A Path Tracking Criterion for an LHD Articulated Vehicle

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Abstract

A path tracking criterion for the so-called LHD (Load-Haul-Dump) truck used in underground mining is proposed in this paper. It exploits the particular configuration of this vehicle, composed of two units connected by an actuated articulation. The task is to follow the path represented by the middle of the tunnel maintaining the whole vehicle at a reduced distance from the path itself, in order to decrease the risk of crashes against the walls of the tunnel. This is accomplished via feedback through the synthesis of an appropriate path tracking criterion. The criterion is based on keeping track of the distances of the midpoints of both axles of the vehicle from their orthogonal projections on the path, using two different moving frames simultaneously. Local asymptotic stability to paths of constant curvature is achieved by means of linear state feedback.

1 Introduction

In modern underground mining, the operation of transporting the ore from the stope to the dumping point is performed by a truck called LHD (Load-Haul-Dump) navigating through narrow tunnels. Compared to the usual work place for autonomous robots out of laboratories, the underground gallery is a semi-structured environment: the walls usually define in a clear way the limits of the free space and an accurate map of the whole gallery is often available a priori, describing the path in terms of its curvature. The simplified work environment, together with obvious safety reasons, motivates the numerous attempts to provide the mining truck of autonomous guidance capabilities, see [Hurteau 92, Juneau 93, Makela 95, Saint-Amant 88, Scheding 97]. Although well structured, the tunnel is usually very narrow: in the narrowest parts it can be only 10 % wider that the mining truck running into it. Therefore, it is critical for the autonomous navigation of the vehicle to have a reliable control of the lateral displacement with respect to the path, naturally represented by the center of the tunnel, in order to avoid crashes against the walls of the tunnel.

For an underground tunnel, the radius of curvature is the only measure easily attainable to describe the path and it is naturally given with respect to the length covered along the path. Cartesian coordinates can be obtained by integration but they are subject to drift errors and their usefulness is indeed doubtful since a gallery can have a length of several kilometers.

The LHD is an articulated vehicle composed of two bodies connected by a kingpin hitch. Each body has a single axle and the wheels are all non-steerable. The steering action is performed on the joint, changing the angle between the front and rear part by means of hydraulic actuators. Both the shape and the steering mechanism are intended to improve the curve negotiation skills of the vehicle. In fact, it is intuitively easy to understand that a vehicle without articulation, say a car-like vehicle, would be more cumbersome i.e. would span a larger area than the LHD when steering. How to explain and exploit this difference in mathematical algorithms for the problem of tracking a given path has been the subject of several research contributions [DeSanctis 97, Hemami 97, Polotski 97].

For a generic wheeled vehicle, the path following problem is a well studied problem, see [Canudas de Wit 97, Chapter 9] [Canudas de Wit 98]. It is essentially based on the idea that it is possible to define at each time instant a notion of *lateral distance* of the vehicle from the path and that the lateral dynamic associated with that distance can be studied independently from the longitudinal dynamic (expressing how fast the path is covered). All the several approaches proposed in the literature to solve the path following problem for wheeled vehicles are essentially based on a common feature: a point on the vehicle (the guidepoint) is selected and a tracking criterion is defined for this guidepoint. The task of the controller, then, is to have the corresponding tracking error converging to zero. Different selections of the guidepoint can be made according to the characteristics of the system. Focusing on the mining truck, if the sensor is a video camera [Hurteau 92] or a laser range finder [Juneau 93], it is convenient to take the midpoint of the front axle, where the sensor is normally mounted; alternatively the midpoint of the rear axle can be considered [DeSanctis 97]. Other selections can be the center of mass or some geometric center like the flat output of the system [Polotski 97]. In fact, the two-unit articulated vehicle was proven to be flat in [Rouchon 93] but, unlike the standard n-trailer, the flat output is not a physical point laying on the vehicle, therefore its usefulness for motion planning and trajectory following purposes is less straightforward. Similarly, the tracking error can be an euclidean distance between the guidepoint and the corresponding point on a virtual vehicle moving along the path according to some law [Kanayama 90] or a combination of angular and position errors (in [Hemami 98] it is based on tangent linearization of the error dynamics with the curvature of the path assumed as disturbance) or, as we will use below, a distance between a point and its orthogonal projection on the path [Samson 95]. Anyway, in all cases treated in the literature, the criterion is based on a single point of the vehicle without care of how much off-tracking the rest of the vehicle is accumulating with respect to the path. In fact, one of the main problems for the navigation of the truck is that the rear and front parts tends to follow different trajectories when the truck is bending. Take for example the case of the guidepoint on the midpoint of the rear axle as in [DeSanctis 97]: for a path composed of straight lines and arcs of circles, while passing form line to arc, the steering action will start only when the guidepoint reaches the nonzero curvature path but, by that time, the front part of the truck will be already far off the path if the curvature is high enough. Moreover, also at steady state along an arc of circle, the front point can have off-tracking if the two units of the vehicle have different lengths [Bushnell 94]. Also the kingpin hitch contributes to increase the difficulty of defining a suitable tracking law. In fact, choosing as guidepoint the articulation joint (that can correspond to another geometric center of the vehicle) is misleading because as soon as the steering angle is different from zero both units will follow a trajectory narrower than the guidepoint. Similar considerations and similar problems hold also for the other tracking errors mentioned above.

