Homework # 2

PhD course. Systems Biology: nonlinear mechanistic models

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1. Equilibria and stability of a simple biochemical reaction network

Consider the biochemical networks given by the following reactions:

$$2X_{1} \xrightarrow{k_{1}} X_{2} + X_{3}$$

$$X_{2} + X_{3} \xrightarrow{k_{2}} X_{4}$$

$$X_{2} \xrightarrow{k_{3}} X_{4}$$

$$X_{4} \xrightarrow{k_{4}} 2X_{1}$$

$$(1)$$

- Write down the mass-action ODEs and describe the behavior of this network, for instance computing the equilibria and their stability character. You can use deficiency theory and/or the ERNEST Matlab toolbox¹. Add also comprehensive simulations of the behavior of the system.
- Describe what happens in the following cases:
 - (a) $k_3 = 0$ (i.e., 3rd reaction is absent);
 - (b) a constant inflow reaction

"outside"
$$\xrightarrow{k_5} X_2$$

is added to the network (1);

(c) the following reaction is added to the network (1)

$$2X_1 \xrightarrow{k_5} X_2$$

¹See my webpage, search for "ERNEST" under the publications, and download the toolbox. When you run it, the first lines of the result concern deficiency theory, then other, more complex tests are performed, which were not discussed in class.

2. Metabolic network and Flux Balance Analysis

The file MetabolicNetwork.mat contains a Matlab structure (called network) with the metabolic network of central metabolism of the bacterium *E.coli*. The stoichiometric matrix is the field network.S: columns correspond to reactions and rows to metabolites. The fields network.lower_bound and network.upper_bound contain lower and upper bounds on the reaction fluxes. The structure also has a "biomass" reaction, indexed by network.index_biomass. This reaction will be used as cost function to be used for the optimization of growth.

- How many conservation laws does the network has and what are they?
- What is the maximal biomass achievable by this model (i.e. what is the value of the cost function in the flux vector that solves the LP problem)?
- How many reactions are active at the optimum you found?
- Is the optimal flux vector unique? Can you write an algorithm that checks the existence of degenerate optimal solutions?
- FACULTATIVE: Can you even estimate how many degenerate optima there are?
- FACULTATIVE: A "knock-out" can be represented in this model as imposing 0 flux on a certain reaction. This can be obtained by putting lower and upper bound equal to 0 for that reaction. In some cases, when a reaction is missing the organism cannot grow (that reaction is hence called "essential" for the growth). How many essential reactions are there in the E.coli central metabolism?