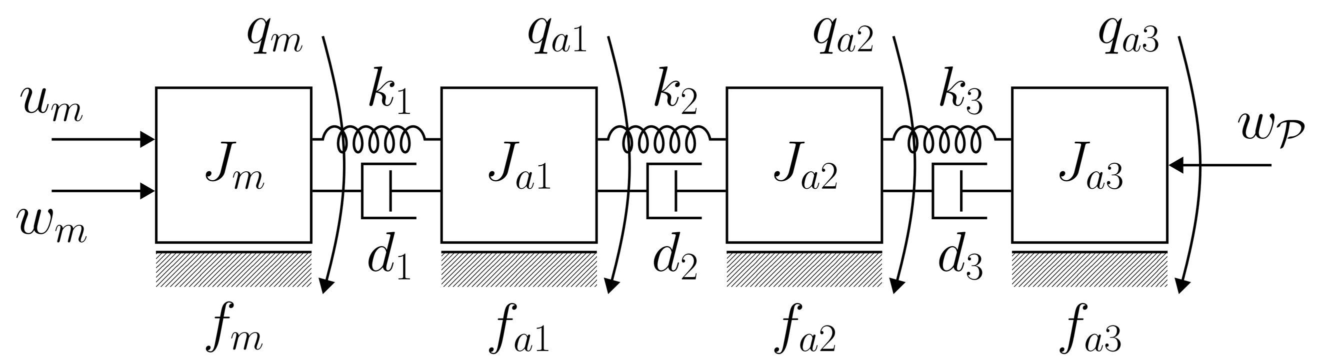
Loop shaping is a method where the plant $G(s)$ is shaped to look like the desired open loop $K(s)G(s)$. No account for model errors is used in the design. Instead a general error description is used in the synthesis step. The method can be summarized in four steps:



1. Pre- and post-multiply $G(s)$, such that
$$G_s(s) = W_2(s)G(s)W_1(s)$$
has the desired properties.
2. Calculate the controller $K_s(s)$ using left coprime factorization (`ncfsyn` in MATLAB).
3. The final controller is given by
$$K(s) = W_1(s)K_s(s)W_2(s).$$
4. If performance not satisfied, change $W_1(s)$ and $W_2(s)$.

Robot Joint Model

The joint model is a four-mass model of a single flexible joint. The joint corresponds to joint 1 of a serial 6 DOF industrial manipulator.



Torque disturbance signals on the motor torque w_m and tool position $w_\mathcal{P}$ excite the model. Measurements are the motor position q_m and the tool acceleration $\ddot{\mathcal{P}}$. A linear model with 8 states is used for controller synthesis.