It has been shown by several authors [Sampei 91, Samson 95] that for a path described in terms of curvature as function of the curvilinear abscissa it is convenient to introduce a local frame, called the Frenet frame, moving along the path itself. The main property of this frame is that it decouples the longitudinal motion along the path from the lateral motion defined in terms of the distance of a prespecified point (the guidepoint) from its orthogonal projection on the path. The Frenet frame gives only a local representation i.e. it is well defined only in a "tube" around the path to follow or in a tunnel, as in our case. Usually, regularity assumptions are needed for the path in order to use the moving frame. However, the tunnel can also be build in such a way to be described by means of only line segments and arcs of circle i.e. by means of continuous path of discontinuous curvature.

Our problem can then be formulated as follow: we want to define an algorithm so that an articulated vehicle can follow a C^1 path in a stable way and with a reduced off-tracking i.e. with a reduced maneuvering space spanned by the vehicle around the path.

The proposed solution consists in redefining the tracking error of the path following problem not based only on one single distance but on the *sum* of the signed distances of the midpoints of both axles of the vehicle from their orthogonal projections on the path. A sum of distances is chosen instead of a quadratic function or of an infinity norm because of its underlying linear structure. Choosing distances with sign implies that the tracking error is not a norm; however a sign associated with the tracking error is needed in order to decide the direction of the steering input. Due to the presence of the nonholonomic constraints, the possible cancellations that can occur when the distances are signed but the whole state is out of its equilibrium point (see [Hemami 98] for a description of the different cases) correspond to transient "snapshots" of the dynamic evolution of the system out of its steady state (which is achieved only asymptotically) i.e. to passages through zero of the tracking error during the transient.

Stability can be proven locally for paths of constant curvature. Uniqueness of the solution follows from the controllability of the system that was proven for an articulated vehicle in [Laumond 93].

The emphasis on reducing the off-tracking has also a "dual" point of view: the space spanned by the vehicle is a critical factor in the construction of the tunnel. In fact it can be said that the tunnel width is determined by crash-safe considerations. Therefore, improving the exact knowledge of the trajectories in the critical situations like curves means being able to give more precise specs for the construction. Of course, making a larger tunnel costs more, so an "optimal" trajectory can help in reducing the width of the tunnel keeping the same safety constraints.

2 Kinematic model and Frenet frames

A typical configuration for a mining truck is the one shown in Fig. 1. Since the

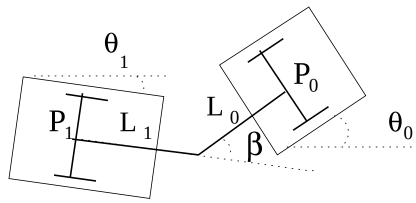

Figure 1: Two-unit articulated vehicle.

operative speed for such a vehicle is low enough, we will neglect the dynamic effects due to acceleration and braking as well as the slippage of the wheels and other effects deviating from the exact kinematic model. So, for example, we allow the two wheels of each axle to rotate independently and we implicitly assume that the steering action on the joint is coordinate with a differential drive on the actuated wheels. This is obviously "transparent" to our model if we make the assumption that the two wheels of each axle can be lumped together in the midpoint of the axle.

