Teaching portfolio

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Contents

Note: Course evaluations are supplied as separate documents

1 Teaching merits

1.1 Teaching and planning experience

I have been enrolled in higher education teaching since 2005, and have been responsible for the course Signals & Systems II (KTH EQ1100) twice, and Optimal Filtering (KTH EM3200/EQ2800) once. I have also been teaching assistant in the courses Signal Theory (KTH 2E1423), Radio Basics (KTH EQ1000), Optimal Filtering (KTH EM3200/EQ2800), Elektroprojekt del I (KTH EH1010), Elektroprojekt del II (KTH EH1020), and Digital Signal Processing (KTH EQ2300) multiple times. Further, I have co-supervised one now graduated Ph.D. student (Ph.D. J-O. Nilsson) [13, 14, 17, 34–37, 39, 41–43, 53], and am currently the de-facto supervisor of the three Ph.D. students (J. Wahlström [1, 4–6, 8, 9, 24, 26, 27], R. Larsson, and J. Trehag), and co-supervisor for H. Carlsson. I have supervised 8 Master's thesis, 1 Bachelor thesis, and 2 summer internship workers; the master thesis *Traffic Flow Enhancement through Guidance for Driver Based on GPS Aided Smartphones* by Frej Sojé-Berggren, was awarded the best student work by the Swedish Radio Navigation Board.

I have actively been enrolled in the development of the course material and structure of the course Signal Theory, Radio Basics, Elektroprojekt del I, Elektroprojekt del II, and Optimal Filtering. The work with the course development of Elektroprojekt del II was documented in the conference paper [33] on CDIO (Conceive-Design-Implement-Operate) pedagogics. Further, I have developed and taught parts of the Sensing and Perception module of the Autonomous Systems course within the newly launched Wallenberg Autonomous Systems Program (WASP). To develop my insight in teaching theory, I have participated in the pedagogical courses Learning and Teaching (KTH LH201V) and Research Supervision (KTH LH207V).

1.1.1 Course responsible

I have been course responsible with responsibility for lectures, homework assignments, and final exams for the courses

- Signals and Systems II, EQ1100, KTH, (*First cycle*), 7.5 credits, Fall 2010. (32 students)
- Signals and Systems II, EQ1100, KTH, (*First cycle*), 7.5 credits, Spring 2012. (55 students)
- Optimal Filtering, EM3200, KTH, (*Third cycle*), 10 credits, Fall 2014. (16 students)
- Optimal Filtering, EQ2800, KTH, (*Second cycle*), 6 credits, Fall 2014. (6 students)

The course Signals & Systems II (KTH EQ1100) was compulsory for all students at the electrical engineering and microelectronics programs at KTH Royal Institute of Technology. No course evaluations have been attached due to crashed course evaluation database; restoration of database has not been possible. Contact the Director of studies Prof. M. Jansson at `janssonm@kth.se` for more information.

The course Optimal Filtering (KTH EM3200) is a Ph.D. level course on optimal linear estimation principles, i.e., Wiener and Kalman filtering. The course analysis from the 2014 version of the course is attached as the separate document `Optimal_Filtering_Course_Eval.pdf`. The master level course Optimal Filtering (KTH EQ2800) is a reduced version of EM3200.

1.1.2 Teaching assistant

I have been teaching assistant, with responsibility for giving exercises and laboratory sessions, in the following courses.

- Radio Basics (EQ1000) KTH, (*First cycle*), 1.5 credits, 2007, 2014, 2015, 2016.
- Elektroprojekt del I (EH1010) KTH, (*First cycle*), 4.5 credits, 2016, 2017.
- Elektroprojekt del II (EH1020) KTH, (*First cycle*), 4.5 credits, 2011.
- Optimal Filtering (EQ2800) KTH, (*Second cycle*), 6 credits, 2008.
- Signal Theory (2E1423) KTH, (*First cycle*), 7.5 credits, 2006.
- Signal Theory (2E1423) KTH, (*First cycle*), 7.5 credits, 2007.
- Digital Signal Processing (EQ2300), KTH, (*Second cycle*), 7.5 credits, 2007.
- Project in Wireless Communication (EQ2440), KTH, (*Second cycle*), 12 credits ,2016.

As a teaching assistant I have also assisted the course responsible in the design of the laboratory, homework, and final-examination exercises, as well as correcting technical reports and exams.

1.1.3 Contribution to external courses

In addition to the earlier listed courses I have also taught the following parts of externally given courses.

- Signal processing for foot-mounted navigation, GETA Short course on Wireless Localization, Aalto University, Finland, 2012. ($\sim$ 20 students)
- Localization and positioning, Sensing & Perception module, WASP Autonomous Systems course, Sweden, 2016. ($\sim$ 50 students)

Both lecture and examination material have been developed. See attachment A.7 and A.8.

1.1.4 Courses development

I have been a part of the development of the course structure, learning outcomes, learning activities, examination moments, etc. for the following courses.

- Signal Theory (2E1423) KTH, 7.5 credits, *First cycle.*
  - The course Signal Theory (new name: Signal Processing EQ1240) gives a broad overview of modeling using stochastic processes in electrical engineering applications. To assist the students to recapitulate the theory before each tutorial, the material in Attachment A.2 was developed.
- Radio Basics (EQ1000) KTH, 1.5 credits, *First cycle.*
  - The course Radio Basics gives an introduction to simpler systems for analog and digital radio communication, with hands on experience in using simpler analog radio equipment. The instructions for the laboratory exercises that I have developed are attached in Attachment A.4.
- Elektroprojekt del I (EH1010) KTH, 7.5 credits, *First cycle.*

  - The course Elektroprojekt, del I is a compulsory project course in the first year of the electrical engineering program at KTH. The course introduces the students to project management, technical writing, and technical communication. Together with the course responsible J. Liljesköld and L. Gingnell, I developed a new type of project tasks for the 2017 version of the course. We also developed and introduced learning activities about peer-reviewing and video communication[1].

- Elektroprojekt del II (EH1020) KTH, 4.5 credits, *First cycle.*
  - The course Elektroprojekt, del II is a compulsory project course in the second year of the electrical engineering program at KTH. It involves teachers from several parts of the electrical engineering program and usually have approximately 45 students. In 2011, I and four other teachers developed a new version of the course using a CDIO based framework. The course development has been documented in the conference paper [33]. An example of the laboratory instructions that have been developed are attached in Attachment A.4.
- Optimal Filtering, EM3200, KTH, 10 credits, *Third cycle.*
  - The course Optimal Filtering is a Ph.D. course on optimal linear estimation principles. i.e. Wiener and Kalman filtering, both in continuous and discrete time. To help the students bridge the gap between the taught filter theories and the practical implementation of the filter algorithms, I developed a new project assignment; see attachment A.6. The assignment is about positioning of a vehicle using real-world GPS-receiver pseudo range measurement data. The use of real-world data trains the students in making engineering trade-offs when it comes to filter tuning and filter model selection.
- Autonomous Systems, WASP, 12 credits, *Third cycle.*
  - Autonomous Systems is a WASP Ph.D. program core course whose purpose is to give a broad understanding of the wide area of autonomous systems and the foundational knowledge in the topic areas required to understand and develop autonomous systems. The course consists of four modules Control and Decision, Sensing and Perception, Learning and Knowledge, and Interaction and Collaboration. The course was first given during the fall 2016 and I developed and taught the "Localization and Positioning" part of the Sensing and Perception module. The developed material includes lecture slides[2], a homework assignment (attachment A.7), and a project assignment (attachment A.8). The evaluation of the part of the course that I developed and taught is attached as the separated document `WASP_Eval.pdf`.

1.2 Experience as supervisor

1.2.1 Ph.D. supervision

Acted as a co-supervisor and de-facto[3] main supervisor for the following Ph.D. students.

Graduated

[1] An example of the outcome of the video communication learning activity can be found at `https://www.youtube.com/playlist?list=PL2v3tBVOPFedUx82Fs1hPz7P4-8FHYen6`

[2] `http://wasp-sweden.org/custom/uploads/2016/10/isaac-skog-localization-20161005.pdf`

[3] At the time of hiring the Ph.D students J. Wahlström and R. Larsson, I had not been appointed docent and could not be registered as a main supervisor.