Design of Controllers

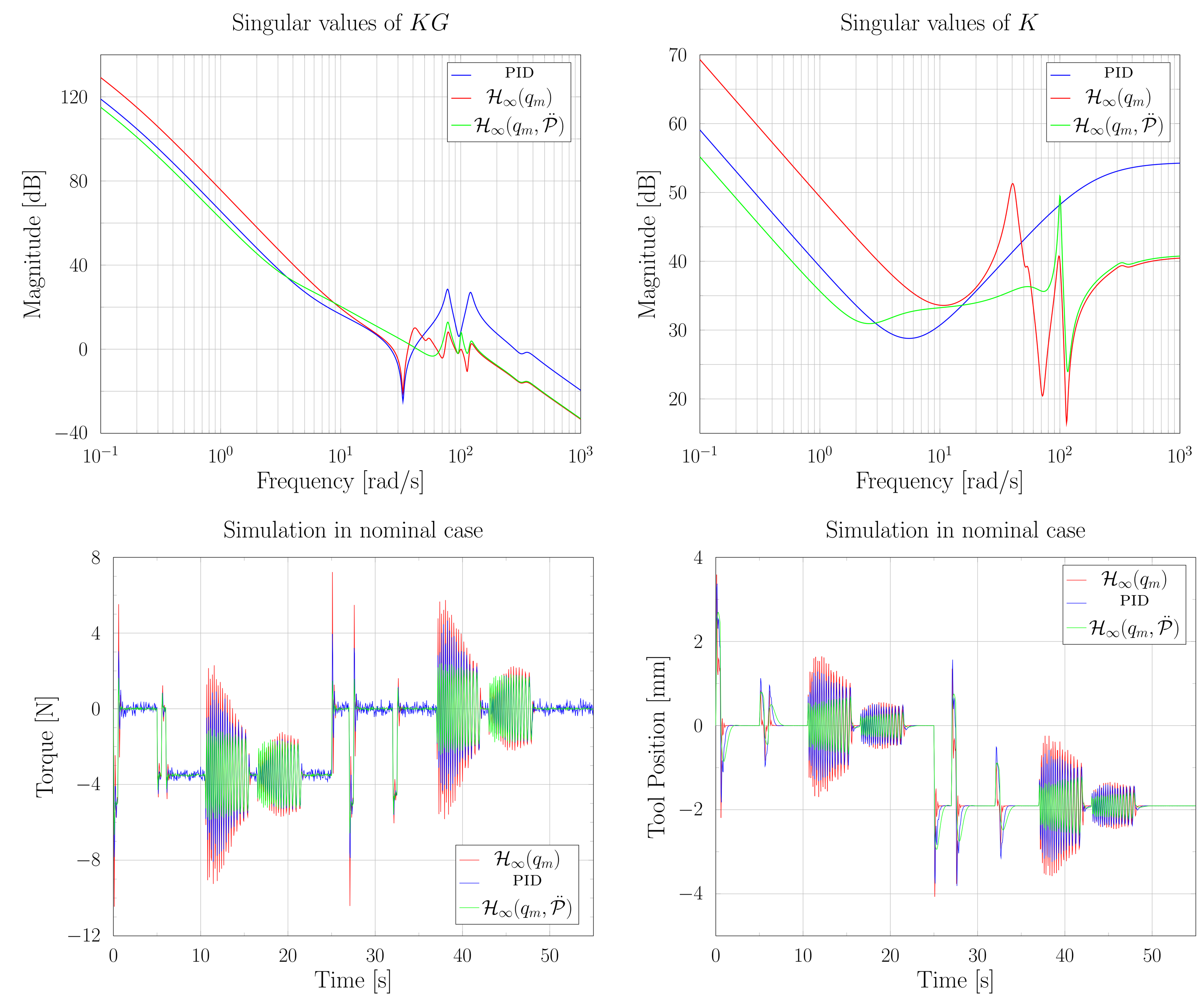
Two controllers are designed; 1) Loop Shaping using q_m, and 2) Loop Shaping using q_m and $\ddot{\mathcal{P}}$.

- **Loop Shaping using q_m ($\mathcal{H}_\infty(q_m)$):** SISO system with an integrator. Must have an integrator in K because disturbance model has an integrator.
$$W_1(s) = 1, \quad W_2(s) = 100\frac{s+10}{s}H(s).$$
- **Loop Shaping using q_m and $\ddot{\mathcal{P}}$ ($\mathcal{H}_\infty(q_m, \ddot{\mathcal{P}})$):** MIMO system with an integrator. Integrator in K placed in q_m channel. High gain in $\ddot{\mathcal{P}}$ channel, a LP-filter with gain < 1 required.
$$W_1(s) = 50, \quad W_2(s) = \text{diag}\left(\frac{s+3}{s}, \frac{0.2}{(s+5)^2}\right).$$

Results

The controllers are evaluated in a simulation model for

- Nominal conditions. Same model for simulation and controller synthesis.
- Gain error of 2.5 added, $G_p(s) = 2.5G(s)$.
- Time delay error of $T = 2\,\text{ms}$, $G_p(s) = G(s)e^{Ts}$. In the nominal case, the time delay is $T = 0.5\,\text{ms}$.
- Model order reduction of K.



- Nominal performance for PID and $\mathcal{H}_\infty(q_m)$ similar.
- For $\mathcal{H}_\infty(q_m, \ddot{\mathcal{P}})$, the nominal performance improves.
- Increased time delay does not affect the performance that much for the three controllers.
- Adding a gain error makes the motor torque from the PID controller oscillate.
- The gain error does not affect $\mathcal{H}_\infty(q_m)$ and $\mathcal{H}_\infty(q_m, \ddot{\mathcal{P}})$. It only makes the motor torque decrease.
- The model order of $\mathcal{H}_\infty(q_m)$ can be decreased a factor 2 without changing the performance significantly.
- Model order reduction for $\mathcal{H}_\infty(q_m, \ddot{\mathcal{P}})$ not working. An unstable closed loop system is obtained.

Future Work

- Investigate other sensors
  - Encoder measuring q_{a1}
- Use estimate of $\mathcal{P}$ in the controller
  - Estimated using EKF or PF
- Extended robustness analysis
  - Structured singular values
- More than one joint
  - Linearization in one position
    - Gain scheduling
    - Linear parameter varying (LPV) methods
  - Exact linearization