At kinematic level, we have two nonholonomic constraints acting on the front and rear axles, due to the assumption of rolling without slipping of the wheels. The corresponding one-forms are:

$$\begin{aligned} \dot{x}_0 \sin\theta_0 - \dot{y}_0 \cos\theta_0 &= 0 \\ \dot{x}_1 \sin\theta_1 - \dot{y}_1 \cos\theta_1 &= 0 \end{aligned}$$

where (x_i, y_i), $i = 0, 1$ are the cartesian coordinates and θ_i the orientation angles of the midpoints P_0 and P_1 of the axles of the vehicle.

From the geometry of the vehicle we can obtain a relation between the cartesian coordinates:

$$\begin{cases} x_0 = x_1 + L_1 \cos\theta_1 + L_0 \cos\theta_0 \\ y_0 = y_1 + L_1 \sin\theta_1 + L_0 \sin\theta_0 \end{cases}$$

where L_0 is the distance between P_0 and the articulation point and L_1 the distance between the articulation point and P_1.

We can associate to both P_i a velocity vector v_i and express the motion of P_i as:

$$\begin{cases} \dot{x}_i = v_i \cos\theta_i \\ \dot{y}_i = v_i \sin\theta_i \end{cases}$$

When only the kinematics of the vehicle is considered, the physical inputs of the system can be considered to be one of the speed vectors, for example v_1 if the actuated wheels are the rear wheels, and the rate of the angle between the two bodies of the vehicle where the hydraulic actuator is placed, i.e. the steering speed $\omega = \dot{\beta}$ where $\beta \triangleq \theta_0 - \theta_1$.

A set of variables that describes the configuration space of the truck is given for example by (x_1, y_1, θ_1) with the kinematic equations:

$$\begin{aligned} \dot{x}_1 &= v_1 \cos\theta_1 && (1) \\ \dot{y}_1 &= v_1 \sin\theta_1 && (2) \\ \dot{\theta}_1 &= \frac{v_1 \sin\beta - L_0\dot{\beta}}{L_0 + L_1\cos\beta} && (3) \end{aligned}$$

The relation between the angle θ_0 and θ_1 is simply $\theta_1 = \theta_0 + \beta$ and a dynamic equation for θ_0 can be obtained consequently:

$$\dot{\theta}_0 = \frac{v_1 \sin\beta + L_0 \cos\beta\dot{\beta}}{L_0 + L_1\cos\beta}$$

Alternatively, v_0 can be taken as input. From the geometry of the vehicle, the relation between the two speed vectors is given by:

$$v_1 = v_0\cos\beta + L_0\sin\beta\dot{\theta}_0 = \frac{(L_0 + L_1\cos\beta)\, v_0 + L_0L_1\sin\beta\dot{\beta}}{L_1 + L_0\cos\beta}$$

which explains the physical situation of v_0 and v_1 having opposite signs in case the rate of change of the steering angle is high enough.

The system (1)-(3) was proven to be controllable in [Laumond 93] using tools from differential geometry, like the rank of the Control Lie Algebra generated by the vector fields associated with the inputs.

The typical work environment for such a system is an underground tunnel of arbitrary length. A common and convenient representation for such an environment is in terms of a curvature function associated with the length of a curve representing for example the middle of the tunnel. Translating this into the cartesian coordinates of an inertial frame is not possible analytically because of the absence, except for trivial cases, of a closed form in the line integral expressing the length of the path covered. Furthermore, since the tunnel can be arbitrarily long, the use of an inertial frame is an awkward solution. To overcome these problems, different representations based on local coordinates have been proposed like the use of error coordinates [Kanayama 90] or the use of moving frames (see [Canudas de Wit 98] for a survey).

A particularly convenient local representation is given by a Frenet frame i.e. a frame moving on the path to follow (see [Sampei 91, Micaelli 93]). The main advantage of the Frenet frame is that it naturally decouples the lateral dynamics (expressing the distance of the point of interest from the path) from the longitudinal dynamics (i.e. the length covered along the path).