- John-Olof Nilsson, (Ph.D. Signal Processing, KTH)
  - Thesis title: Infrastructure-free pedestrian localization
  - Main supervisor: Prof. Peter Händel
  - Period: 2008 – 2013
  - Joint publications: [13, 14, 17, 34–37, 39, 41–43, 53]

<u>Ongoing</u>

- Johan Walhström, (M.Sc. Engineering Physics, KTH)
  - Main supervisor: Prof. Peter Händel (De-facto supervisor: Dr. Isaac Skog)
  - Period: Feb. 2014 – (ongoing) (Date scheduled for Ph.D. defense: 2017-12-20)
  - Joint publications: [1, 4–6, 8, 9, 24, 26, 27]
- Robin Larsson, (M.Sc. Engineering Physics, KTH)
  - Main supervisor: Prof. Peter Händel (De-facto supervisor: Dr. Isaac Skog)
  - Period: Aug. 2014 – (ongoing) (50% activity level)
  - Joint publications: –
- Håkan Carlsson, (M.Sc. Engineering Physics, Chalmers)
  - Main supervisor: Assistant. Prof. Joakim Jaldén
  - Period: Mar. 2016 – (ongoing)
  - Joint publications: –
- Jacob Trehag, (M.Sc. Electrical Engineering, KTH)
  - Main supervisor: Prof. Peter Händel
  - Period: Jan. 2017 – (ongoing)
  - Joint publications: –

<u>Other</u>

Apart from formalized supervision efforts, after my Ph.D. degree I have supervised the following Ph.D. students.

- Dave Zachariah, Ph.D. Signal Processing, KTH, 2013.
  - Thesis title: Estimation for Sensor Fusion and Sparse Signal Processing
  - Main supervisor: Prof. Magnus Jansson
  - Joint publications: [13, 15, 32, 53]
- Ghazaleh Panahandeh, Ph.D. Signal Processing, KTH, 2014.
  - Thesis title: Selected Topics in Inertial and Visual Sensor Fusion: Calibration, Observability Analysis and Applications
  - Main supervisor: Prof. Magnus Jansson
  - Joint publications: [40]

1.2.2 M.Sc., B.Sc. and internship supervision

I have been supervisor for the following students during their M.Sc. thesis, B.Sc. thesis, and summer internship work.

M.Sc. supervision

- Johan Lovén, *Data Compression in a Vehicular Environment* (2014)
- Frej Sojé-Berggren, *Traffic Flow Enhancement through Guidance for Driver Based on GPS Aided Smartphones*[4] (2011)
- Sujatha Rajagopal, *Personal Dead Reckoning System with Shoe Mounted Inertial Sensors* (2008)
- Kristoffer Kjellgren, *GPS-aided Dead Reckoning Navigation* (2008)
- Claudia C. Meruane Naranjo, *Analysis and Modeling of MEMS based Inertial Sensors* (2008)
- Jon Gretar Gudjonsson, *Acustic localization in Wireless Sensor Networks* (2008)
- John-Olof Nilsson, *Navigation system for a micro-UAV* (2008)
- Adrian Schumacher, *Integration of a GPS aided strap-down inertial navigation system for land vehicles* (2006). Joint publication based on master thesis [48].

B.Sc. supervision

- Robert Lindberg, *Höjdkorrigerad inomhusnavigering* (2011)

Internship supervision

- Ioannis Karagiannis, *Smartphone based elevator positioning* (2015)
- Thomas Gaudy, *Smartphone based elevator positioning* (2015)

Parts of the outcome of the internship work is presented in the paper [3].

1.3 Pedagogical studies and development work

1.3.1 Courses in teaching and supervision pedagogy

As a part of the requirements at KTH to act as a teacher I have participated and passed the following higher education pedagogic courses.

- Learning and Teaching (LH201V) KTH, 7.5 credits, 2010.
- Research Supervision (LH207V) KTH, 3 credits, 2013.

The course Research Supervision (KTH LH207V) or equivalent is mandatory to be appointed Docent. The career plan for assistant professor at KTH specifies that higher education pedagogic courses corresponding to 15 credits or more should be completed.

1.3.2 Contributes to pedagogical conferences

The development of the Elektroprojekt, del II course (KTH EH1020) has been documented in the conference paper [33] on CDIO educational frameworks. The paper is included in Attachment A.3.

1.4 Summary of teaching experience

[4] Awarded the Swedish Radio Navigation Board (Radionavigeringsnämnden) yearly prize for best student work. `http://www.kth.se/ees/nyheterochpress/nyheter/studierna-visade-vagen-till-vinnande-app-1.288993`

Table 1: Summary of teaching experience between 2006 – 2017.

Time	Course	Nr stud.	Credits	Cycle	Language	Roll	Type of teaching
2006	Signal Theory (KTH 2E1423)	≈ 60	7.5	1	Swedish	Teaching assistant	Problem solving sections, labs
2007	Signal Theory (KTH 2E1423)	64	7.5	1	Swedish	Teaching assistant	Problem solving sections, labs
2007	Digital Signal Processing (KTH EQ2300)	N/A	7.5	2	English	Teaching assistant	Problem solving sections, labs
2007	Radio Basics (KTH EQ1000)	15	1.5	1	Swedish	Teaching assistant	Labs
2010	Signals & Systems II (KTH EQ1100)	32	7.5	1	Swedish	Course responsible	Lectures
2011	Elektroprojekt II (KTH EH1020)	≈ 60	4.5	1	Swedish	Teacher	Labs
2012	Signals & Systems II (KTH EQ1100)	55	7.5	1	Swedish	Course responsible	Lectures
2012	GETA Short course on Wireless Localization	≈ 20	1.5*	N/A	English	Invited lecture	Lectures
2014	Optimal filtering (KTH EM3200)	16	10	3	English	Course responsible	Lectures
2014	Optimal filtering (KTH EQ2800)	6	6	2	English	Course responsible	Lectures
2014	Radio Basics (KTH EQ1000)	22	1.5	1	Swedish	Teacher	Labs
2015	Radio Basics (KTH EQ1000)	50	1.5	1	Swedish	Teacher	Labs
2016	Radio Basics (KTH EQ1000)	26	1.5	1	Swedish	Teacher	Labs
2016	Project in Wireless Communication (KTH EQ2440)	6	12	2	English	Teacher	Supervision of project groups
2016	Elektroprojekt I (KTH EH1010)	≈ 60	4.5	1	Swedish	Teacher	Supervision of project groups
2016	Autonomous Systems (WASP Ph.D. program core course)	≈ 50	12*	N/A	English	Invited lecture	Lectures
2017	Elektroprojekt I (KTH EH1010)	≈ 60	4.5	1	Swedish	Teacher	Lectures, Supervision of project groups

* Only parts of the course has been taught by I. Skog.

2 Teaching approach

2.1 Theoretical knowledge

My insight in teaching theory is based on the pedagogical courses

- Learning and Teaching (LH201V) KTH, 7.5 credits, 2010, and
- Research Supervision (LH207V) KTH, 3 credits, 2013,

in which I have participated.

The aim with the course Learning and Teaching (KTH LH201V) is to teach how one may support and facilitate the students' learning in higher education. From the course, I have gained knowledge about: (i) how to formulate expected learning outcomes that are well adjusted to the educational context, and develop learning and assessment activities that support the student to reach the expected learning outcomes; (ii) how to integrate learning and assessment activities into courses so that they are in line with the objectives of the education program; (iii) how to use the course analysis as a tool for course development; and (iv) how plagiarism can be prevented. This theoretical knowledge was converted into practice when I, in collaboration with colleagues at the Electrical Engineering program, developed the course Elektroprojekt del II (KTH EH1020). The course development is described in the conference paper [33]; see attachment A.3. The gained theoretical knowledge about how to formulate expected learning outcomes, and develop learning and assessment activities, was once again converted into practice in the course development I did within Autonomous System course of the newly launched Wallenberg Autonomous Systems Program (WASP) Ph.D. education program; see attachments A.7 and A.8.

The aim with the course Research Supervision (KTH LH207V) is to teach how to reflect on and develop in the role of research supervisor. From the course, I have gained knowledge about: (i) the framework for doctoral education given by the Higher Education Ordinance and the KTH policies; (ii) methods for organizing supervision and using the individual study plan as a guiding instrument; and (iii) how the relation between student and supervisors can be clarified, as well as how gender and ethnicity may affect it.

2.2 Basic teaching outlook

2.2.1 Teaching outlook

What I see as my main goal when designing teaching and learning activities at the undergraduate level is that students should reach such a knowledge level that further knowledge retrieval is done out of curiosity and not because of external motivation criterion such as exams and homework. That is, to make the students feel an intrinsic value in gaining a deeper understanding of various electrical engineering and signal processing phenomena.