A point P in the plane is isomorphically described by a Frenet frame moving on a given path γ when the path itself is a sufficiently smooth continuous curve with a lower bound $r_{\gamma_{min}}$ in the radius of curvature and the point P is located at an absolute distance $|z_i| < r_{\gamma_{min}}$ from its orthogonal projection on the path (the origin of the Frenet frame). With "sufficiently smooth" path we mean that the path must be a simple curve $\in C^1$. The continuity of the curvature function is not required and so also simple paths, composed of straight lines and arcs of circle, can be considered. In our case we consider two Frenet frames moving on the curve to follow, corresponding to the projections on γ of the two points P_0 and P_1 of our vehicle. In our frames,

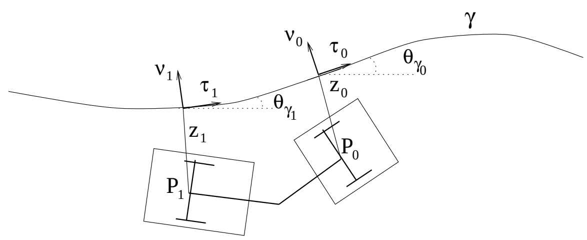

Figure 2: Frenet frames associated with P_0 and P_1.

we assume to have chosen a base with the conventions of Fig. 2 where $\vec{\tau}$ and $\vec{\nu}$ are the unitary vectors respectively tangent and normal to the oriented curve γ. Each of the curvilinear frames is represented by two coordinates $(s_{\gamma_i}, \theta_{\gamma_i})$ where s_{γ_i} is the curvilinear abscissa i.e. the line integral along the path to follow, up to the actual projection of the point P_i on γ:

$$s_{\gamma_i}(t) = s_{\gamma_i}(0) + \int_0^t \frac{v_i(\tau)\cos(\theta_i(\tau) - \theta_{\gamma_i}(\tau))d\tau}{1 - \kappa_\gamma(s_{\gamma_i}(\tau))z_i(\tau)}$$

and θ_{γ_i} is the orientation of the frame with respect to the inertial frame. In the Frenet frame, the point P_i is represented by the signed distance z_i between the point itself and its projection and by the relative orientation angle $\tilde{\theta}_i \triangleq \theta_i - \theta_{\gamma_i}$. The three equations describing the dynamics of the point P_i in the local frame are (see [Canudas de Wit 97] or [Micaelli 93] or any book on classical mechanics):

$$\begin{aligned}
\dot{s}_{\gamma_i} &= \frac{v_i\cos(\theta_i - \theta_{\gamma_i})}{1 - \kappa_\gamma(s_{\gamma_i})z_i} \\
\dot{z}_i &= v_i\sin(\theta_i - \theta_{\gamma_i}) \\
\dot{\tilde{\theta}}_i &= \dot{\theta}_i - \frac{v_i\cos(\theta_i - \theta_{\gamma_i})\kappa_\gamma(s_{\gamma_i})}{1 - \kappa_\gamma(s_{\gamma_i})z_i}
\end{aligned}$$

$i \in \{0, 1\}$. Each of the curvilinear frames is well defined essentially in a "tube" around the path to be followed. The width of this tube, as mentioned above, is at least $r_{\gamma_{min}}$ in the worst case, when the curvature of the path reaches its permitted maximum ($\max|\kappa_\gamma| = 1/r_{\gamma_{min}}$). Obviously, when the path is a straight line, the width of the tube tends to infinity. In order to assure well posedness of our control problem, we must have that $|z_i| < r_{\gamma_{min}}$, $i \in \{0,\, 1\}$. Convergence to the path assures that when the truck starts enough close to the path, then it remains inside the allowed region.

For this system, the main issue of the autonomous navigation is by far to *keep to the middle* of the tunnel, i.e. to keep control of the lateral dynamics. As seen above, the Frenet frame provides a natural way to describe the lateral displacement of a point from the path, since z_i represents the signed distance of P_i from its orthogonal projection on γ. This property has been used by several authors as tracking criterion for the path following problem. We will also use it, but redefining the tracking error as the sum of the two signed distances corresponding to the two points P_0 and P_1. In other words, we substitute the tracking criterion normally used $z_1 \to 0$ (or an equivalent one) with