This goal is based on the motto that *increased knowledge about a subject creates an increased interest in the topic*[5], and an increased interest usually leads to the gathering of more knowledge, further increasing the interest in the topic, and so on. My task as a teacher is thus, to help the student to enter and take the first steps in this interest-knowledge spiral by assisting the student to gain an initial interest in the topic, and create learning activities that assist the student in the acquisition of new and deeper knowledge within the topic.

To create an initial interest and motivation for the student to start acquiring knowledge, I usually try to: (i) show how electrical engineering techniques, and especially signal processing techniques, are used in consumer electronics like Smartphones, Digital TV, GPS navigators, etc.;

[5] J. Biggs, and C. Tang, Teaching for Quality Learning at University, Third Edition, McGraw-Hill, 2007.

and (ii) convey that all students can achieve the course objectives, but that it may require a lot of work.

Two examples of how I have applied this strategy are found in the course Radio Basics (KTH EQ1000) and the project course Elektroprojekt II (KTH EH1020). In the course Radio Basics (KTH EQ1000), I developed a laboratory exercise (See attachment A.1) where the students are introduced to some of the engineering problems faced when designing, e.g., a WIFI access point, by solving different tasks using (analog) communication radios. In the project course Elektroprojekt II (KTH EH1020) where the students build their own loudspeaker, I together with M. Bengtsson designed a laboratory exercise (See attachment A.4), where the students through signal processing can try to enhance the sound quality of the load speakers the design in the course.

2.2.2 Supervision outlook

What I see as my main goal as a supervisor is to foster an independent researcher confident in both theoretical and experimental research. Since all Ph.D. students are individuals who require different types of learning activities and support to become independent researchers, the supervision has to be adapted to the individual. But I have a few general guidelines that I try to follow in my role as a supervisor. These are: (i) explain to the student that the goal of the Ph.D.-education is to make him or her an independent researcher, and to make sure that the student understands what that means and how that relates to the goals stated in the individual study plan; (ii) invest a lot of time in the beginning to get the Ph.D.-student on the right track, as this will save time in the long run; (iii) start off with some low-risk project where I as a supervisor am pretty sure of the outcome, and then as the students' research skills develop, target a high-risk project with a more uncertain outcome; (iv) get the student at an early stage involved in the process of scientific writing and use the writing process as a learning activity for training skills such as formulating research questions, designing experiments, evaluating results, etc.; (v) provide prompt feedback on the students deliverables; and (vi) stimulate student-to-student interaction, through, for example, joint publications and seminars, so that senior students (that hopefully are closer to becoming independent researchers) can foster newer students.

I implemented the ideas of starting of with a low-risk project and using the writing process as a learning activity for training skills such as formulating research questions, designing experiments, etc., for the first time in the supervision of J. Wahlström. He was given a problem I had partly studied within my spin-off company and, within less than 8 months after his start as a Ph.D. student, submitted the journal paper [8].

To foster the Ph.D.-student in both theoretical and experimental research, I find it highly important to get the student involved in research projects where hardware is built and real-world experiments are conducted. This assists the student to: (i) map theory and practices, and learn to make educated engineering tradeoffs; (ii) pose new and relevant research questions from the problems encountered in the prototyping and the experiments; and (iii) assess the strengths and weaknesses of both theoretical and experimental results presented in scientific publications.

An example of how I have implemented this in practise is in the supervision of Ph.D. J-O. Nilsson, where he and I developed the first version of the OpenShoe research platform [35]. Based upon the experimental results obtained with the OpenShoe platform he identified several limitation with commonly used concept in foot-mounted inertial navigation, see [36].

2.3 Teaching proficiency

2.3.1 Course responsible

After being responsible for the course Signals & Systems II (EQ1100) twice and the course Optimal Filtering (EM3200 and EQ2800) once, I am now more relaxed when giving lectures and more responsive to feedback that the students give. I have also gained a better understanding of the course administration (homepage, course registrations, lab registrations, etc.). During my work with the design of the homework assignments and final exams I have, thanks to the pedagogical course Learning and Teaching (LH201V), learned to better align the learning activities and the assessment of the intended learning outcomes.

In 2010, when I for the first time gave the course Signals & Systems II, I did not do any course development, since I was fully occupied with getting to grips with the whole course and its structure. In 2012, when I gave the course a second time, I introduced multiple choice concept questions during the lectures to assess if the students had assimilated the taught theories; see Attachment A.5 for an example of the questions used.

In 2014, when I for the first time taught the course Optimal Filtering, I only did some minor course development; I was fully occupied with reading up on the course content. The course has been running for several years and has a well-developed set of lectures and homework assignments. The course content is mainly theoretically, and I felt that the students had a hard time to map the taught theories to real-world engineering problems. (This was also mentioned in the feedback from students in the course analysis, see separately attached course evaluation.). I therefore developed the a new project assignment, see Attachment A.6, that try to illustrate how the taught theories relate to the functioning of a GPS-receiver.

2.3.2 Teaching assistant

My skills as teaching assistant, i.e., to give problem solving sections, correcting homework assignments, and evaluating laboratory reports, have developed in parallel with that I have gone from being a novice to becoming an expert in the field of signal processing. Today, when presenting problem exercises, I can better map them to real-world problems and point out the fundamental underlying theories and concepts to the students. Thanks to the course Learning and Teaching (KTH LH201V), I have gained a better understanding on how to provide timely and constructive feedback on homework assignments and laboratory reports.

2.3.3 Ph.D. supervision

Thanks to the course Research Supervision (KTH LH207V), I have gotten a better and wider picture of the role of the Ph.D. education and the Higher Education Ordinance. Further, I have started to use the individual study plan as an instrument to structure the Ph.D. supervision and clarify the different educational steps to the Ph.D. students. In the interaction with the Ph.D. students, I am now more aware of the asymmetric power relationship between the supervisor and the student, and how it changes with time. My skills in providing constructive and timely feedback have developed. The quality of my supervision is manifested through the number of publications with supervised and assisted Ph.D. students as first author, [1, 2, 4–6, 8, 13, 15, 24, 26, 27, 32, 35–37, 40–42, 53].

2.3.4 M.Sc. and B.Sc. supervision

Today, thanks to the experience I have gained from the supervision of eight master's and bachelor's thesis, I'm much more structured as supervisor and can more efficiently steer the student's work efforts to where it is needed. Further, from the course Learning and Teaching (LH201V), I have gained insight on how to provide timely and constructive feedback to the thesis worker, and how the M.Sc. thesis work assessment criterions map to the intended learning outcomes; especially the intended learning outcomes on the education program level. I have also become more aware of plagiarism and how to work against it. The quality of my supervision is manifested by the fact that the thesis work *Traffic Flow Enhancement through Guidance for Driver Based on GPS Aided Smartphones* by Frej Sojé-Berggren, was awarded the Swedish Radio Navigation Board (Radionavigeringsnämnden) yearly prize for best student work[6].

[6] http://www.kth.se/ees/nyheterochpress/nyheter/studierna-visade-vagen-till-vinnande-app-1.288993

A Appendices

A.1 Laboratory instructions for Basic Radio Course, EQ1000

EQ1000 Grundläggande radioteknik - övning 1

May 8, 2008

EQ1000 Lärarinstruktion

- Det poängteras att det är en gruppövning med syfte att träna praktiskt handhavande av radioutrustningen och att uppgifterna enklast löses via radio. Två grupper (GRUPP1 och GRUPP2) kommer att mot varandra och det gäller att vinna. I den första övningen ska dessa två grupper formas. Övningen innebär *i.)* Förflyttningar inom KTH Campus mha karta *ii).* Användande av radio för att lösa uppgifterna *iii.* Lärarkanal för "nödfall" kanal FM9 gäller hela dagen. I övrigt är all information fri. Det kan även finnas andra användare (respektera dessa).
- Studenterna delas upp i grupper om normalt 2 personer och förses med varsin handapparat (slumpvis kanal och modulationssätt, låg uteffekt) samt en karta visande gruppens första mål. Grupperna skickas (*en grupp i taget*) iväg till sina samlingspunkter - grupper med längst avstånd skickas iväg först.
- återsamling GRUPP1: vån8Qhus samt GRUPP2: husQvån8, det vill säga samma lokal.
- c. Ledtråden ovan ger återsamling i gemensam lärosal. Den grupp som kommer först med alla medlemmar får bonus i form av längre antenner. d. Återkoppling: handskakning, klart slut, frekvensplanering, störningar e. Kaffe

GRUPP1: vån8Qhus

v s	h 8	n u	Q	å
1 8	6 4	3 7	5	2

GRUPP2: husQvån8

h 8	u n	så	Q	v
1 8	2 7	36	4	5

1

EQ1000 Instruktioner till grupp om 2 studenter

Välkommen till övning nr 1. Nedan finns en karta över KTH campus. På kartan finns en plats markerad. Bege er dit *skyndsamt* för att att inhämta ytterligare instruktioner.

OBS! Om problem uppstår, kontakta övningsledarna på kanal 9 FM.

Lycka till!!!

EQ1000 Geografiskt utplacerad instruktion

Nedan har ni blivit tilldelade en grupptillhörighet (GRUPP 1 eller GRUPP 2), ett antal bokstäver/siffror, samt deras placering i ett ord (tex har bokstav *i* placering nr 2 i ordet *signal*).

Ordet som söks beskriver gruppens återsamlingsplats, till vilken ni ska bege er så fort som möjligt.

För att lösa uppgiften har ni blivit tilldelade en radio per person. Använd radion för att försöka komma i kontakt med era gruppkamrater. Utbyt sedan information med varandra för att få fram er grupps återsamlingsplats. Hur gör ni detta? Tänk på att radion har 40 olika kanaler och två modulationssätt - hur kommer ni överens och hur låter ni bli att störas av den andra gruppen?

OBS! tänk på att kanal FM9, endast får användas vid kontakt med övningsledarna.

Den grupp som först lyckas samla alla medlemmar på avsedd plats vinner!

Grupp:

Bokstäver:

Bokstävernas placering i ordet:

Återsamlingsplats:

Namn 1:

Namn 2:

Fyll i era namn och ta med lappen till samlingsplatsen!

EQ1000 Grundläggande radioteknik - övning 2

May 9, 2008

EQ1000 Lärarinstruktion

- minst 1 stycken vid ”basen” på parkeringen
- 15 minuter att organisera grupperna.
- 30 minuter för lösa uppgiften.
- KTH-frågor geografiskt i kluster. Flest rätt svar då tiden är ute vinner.
- Rättning och återsamling
- Återkoppling: Samband, repeater, coopcom, avlyssning, räckvidd. Speciellt antennens påverkan på räckvidden.
- Den grupp som har flest rätt får en basstation.
- Avbrott för lunch, med återsamling
- : tidsbudget: intro: 15 minuter, genomförande 30 minuter: återsamling och rättning: 15 minuter, genomgång 30 minuter: totalt: 1.5 timme.

EQ1000 Instruktioner

Välkommen till övning nr 2. Ni har nu 15 minuter på er att organisera er inom gruppen och diskutera hur ni på ett effektivt sätt ska lösa följande uppgift.

Uppgift

Gruppen ska börja med att välja minst en person som ska stanna kvar vid "basen" medan de andra kommer att samla in information på campus. Personen vid basen får uppgifterna om 15 minuter, övriga bör då vara väl spridda på campus!

Personen vid basen kommer att tilldelas ett frågeformulär med ett antal frågor rörande KTH campus. Basen ska sedan via radio dirigera springarna (förmedla frågorna) så effektivt som möjligt för att på kortast möjliga tid besvara så många frågor som möjligt (max tid är 30 minuter). Varje rätt svar ger 2 poäng.

All kommunikation mellan springarna och basen får endast ske via radio och frågeformuläret får inte lämna basen. Vidare gäller det att

- tiden startar då basen får frågeformuläret
- tiden stoppas då hela gruppen är åter i övningssalen.

För varje minut gruppen överstiger maxtiden om 30 minuter dras 3 poäng av från poängsumman.

OBS, Tänk på att strukturera radiokommunikationen på ett effektivt sätt och att det är möjligt för den andra gruppen att avlyssna er kommunikation. Om problem uppstår, kontakta övningsledarna på kanal 9 FM.

Lycka till!!!

Frågor och svars-formulär

1. När är klockan i klocktornet gjuten?
2. Hur många bänkar finns det utanför Lindstedtsväg 11?
3. Mellan vilka tider är grindarna mellan KTH campus och Sophiahemmet stängda?
4. Hur många lindar finns det längs cykelbanan utanför Drottning Kristinas väg 2?
5. Vilken planet finns det en model av på KTH?
6. Vilken typ av brandssläckare finns längst in i bibliotekets källare?
7. Vem har gjort det obskyra konstverket på södersidan av byggnaden som hyser Marcus Wallenberg laboratoriet?
8. Hur många fönster finns på kemis innergård (avrunda till närmsta tiotal)?
9. Hur många gula "ventilationstuber" finns utanför Teknikringen 50?
10. KTHs vaktmästare är väldigt miljömedvetna, därför komposteras allt trädgårdsavfall. Vilka typer av komposter finns? (*Tips: Komposten ligger bakom V huset.*)
11. Vanligen brukar klockor vara runda, men vilken form har klocka utanför KTH-hallen?
12. Hur många P-platser finns utanför KTH-hallen?
13. Vem ligger begraven utanför maskinsektionens lokaler (Brinellsväg 85)?
14. Mellan Q-byggnaden och Albanova finns ett dagis. I tillhörande lekplats finns ett antal bildäck. Hur många bildäck är det?
15. På plan 2, i Albanovas stora traphuset finns en märklig tavla av Karin Granquist. Vad föreställer tavlan?
16. Glass är gott, men vilka glassar finns till försäljning på restaurangen syster och bror?
17. Vad heter Försvarshögskolans och Utrikespolitiska institutets bibliotek?
18. Mellan vilka tider är Tåggrillen öppen?
19. Vilka öppetider har kårbokhandeln?
20. Vilken adress låg ingången till KTHs gamla stolthet kärnreaktorn R1 på (akta strålningen!). Tips: numera står det korrosionslära på dörren och huset är brungult.
21. Vad kostar det att parkera utanför Q. (svaret är inte "350kr i böter")
22. När byggdes L-huset (Lantmäteri)? Tips: informationstavla på huset.

23. Det finns många som tycker om träd på KTH men nog inga andra som tycker om träd så mycket som Svenska träskyddsföreningen. De har lokaler långt ned på Drottning Kristinas väg. Vilken färg har deras logga?

24. För att vara så kort har Teknikringen ganska många nummer. Vilket är det sista numret på den gatan och vad finns där?

25. Det finns 3 st fontäner på KTH, alla med någon form av utsmyckning. En har dock en något målat (skrivet) i botten. Vad står det? Tips: Nymbl... Bonus: 1p för varje extra fontän utöver 3 som hittas.

26. Vad för registreringsnummer har PQ-bussen (Promenadorquesterns buss)?

27. SL har visst bestämt sig för att tåg är rymningsbenägna. För att stävja rymmandet bland tågen har de nu satt upp ett nytt högt mellan KTH och bangården vid Östra Station. Hur många taggtrådar har detta stängsel på sig?

28. KTH har hittills endast haft en nobelpristagare. Denna har ett laboratorium uppkallat efter sig. Vilken adress ligger detta laboratorium på?

29. Den kortaste vägen från Q till Albanova med icke bemotrade fortskaffningsmedel går via en bro över Roslagsbanan. Denna bro är belaggd med plankor, hur många i bredd?

30. Under senare år har ytterligare en bro byggds över Roslagsbanan uppe vid Maskin. Hur många bropelare stöttas den bron av?

Facit

Fråga	**Svar**
När är klockan i klocktornet gjuten	1927
Hur många bänkar finns det utanför Lindstedtsväg 11	8 st
Mellan vilka tider är grindarna mellan KTH campus och Sophiahemmet stängda.	20.00–06.30
Hur många lindar finns det längs cykelbanan utanför Drottning Kristinas väg 2.	6 st
Vilken planet finns det en model av på KTH.	Venus
Vilken typ av brandssläckare finns längst in i bibliotekets källare? Pulver-, skum- eller vatten-släckare.	Pulversläckare
Vem har gjort det obskyra konstverket på södersidan av byggnaden som hyser Marcus Wallenberg laboratoriet?	Olov Tällström
Hur många fönster finns på kemi's innergård (avrunda till närmsta tiotal)	ca 80 st
Hur många gula "ventilations tuber" finns utanför teknikringen 50?	3 st
KTH's vaktmästare är väldigt miljömedvetna, därför komposteras allt trädgårdsavfall. Vilka typer av komposter finns? (Tips: Komposten ligger bakom V huset.)	Löv, gräs och eventuellt grenar.
Vanligen brukar klockor vara runda, men vilken form har klocka utanför kth hallen?	Fyrkantig
Hur många P-platser finns utanför kth hallen?	35 st (alt. 30 st)
Vem ligger begraven utanför maskinsektionens lokaler (Brinellsväg 85)?	Godtycklig nollan
Mellan Q-byggnaden och Albanova finns ett dagis. I tillhörande lekplats finns ett antal bildäck. Hur många bildäck är det?	11 st
På plan 2, i Albanovas stora traphuset finns en märklig tavla av Karin Granquist. Vad föreställer tavlan?	Kludd
Glass är gott, men vilka glassar finns till försäljning på restaurangen syster och bror?	88 an, Nogger, Magnum, Piggilin och Lakrits puck.
Vad heter försvarshögskolans och utrikespolitiska institutets bibliotek?	Anna Lind biblioteket.
Mellan vilka tider är tåg grillen öppen?	xxxx