$$z_0 + z_1 \to 0 \tag{4}$$

Using the Frenet frame, the control of the longitudinal dynamics, i.e. how fast the path is covered, becomes an independent problem and relatively secondary with respect to the lateral control. Therefore, it will be neglected in this study. This is equivalent to drop one of the degrees of freedom of the control design that is to say, in our case, to consider the input speed v_1 as a given (open-loop) function or more simply a constant value. Furthermore, also the dynamic equations for the length covered along the path can be neglected: in fact s_{γ_0} and s_{γ_1} enter into the model only when the curvature κ_γ is varying. Since the stability analysis of our path tracking algorithm will be carried out along an arc of circle, the model is completely independent from s_{γ_0} and s_{γ_1}. In order to keep track of both the distances z_0 and z_1 simultaneously, we must increase the dimension of the state. Choosing as state vector $\mathbf{p} \triangleq \left[z_0\ z_1\ \tilde{\theta}_0\ \tilde{\theta}_1\ \beta\right]^T$ and rearranging the dynamic equations we get:

$$\begin{aligned}
\dot{\mathbf{p}} \;=\; \begin{bmatrix} \dot{z}_0 \\ \dot{z}_1 \\ \dot{\tilde{\theta}}_0 \\ \dot{\tilde{\theta}}_1 \\ \dot{\beta} \end{bmatrix} &= \begin{bmatrix} \frac{L_1+L_0\cos\beta}{L_0+L_1\cos\beta}\sin\tilde{\theta}_0 \\ \sin\tilde{\theta}_1 \\ \frac{\sin\beta}{L_0+L_1\cos\beta} - \frac{(L_1+L_0\cos\beta)\kappa_\gamma(s_{\gamma_0})\cos\tilde{\theta}_0}{(L_0+L_1\cos\beta)\left(1-\kappa_\gamma(s_{\gamma_0})z_0\right)} \\ \frac{\sin\beta}{L_0+L_1\cos\beta} - \frac{\kappa_\gamma(s_{\gamma_1})\cos\tilde{\theta}_1}{1-\kappa_\gamma(s_{\gamma_1})z_1} \\ 0 \end{bmatrix} v_1 \\
&+ \begin{bmatrix} -\frac{L_0L_1\sin\beta\sin\tilde{\theta}_0}{L_0+L_1\cos\beta} \\ 0 \\ \frac{L_1\cos\beta}{L_0+L_1\cos\beta} + \frac{L_0L_1\sin\beta\kappa_\gamma(s_{\gamma_0})\cos\tilde{\theta}_0}{(L_0+L_1\cos\beta)\left(1-\kappa_\gamma(s_{\gamma_0})z_0\right)} \\ -\frac{L_0\cos\beta}{L_0+L_1\cos\beta} \\ 1 \end{bmatrix} \omega
\end{aligned} \tag{5}$$

or, in more compact form:

$$\dot{\mathbf{p}} = \mathcal{A}(\mathbf{p}) + \mathcal{B}(\mathbf{p})\omega \tag{6}$$

which emphasizes the presence of a drift term.

Clearly, considering both reference systems in P_0 and in P_1 gives a redundant description of the system. In order to complete this overparametrized state representation, one has to introduce three constraints expressing the fact that the vehicle is a rigid body. These three constraints are given by line integrals that depend on the geometry of the truck and on the curvature of the path between the two projections of P_0 and P_1 on the path. They are obviously holonomic i.e. they reduce the configuration space of the system down to the original number of variables (4 when the input is the steering speed). They cannot be expressed in a purely algebraic form since, except for the trivial cases, line integrals do not have a closed form expression. If E is a generic point of the body $[P_0,\ P_1]$ and dl is the increment along the body, they can be written as:

$$s_{\gamma_0} = s_{\gamma_1} + \int_0^{L_0+L_1} \frac{\cos\tilde{\theta}_E(l)dl}{1-\kappa_\gamma(s_{\gamma_E}(l))z_E(l)} \tag{7}$$

$$z_0 = z_1 + \int_0^{L_0+L_1} \sin\tilde{\theta}_E(l)dl \tag{8}$$

$$\tilde{\theta}_0 = \tilde{\theta}_1 + \beta - \int_0^{L_0+L_1} \kappa_\gamma(s_{\gamma_E}(l))\dot{s}_{\gamma_E}(l)dl \tag{9}$$

Eq. (7) is a line integral independent from the speed of the vehicle, function only of the position and orientation with respect to the path. In eq. (9), we have used the relation $dl = v_l\, dt$ where v_l assumes the meaning of the "velocity" with which the point E moves along the body of the vehicle. In the simplified case of a path that is a straight line corresponding, for example, to the x axis in cartesian coordinates ($s_\gamma = x$), the rigid body constraints simplify to:

$$\begin{aligned} s_{\gamma_0} &= s_{\gamma_1} + L_0\cos\theta_0 + L_1\cos\theta_1 \\ z_0 &= z_1 + L_0\sin\theta_0 + L_1\sin\theta_1 \\ \tilde{\theta}_0 &= \theta_0 = \theta_1 + \beta = \tilde{\theta}_1 + \beta \end{aligned}$$

The constraints correspond to the assumption of rigid body, so they continue to hold also in the case of $v_1 = 0$ i.e. no motion at all for the system.