EQ1000 Grundläggande radioteknik - övning 3

May 5, 2008

EQ1000 Lärarinstruktion

- basstation: Två personer per grupp (med radio) stängs in i forskningslabbet respektive klocktornet med tillgång till basstation. samtidigt lämnas 2 personer ur andra gruppen på Borggården respektive Q-parkeringen, med instruktioner.
- När basstation och sammanställare är på plats så släpps övriga studenter i väg från lärosalen.

	sammanställare	basstation	övriga
GRUPP1	Borggården	Forskningslabb	väntar i sal
GRUPP2	Q-parkering	Klocktorn	väntar i sal

EQ1000 Övning 3

Instruktioner till sammanställaren för Grupp 1

Fyll i rutnätet nedan med rätt bokstav på rätt plats. Er basstation har information om 50 st av bokstäverna. Motståndargruppen har information om de resterande 50 bokstäverna. Er basstation vet inte vad som ska göra, så den måste informeras. Er bas lyssnar på kanal 10 FM. Några tips:

1. Försök undvika att motståndarlaget lyckas snappa upp information om era bokstäver.
2. Det är tillåtet att agera störsändare för att försvåra motståndarnalagets kommunikation. Tänk dock på att ni samtidigt vill komma åt motståndarlagets information, vilket görs genom att lyssna på deras sändningar.

OBS !!! De två personerna i "sammanställar"-gruppen samt detta papper får på inget vis lämna borggården innan övningen är slut. Övriga lagmedlemmar får dock röra sig fritt på KTH-campus, dock ej bege sig till sin egen basstation.

Poängbedömning sker enligt följande:

- Korrekt överförd bokstav från egen basstation, utan att motståndarlaget lyckas snappa upp (avlyssna) bokstaven ger 1 poäng.
- Uppsnappadet av motståndarlagets bokstav, samtidigt som ni lyckas förhindra deras förmedlingen av bokstaven mellan deras basstation och informationssammanställare ger 1 poäng.

EQ1000 Övning 3

Instruktioner till sammanställaren för Grupp 2

Fyll i rutnätet nedan med rätt bokstav på rätt plats. Er basstation har information om 50 st av bokstäver, motståndarlaget har information om de resterande 50 bokstäverna. Er basstation vet inte vad som ska göra, så den måste informeras. Er bas lyssnar på kanal 25 FM. Några tips:

1. Försök undvika att motståndarlaget lyckas snappa upp information om era bokstäver.
2. Det är tillåtet att agera störsändare för att försvåra motståndarna lagets kommunikation. Tänk dock på att ni samtidigt vill komma åt motståndarlagets information, vilket görs genom att lyssna på deras sändningar.

OBS !!! De två personerna i "sammanställar"-gruppen samt detta papper får på inget vis lämna Q-husets parkering innan övningen är slut. Övriga lagmedlemmar får dock röra sig fritt på KTH-campus, dock ej bege sig till sin egen basstation.

Poängbedömning sker enligt följande:

- Korrekt överförd bokstav från egen basstation, utan att motståndarlaget lyckas snappa upp (avlyssna) bokstaven ger 1 poäng.
- Uppsnappadet av motståndarlagets bokstav, samtidigt som ni lyckas förhindra deras förmedlingen av bokstaven mellan deras basstation och informationssammanställare ger 1 poäng.

Instruktioner till basstationen för Grupp 1

Invänta instruktioner från din grupp på kanal 10 FM.

F			A	I		P			
L	F		O	L	V	J		G	P
C		Z	U	X		Z		G	
	R			P		C			
		O	I	F	I	R		M	
	J			B	S	J	M		
	F	V	M	O		R	J	M	R
	P			T	Q	J		G	
T			Q					H	
		Å							U

Instruktioner till basstationen för Grupp 2

Invänta instruktioner från grupp på kanal 25 FM.

	H	M			P		E	V	Y
		P					B		
	U				P		S		C
L		J	E		V		V	L	Y
E	W						T		L
Q		Y	Q					G	J
F					M				
K		M	P				C		I
	E	Q		G	P	N	M		O
Y	J		K	I	K	H	K	N	

Instruktioner till assistenter

Vi denna övning ska varje lag delas upp i tre styckena undergrupper. En basstations grupp bestående av 2 personer, en sammanställnings grupp om 2 personer, och en lyssnar och stör grupp bestående av resterande gruppmedlemmar.

Personerna i basstationsgrupperna skickas därefter till respektive basstation tillsammans med tillhörande instruktioner (dessa får inte ses av de andra lagmedlemmarna). Väl där ska de invänta ytterligare information från resten av laget. Lag nr 1's basstation är Q-huset och lag nr 2's basstation är klocktornet.

Där efter ska resterande del av lag nr 1 bege sig till borggården och lag 2 ska bege sig till parkeringen utanför Q-huset. Väl på plats får de pappret "Instruktioner till sammanställarna för lag nr x".

Nu är det bara att starta övningen.

Tips på återkoppling efter övningen

- Vilka strategi använde de olika lagen?
- Vilka moment ingick i övningen? Upprättande av kommunikations protokoll för överförandet av bokstavs informationen (digital kommunikation). Informationen som sänds via etern är fri att avlyssna för personer med "rätt" utrustning (Jämför polisens gamla system och RAKEL systemet). Hur fungerade det att störa och samtidigt upprätthålla kommunikation med sina egna (telekrigföring)?

A.2 Theory recapitulation slides for Signal Theory, 2E1423

TEORI SAMMANFATTNING, ÖVNING ♯ 1

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1. STOKASTISKA VARIABLER

Nedan följer en kort sammanfattning av en- och tvådimensionella stokastiska variabler och några av de grundläggande begreppen som används vid karaktärisering av dessa.

1.1 Kontinuerliga stokastiska variabler

Låt X vara en kontinuerlig stokastiskt variabel. Fördelningsfunktionen $F_X(x)$, eller täthetsfunktionen $f_X(x)$ beskriver X fullständigt.

Fördelningsfunktionen:

$$F_X(x) = Pr(X \leq x) = \int_{-\infty}^{x} f_X(a)da$$

$$\lim_{x\to\infty} F_X(x) = 1$$
$$\lim_{x\to-\infty} F_X(x) = 0$$

Täthetsfunktionen:

$$f_X(x) = \frac{d\,F_X(x)}{dx}$$

$$\int_{-\infty}^{\infty} f_X(x)dx = 1$$

Väntevärde:

$$m_x = E[X] = \int_{-\infty}^{\infty} x f_X(x)dx$$

Väntevärdet av en funktion $g(X)$ ges av

$$E[g(X)] = \int_{-\infty}^{\infty} g(x) f_X(x)dx$$

Om $g(X)$ är en linjär funktion av formen $g(X) = aX + b$ medför detta att

$$E[aX+b] = aE[X] + b$$

Varians:

$$\begin{aligned} \sigma_x^2 &= var[X] = E[(X-E[X])^2] = \\ &= E[X^2 - 2XE[X] + E[X]^2] = E[X^2] - m_x^2 \end{aligned}$$

1.2 Diskreta stokastiska variabler

För en diskret stokastiskt variabel X som kan anta värdena x_k med sannolikheterna $p_k = Pr(X = x_k)$ är fördelningsfunktionen, täthetsfunktionen och väntevärdet definierade enligt följande

Fördelningsfunktionen:

$$F_X(x) = \sum_k p_k u(x - x_k)$$

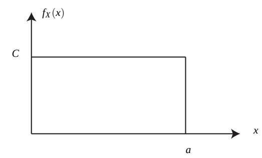
Täthetsfunktionen:

$$f_X(x) = \sum_k p_k \delta(x - x_k)$$

Väntevärde:

$$m_x = E[X] = \sum_k x_k p_k$$

Exempel: X är en kontinuerlig stokastisk variabel, likformigt fördelad mellan $[0, a]$. Beräkna medelvärde m_x och varians σ_x^2.



Lösning: Först beräknar vi C utifrån vetskapen att

$$\int_{-\infty}^{\infty} f_X(x)dx = \int_0^a C\,dx = aC = 1 \;\Rightarrow C = \frac{1}{a}$$

Därefter beräknar vi medelvärdet och det kvadratiska medelvärdet enligt följande

$$m_x = E[X] = \int_{-\infty}^{\infty} x f_X(x)dx = \int_0^a xCdx = C\frac{a^2}{2} = \frac{a}{2}$$

$$E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)dx = \int_0^a x^2 Cdx = C\frac{a^3}{3} = \frac{a^2}{3}$$

Slutligen fås variansen enligt

$$\sigma_x^2 = E[X^2] - m_x^2 = \frac{a^2}{3} - \left(\frac{a}{2}\right)^2 = \frac{a^2}{12}$$

1.3 Flera kontinuerliga stokastiska variabler

Låt X och Y vara två kontinuerliga stokastiska variabler. Den tvådimensionella (simultana) fördelningsfunktionen $F_{X,Y}(x,y)$, respektive täthetsfunktionen $f_{X,Y}(x,y)$ är då definierade enligt följande

Fördelningsfunktionen:

$$F_{X,Y}(x,y) = Pr(X \leq x, Y \leq y)$$

Täthetsfunktionen:

$$f_{X,Y}(x,y) = \frac{d^2 F_{X,Y}(x,y)}{dx\,dy}$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{X,Y}(x,y)dx\,dy = 1$$

De marginella täthetsfunktionerna för X respektive Y få s genom integration över all andra variabler. Det innebär att

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)dy$$

$$f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)dx$$

Oberoende stokastiska variabler:

Om utfallet av X inte beror på utfallet av Y och vice versa är de två variablerna oberoende. Mer stringent, de två stokastiska variablerna X och Y sägs var oberoende om och endast om

$$F_{X,Y}(x,y) = F_X(x)F_Y(y)$$

Om däremot utfallet av X beror på utfallet av Y och vice versa finns det tre enkla mått på detta beroende: korrelationen $r(X,Y)$, kovariansen $Cov(X,Y)$ och korrelationskoefficienten $\rho(X,Y)$. Dessa är definierade enligt följande

Korrelation:

$$r(X,Y) = E[XY] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy\, f_{X,Y}(x,y)dx\,dy$$

Kovariansen:

$$Cov(X,Y) = E[(X-m_x)(Y-m_y)] = r(X,Y) - m_x m_y$$

Korrelationskoefficienten:

$$\rho(X,Y) = \frac{Cov(X,Y)}{\sigma_x \sigma_y}$$

Om $\rho(X,Y) = 0$ så säger man att X och Y är okorrelerade. För oberoende stokastiska variabler gäller att de också är okorrelerade. Det omvända gäller i allmänhet inte (dock för Gaussiska variabler).

2. STOKASTISKA PROCESSER

En stokastisk process kan väldigt förenklat beskrivas som en slumpmässig funktion, alternativt som generalisering av begreppet stokastiska variabler till ett godtyckligt antal dimensioner (en dimension för varje tidpunkt). I princip behövs således ett oändligt antal dimensioner för att fullständigt karaktärisera en stokastisk process. Lyckligtvis räcker det oftast med att beskriva signalen (den stokastiska processen) i termer av dess väntevärde och autokorrelation.

Väntevärde:

Det förväntade utfallet av den stokastiska processen $X(t)$ vid tidpunkten t ges av processens väntevärde vid tidpunkten t, vilket är definierat enligt nedan

$$m_x(t) = E[X(t)] = \int_{-\infty}^{\infty} x\, f_{X(t)}(x)dx$$

Autokorrelationsfunktion:

Autokorrelationen är ett mått på beroende mellan två godtyckliga tidpunkter t_1 och t_2 av den stokastiska processen $X(t)$. Autokorrelationen är definierad enligt följande

$$r_x(t_1,t_2) = E[X(t_1)X(t_2)]$$

2.1 Strikt stationära processer

En stokastisk process sägs vara (strikt) stationär om de simultana fördelningsfunktionerna för alla godtyckligt valda tidpunkter förblir oförändrade vid en godtycklig tidsförskjutning (heltalsförskjutning för en diskret stokastisk process). Det innebär att processens väntevärde är oberoende av tiden och att akf:en endast beror av differensen $t_1 - t_2$.

2.2 Svagt Stationära processer

Det är i allmänhet svårt att avgöra om en processes är strikt stationär. I många tillämpningar räcker det lyckligtvis med att väntevärdet är tidsoberoende och att akf:en endast beror på differensen mellan två tidpunkter. En stokastisk process sägs vara svagt stationär om

$$E[X(t)] = m_x$$
$$E[X(t+\tau)X(t)] = r_x(\tau)$$

Det går också att visa att följande villkor är nödvändiga, men ej tillräckliga för att en funktion ska kunna vara en autokorrelationsfunktionen till en svagt stationär stokastisk process.

1. $r_x(0) \geq 0$
2. $r_x(\tau) = r_x(-\tau)$ för alla τ.
3. $|r_x(\tau)| \leq r_x(0)$ för alla τ.
4. om $|r_x(\tau)| = r_x(0)$ för något $\tau \neq 0$ så måste $r_x(\tau)$ vara en periodisk funktion i τ.
5. om $r_x(\tau)$ är kontinuerlig för $\tau = 0$ så är $r_x(\tau)$ kontinuerlig för alla τ.

TEORISAMMANFATTNING, ÖVNING ♯ 2

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1. ERGODICITET

Något förenklat innebär ergodicitet att en stokastisk process statistiska egenskaper kan observeras utifrån enskilda realiseringar av processen, dvs ergodicitet behandlar likheten mellan tids- och ensemblemedelvärden. Följande ergodicitetsbegrepp gäller för svagt stationära stokastiska processer.

1.1 Partiell ergodicitet map väntevärdet

Om tidsmedelvärdet $\hat{M}_x(N) = \frac{1}{2N+1}\sum_{n=-N}^{N} X(n)$ konvergerar mot ensemblemedelvärdet i kvadratisk mening när antalet datapunkter N växer sägs den stokastiska processen $X(n)$ vara ergodisk med avseende på väntevärdet. Det vill säga att

$$E\left[(\hat{M}_x(N) - m_x)^2\right] \to 0 \qquad \text{då } N \to \infty$$

Detta innebär att givet en tillräckligt lång realisation $x_i(n)$ av den stokastiska processen $X(n)$ kan vi skatta processens väntevärde $m_x = E[X(n)]$ med godtycklig noggrannhet.

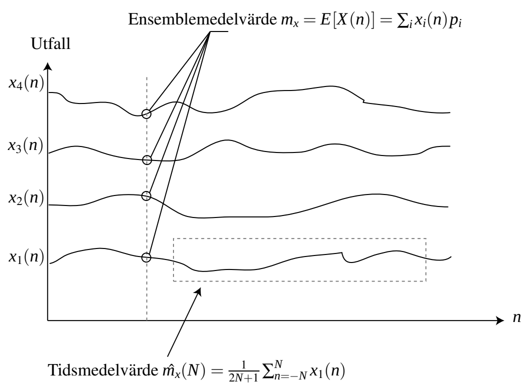

Figure 1: *Skillnaden mellan ensemble- och tidsväntevärde.*

Med de kommande två satserna följer en mer stringent definition av när en svagt stationär process är ergodisk med avseende på väntevärdet.

Sats: *Partiell ergodicitet map väntevärdet*
Betrakta den svagt stationära stokastiska processen $X(n)$ med väntevärde m_x och akf:en $r_x(k)$. Processen $X(n)$ är partiellt ergodisk med avseende på väntevärdet om och endast om

$$\lim_{k \to \infty} r_x(k) = m_x^2$$

förutsatt att gränsvärdet existerar.

Sats: *Partiell ergodicitet map väntevärdet*
En svagt stationär stokastisk process $X(n)$ är partiellt ergodisk med avseende på väntevärdet m_x om och endast om

$$\lim_{N \to \infty} \frac{1}{1+2N} \sum_{k=-2N}^{2N} \left(1 - \frac{|k|}{1+2N}\right)(r_x(k) - m_x^2) = 0$$

1.2 Partiell ergodicitet map autokorrelationsfunktionen

Om en stokastisk process $X(n)$ är partiellt ergodisk med avseende på akf:en innebär det, något förenklat att man kan skatta processens akf med godtycklig noggrannhet givet en tillräckligt lång realisation av processen. En svagt stationär stokastisk process är partiellt ergodisk med avseende på autokorrelationsfunktionen om följande sats är uppfylld.