3 Stability analysis

For a car-like vehicle, the path following with positive speed implies that the open loop equilibrium point is "naturally" stable whereas backward motion implies that the same equilibrium is open loop unstable (in fact, the former resembles a normal pendulum and the latter an inverted pendulum, as degree of difficulty of the control design). For a mining truck, the path following problem has always an unstable equilibrium point due to the steering action performed on the articulation joint.

We use Lyapunov linearization method to show that the system can be locally asymptotically stabilized to a path of constant curvature. Since the method is based on tangent linearization of the original dynamics of the system around the equilibrium point, looking at eq. (5) it is easily seen that the system has a steady state only when κ_γ is constant i.e. when the path is a straight line or an arc of circle. However, the simulations of Section 4 show that, in practice, the controller assures stability also with varying curvature. Moreover, the fact that linearization does not provide global results is not a limitation in our case since the mining truck has to navigate into a tunnel of reduced width and also the local frames are isomorphically defined only in a region around the path.

Proposition 1 *The system (6) can be locally asymptotically stabilized to a path of constant curvature by means of a linear state feedback.*

Proof

For a path of constant curvature κ_γ, the equilibrium point $\mathbf{p}_e$ can be calculated from the geometry of the problem. Both the relative orientation angles $\tilde{\theta}_i$ must be 0 since the points P_i have to rotate around circumferences concentric with the path. Moreover, we have to impose the condition $z_0 + z_1 = 0$ which will also identify uniquely the value of β at the equilibrium. We have then:

$$\mathbf{p}_e = \begin{bmatrix} z_{0_e} \\ z_{1_e} \\ \tilde{\theta}_{0_e} \\ \tilde{\theta}_{1_e} \\ \beta_e \end{bmatrix} = \begin{bmatrix} \frac{\kappa_\gamma\left(L_0^2-L_1^2\right)}{4} \\ -\frac{\kappa_\gamma\left(L_0^2-L_1^2\right)}{4} \\ 0 \\ 0 \\ \arctan\left(\frac{L_0\kappa_\gamma}{1-\kappa_\gamma z_0}\right) + \arctan\left(\frac{L_1\kappa_\gamma}{1-\kappa_\gamma z_1}\right) \end{bmatrix}$$

The Jacobian matrix calculated around $\mathbf{p}_e$ is:

$$A = \left.\frac{\partial \mathcal{A}}{\partial \mathbf{p}}\right|_{\mathbf{p}=\mathbf{p}_e} =$$

$$= v_1 \begin{bmatrix} 0 & 0 & \frac{L_1+L_0\cos\beta_e}{L_0+L_1\cos\beta_e} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -\frac{(L_1+L_0\cos\beta_e)\kappa_\gamma^2}{(L_0+L_1\cos\beta_e)(1-\kappa_\gamma z_{0_e})^2} & 0 & 0 & 0 & \frac{L_1+L_0\cos\beta_e}{(L_0+L_1\cos\beta_e)^2} - \frac{\left(L_1^2-L_0^2\right)\sin\beta_e\kappa_\gamma}{(L_0+L_1\cos\beta_e)^2(1-\kappa_\gamma z_{0_e})} \\ 0 & -\frac{\kappa_\gamma^2}{(1-\kappa_\gamma z_{0_e})^2} & 0 & 0 & \frac{L_1+L_0\cos\beta_e}{(L_0+L_1\cos\beta_e)^2} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

is a Hurwitz matrix i.e. the closed loop state matrix $A - v_1KB$ with $B \stackrel{\triangle}{=} \mathcal{B}(p_e)$ can be made stable by choosing an appropriate gain K.