Sats: *Partiell ergodicitet map autokorrelationsfunktionen*
Betrakta den svagt stationära stokastiska processen $X(n)$ med akf:en $r_x(k)$. Om och endast om

$$\lim_{l \to \infty} E[X(n+k+l)X(n+l)X(n+k)X(n)] = r_x^2(k)$$

gäller för alla heltalsvärden på k förutsatt att gränsvärdet existerar, så följer att sekvensen $X(n)$ är partiellt ergodisk med avseende på autokorrelationsfunktionen.

TEORI SAMMANFATTNING, ÖVNING ♯ 3

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1. EFFEKTSPEKTRUM

Låt $X(t)$ vara en tidskontinuerlig svagt stationär stokastisk process med autokorrelationsfunktionen $r_X(\tau)$. Hur processens (medel) effekt är fördelad i frekvensled beskrivs då av signalens effektspektrum $R_X(f)$, vilket är definierat som fouriertransformen av akf:en (dvs $R_X(f) = \mathscr{F}[r_X(\tau)]$). Effektspektrumet ger oss således yttligare en möjlighet att karaktärisera/behandla stokastiska processer. Nedan följer en kort sammanfattning av relationen mellan en stokastisk process effektspektrum och dess tids- och korrelationsdomäns egenskaper.

1.1 Tidskontinuerliga processer

För en svagt stationär stokastisk process $X(t)$ med akf:en $r_X(\tau)$ är effektspektrumet definierat som

$$R_X(f) = \mathscr{F}[r_X(\tau)] = \int_{-\infty}^{\infty} r_X(\tau) e^{-j2\pi f\tau} d\tau$$

dvs som den tidskontinuerliga fouriertransformen av akf:en. (Notera att enheten för f är Hertz(Hz).) Då ingen information går förlorad i transformationen så är det möjligt att beräkna en process autokorrelationsfunktion givet dess effektspektrum. Den inversa fouriertransformen ger

$$r_X(\tau) = \mathscr{F}^{-1}[R_X(f)] = \int_{-\infty}^{\infty} R_X(f) e^{j2\pi f\tau} df$$

En process medeleffekt kan beräknas direkt från dess effektspektrum genom att notera att

$$P_X = E[X^2(t)] = \sigma_X^2 + m_X^2 = r_X(0) = \int_{-\infty}^{\infty} R_X(f) df$$

Där den sista likheten kan inses genom att sätta $\tau = 0$ i uttrycket ovan för relationen mellan akf:en och effektspektrumet.

Nedan följer nödvändiga och *tillräckliga* vilkor för att en funktion $R_X(f)$ ska kunna vara effektspektrum till en reellvärd svagt stationär tidskontinuerlig process.

1. $R_X(f)$ är reellvärd
2. $R_X(f)$ är en jämn funkton, det vill säga symmetrisk runt $f = 0$ med $R_X(f) = R_X(-f)$.
3. $R_X(f)$ är icke-negativ, det vill säga $R_X(f) \geq 0$.

Vi har därmed, jämfört med tidigare fått både nödvändiga och tillräckliga villkor för att avgöra om en funktion $r_X(\tau)$ kan vara en autokorrelationsfunktion.

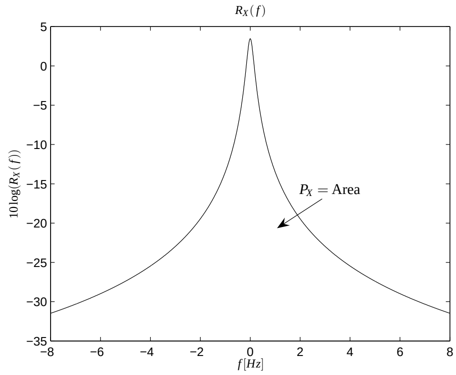

Figure 1: *Effektspektrum $R_X(f)$ för en svagt stationär tidskontinuerlig process $X(t)$ med autokorrelationsfunktionen $r_X(\tau) = e^{-0.9|\tau|}$.*

1.2 Tidsdiskreta processer

Effektspektrumet hos en tidsdiskret stokastisk processs $X(n)$ med autokorrelationsfunktionen $r_X(k)$ definieras på liknande sätt som i det tidskontinuerliga fallet, ända skillnaden är att den tidsdiskreta fouriertransformen används istället. Effektspektrumet för en tidsdiskret stokastisk process ges således av

$$R_X(\nu) = \mathscr{F}_d[r_X(k)] = \sum_{-\infty}^{\infty} r_X(k) e^{-j2\pi\nu k}$$

Där variabeln $\nu = f/f_{sample}$ betecknar signalens normerade frekvens och är således enhetslös. Processens akf $r_X(k)$ kan givet dess effektspektrum $R_X(\nu)$ beräknas med den inversa fouriertransformen enligt

$$r_X(k) = \mathscr{F}_d^{-1}[R_X(\nu)] = \int_{-1/2}^{1/2} R_X(\nu) e^{j2\pi\nu k} d\nu$$

Processens medeleffekt beräknas från effektspektrum som

$$P_X = \sigma_X^2 + m_X^2 = r_X(0) = \int_{-1/2}^{1/2} R_X(\nu) d\nu$$

Nedan följer nödvändiga och *tillräckliga* villkor för att en funktion $R_X(\nu)$ ska kunna vara effektspektrum till en reellvärd svagt stationär tidsdiskret process.

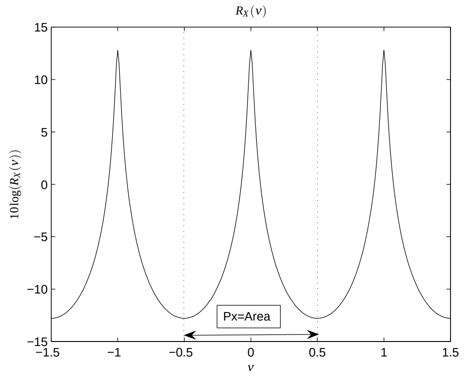

Figure 2: *Effektspektrum $R_X(\nu)$ för en svagt stationär tidsdiskret process $X(n)$ med autokorrelationsfunktionen* $r_X(k) = 0.9^{-|k|}$.

1. $R_X(\nu)$ är reellvärd
2. $R_X(\nu)$ är en jämn funkton, det vill säga symmetrisk runt $\nu = 0$ med $R_X(\nu) = R_X(-\nu)$.
3. $R_X(\nu)$ är icke-negativ, det vill säga $R_X(f) \geq 0$.
4. $R_X(\nu)$ är periodisk med perioden 1, det vill säga $R_X(\nu) = R_X(\nu + p)$ för alla heltal $p = \pm 1, \pm 2, \ldots$

2. KORSKORRELATION OCH KORSSPEKTRUM

Korskorrelatonen är ett mått på hur mycket två olika stokastiska signaler beror av varandra. Korskorrelationen mellan två simultant svagt stationära processer är definierad som

$$r_{YX}(k) = E[Y(n+k)X(n)] \tag{1}$$

och är således en generalisering av autokorrelationsfunktionen. På samma sätt som fouriertransformen av akf:en gav signalens medel effektspektrum så ger fouriertransformen av korskorrelationen $r_{YX}(k)$ det så kallade korsspektrumet $R_{YX}(\nu)$. Korsspektrumet är definierat som

$$R_{YX}(f) = \int_{-\infty}^{\infty} e^{-j2\pi f\tau} d\tau$$

$$R_{YX}(\nu) = \sum_{-\infty}^{\infty} r_{YX}(k) e^{-j2\pi\nu k}$$

Av definitionen framgår att följande håller

$$r_{YX}(\tau) = r_{XY}(-\tau) \quad \text{samt} \quad r_{YX}(k) = r_{XY}(-k) \tag{2}$$

och att

$$R_{YX}(f) = R_{XY}(-f) \quad \text{samt} \quad R_{YX}(\nu) = R_{XY}(-\nu) \tag{3}$$

Notera för övrigt att korskorrelationen inte nödvändigtsvis är symmetrisk och att korsspektrumet inte nödvändigtvis är reellt.

TEORI SAMMANFATTNING, ÖVNING ♯ 4 OCH ♯ 5

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1. FILTRERING

Om insignalen till ett system (filter) är en stokastisk signal (process) är systemets utsignal också en stokastisk signal, vars stokastiska egenskaper uteslutande bestäms av filtrets och insignalens egenskaper.