$\diamondsuit$

It can be noticed that the stability of the closed loop system is independent from the sign of the speed vector v_1 which means that the vehicle can track the path in both directions of motion.

If a saturation is introduced on the steering angle β, then from the expression for the equilibrium point $\mathbf{p}_e$, it is possible to obtain a maximum value of the curvature κ_γ that can be followed at steady state with zero tracking error.

4 Simulations

In Fig. 3, the path to track is an arc of circle. It can be seen that convergence is achieved from a generic admissible initial condition. A similar behavior is obtained

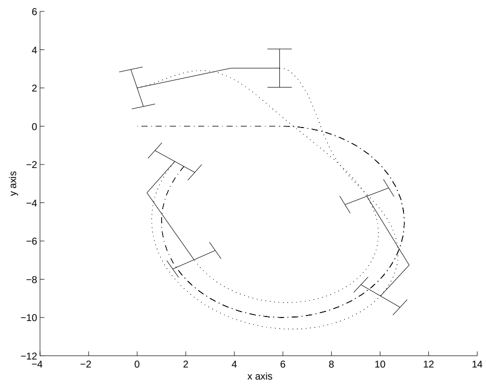

Figure 3: Following an arc of circle ($\kappa_\gamma = -0.2$).

with a negative speed (Fig. 4). The low off-tracking induced by the criterion (4)

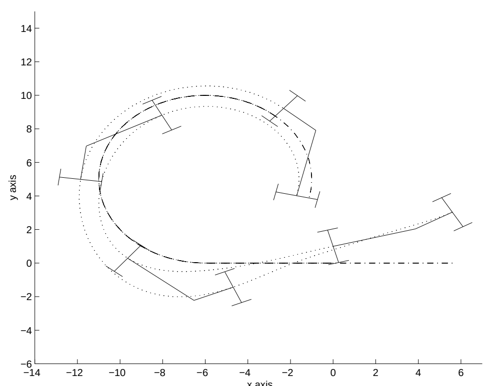

Figure 4: Reversing along an arc of circle ($\kappa_\gamma = 0.2$).

can be appreciated in Fig. 5 for a path composed of line-arc-line segments. Finally, Fig. 6 shows how it is possible to follow in a stable way also a path on nonconstant curvature, here a clothoid (κ_γ grows linearly).

5 Conclusion

How must a mining truck navigate into the narrow underground tunnels that constitute its ordinary work environment avoiding crashes against the walls? The answer to such a question is not so trivial since the standard path tracking methods do not

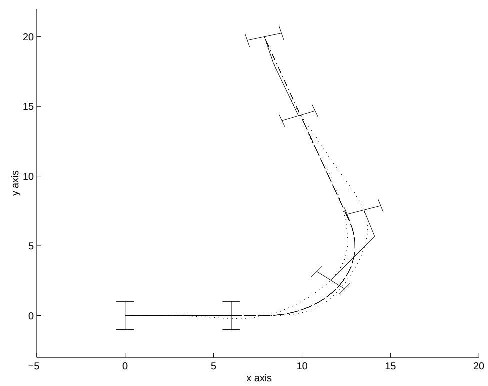

Figure 5: Following a line-arc-line path at steady state: the induced off-tracking is kept low.

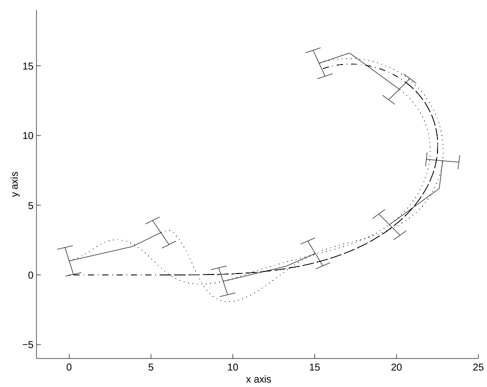

Figure 6: Following a clothoid.

take into account the off-tracking induced in some parts of the vehicle, nor they take advantage of the particular configuration of this articulated truck which is studied to have higher maneuverability than a standard wheeled vehicle.

Our proposed solution consists in considering a feedback law with tracking error based on the sum of the distances of both the units from the path. The distances are expressed using a couple of Frenet frames moving on the path. The moving frames provide a very natural description for the underground gallery usually represented in terms of a curvature function associated with the curvilinear abscissa giving the length covered along the path.

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