1.1 Tidskontinuerliga System

Låt $X(t)$ vara en svagt stationär tidskontinuerlig stokastisk process med autokorrelationsfunktionen $r_X(t)$, effektspektrumet $R_X(f)$ och medelvärdet m_X. $X(t)$ filteras genom det linjära tidsinvarianta systemet med impulssvaret $h(t)$, frekvensfunktionen $H(f)$ och utsignalen $Y(t)$. Se figur 1. Filtrets utsignal $Y(t)$ är i detta fall en svagt stationär process vars statistiska egenskaper kan beräknas enligt följande:

<u>väntevärde</u>

$$m_Y = m_X \int_{-\infty}^{\infty} h(u)du = m_X H(0)$$

<u>akf</u>

$$r_Y(\tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(u)h(v)r_X(\tau - u + v)dudv$$

<u>effektspektrum</u>

$$R_Y(f) = |H(f)|^2 R_X(f)$$

Vidare ges korskorrelationen och korsspektrumet mellan insignalen $X(t)$ och utsignalen $Y(t)$ av

$$r_{XY}(\tau) = \int_{-\infty}^{\infty} h(u)r_X(\tau + u)du$$

$$r_{YX}(\tau) = \int_{-\infty}^{\infty} h(u)r_X(\tau - u)du$$

$$R_{YX}(f) = R_{XY}(-f) = H(f)R_X(f)$$

1.2 Tidsdiskreta System

På samma sätt kan de statistiska egenskaperna hos den svagt stationära utsignalen $Y(n)$ från det tidsdiskret linjärt tidsvariant system med enpulssvaret $h(n)$ bli relaterade till den svagt stationära insignalens $X(n)$ stokastiska egenskaper.

<u>väntevärde</u>

$$m_Y = m_X \sum_{l=-\infty}^{\infty} h(l) = m_X H(0)$$

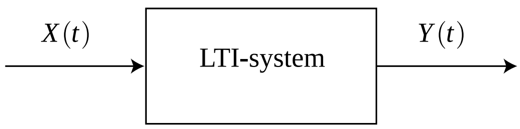

Figure 1: *Linjärt tidsinariant system.*

<u>akf</u>

$$r_Y(k) = \sum_{l=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} h(u)h(v)r_X(k - l + m)$$

<u>effektspektrum</u>

$$R_Y(\nu) = |H(\nu)|^2 R_X(\nu)$$

Vidare ges korskorrelationen och korsspektrumet mellan insignalen $X(n)$ och utsignalen $Y(n)$ av

$$r_{XY}(k) = \sum_{l=-\infty}^{\infty} h(l)r_X(k + l)$$

$$r_{YX}(\tau) = \sum_{l=-\infty}^{\infty} h(l)r_X(k - l)$$

$$R_{YX}(\nu) = R_{XY}(-\nu) = H(\nu)R_X(\nu)$$

Om inprocessen (en tidskontinuerlig eller tidsdiskret stokastisk process) till ett linjärt och tidsinvariant system är gaussisk, så är även utprocessen gaussisk.

2. ARMA PROCESSER

Låt $X(n)$ var insignalen till ett kausalt BIBO-stabilt system $H(\nu)$ med utsignalen $Y(n)$ och frekvensfunktionen

$$H(\nu) = \frac{b_0 + b_1 e^{-j2\pi\nu} + \cdots + b_l e^{-j2\pi l\nu}}{1 + a_1 e^{-j2\pi\nu} + \cdots + a_k e^{-j2\pi k\nu}}$$

Vi kan också beskriva beroendet mellan in- och utsignalen med differensekvationen

$$Y(n) + a_1 Y(n-1) + \cdots + a_k Y(n-k) = b_0 X(n) + b_1 X(n-1) + \cdots + b_l X(n-l)$$

Om $X(n)$ är en vit signal kallas utsignalen $Y(n)$ för en ARMA(k,l)-process (ARMA = Auto Regressive Moving Average). Specialfall av ARMA processer är:

AR(k)-process

$$Y(n) + a_1 Y(n-1) + \cdots + a_k Y(n-k) = X(n)$$

MA(l)-process

$$Y(n) = b_0 X(n) + b_1 X(n-1) + \cdots + b_l X(n-l)$$

Det är vanligt att använda AR-, MA- och ARMA-modeller inom modellbaserad signalbehandling då de flesta spektrum låter sig väl beskrivas med någon av dessa modeller.

2.1 Yule-Walkerekvationerna

Man kan för en AR(N)-process relatera filterkoefficienterna till processens autokorrelationsfunktion enligt det linjära ekvationssystemet

$$\mathbf{R}_Y \mathbf{a} = -\mathbf{r}_Y \tag{1}$$

där autokorrelationsmatrisen $\mathbf{R}_Y$, autokorrelationsvektorn $\mathbf{r}_Y$ och filter-koefficientvektorn $\mathbf{a}$ har följande form

$$\mathbf{R}_Y = \begin{pmatrix} r_Y(0) & r_Y(1) & \dots & r_Y(N-1) \\ r_Y(1) & r_Y(0) & \dots & r_Y(N-2) \\ \vdots & \vdots & \dots & \vdots \\ r_Y(N-1) & r_Y(N-2) & \dots & r_Y(0) \end{pmatrix}$$

$$\mathbf{r}_Y = \begin{pmatrix} r_Y(1) \\ r_Y(2) \\ \vdots \\ r_Y(N) \end{pmatrix} \qquad \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{pmatrix}$$

Detta ekvationssystem är känt som Yule-Walkerekvationerna. Matrisen $\mathbf{R}_Y$ har den speciella strukturen att elementen på de olika diagonalerna alla är lika. En sådan matris kallas för en Toeplitzmatris.

TEORI SAMMANFATTNING, ÖVNING # 6

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1. ESTIMERING

Estimering handlar om att skatta storheter hos en stokastiska process utifrån en enstaka realisering. Man skiljer ofta mellan icke-parametriska respektive parametriska (modellbaserade) metoder.

1.1 Icke-parametriska estimatorer

För icke-parametriska estimatorer gör man inga speciella antaganden om den stokastiska processen, förutom att den är svagt stationär och ergodisk. Nedan följer några exempel på icke-parametriska estimatorer.

1.1.1 Estimering av väntevärde

Utifrån realiseringen $\{x(0),\ldots,x(N-1)\}$ kan väntevärdet m_X skattas som

$$\hat{m}_X = \frac{1}{N}\sum_{n=0}^{N-1} x(n).$$

Skattaren är väntevärdesriktig och konvergerar i kvadratiskt medel[1] mot m_X.

1.1.2 Estimering av akf

Utifrån realiseringen $\{x(0),\ldots,x(N-1)\}$ kan autokorrelationsfunktionen $r_X(k)$ skattas som

$$\hat{r}_{X,B}(k) = \frac{1}{N}\sum_{n=0}^{N-k-1} x(n+k)x(n)$$

eller

$$\hat{r}_{X,U}(k) = \frac{1}{N-k}\sum_{n=0}^{N-k-1} x(n+k)x(n)$$

för $k = 0,\ldots,N-1$.

Skattaren $\hat{r}_{X,B}(k)$ är *inte* väntevärdesriktig. Den uppfyller dock de nödvändiga villkoren för en akf. Skattaren $\hat{r}_{X,U}(k)$ är väntevärdesriktig. Däremot uppfyller den inte med säkerhet de nödvändiga villkoren för en akf. Båda skattarna är asymptotiskt väntevärdesriktiga[2] och konvergerar i kvadratiskt medel.

1.1.3 Estimering av effektspektrum

Utifrån realiseringen $\{x(0),\ldots,x(N-1)\}$ kan effektspektrumet $R_X(\nu)$ skattas som

$$\hat{R}_X(\nu) = \frac{1}{N}|X(\nu)|^2$$

där

$$X(\nu) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi\nu n}$$

är den tidsdiskreta Fouriertransformen av sekvensen. Denna skattning kallas för *periodogrammet*. Den är inte väntevärdesriktig och konvergerar inte i kvadratiskt medel.

Periodogrammet kan ekvivalent skrivas som

$$\hat{R}_X(\nu) = \frac{1}{N}\sum_{k=-(N-1)}^{N-1} \hat{r}_{X,B}(k)e^{-j2\pi\nu k}.$$

Denna skattare kallas då korrelogrammet.

1.2 Parametriska estimatorer

För parametriska estimatorer antar man att $\{x(0),\ldots,x(N-1)\}$ är en realisering av en stokastisk process $X(n)$ som har en viss struktur, som kan beskrivas med en matematiskt modell. Ofta vill man då skatta en eller flera parametrar i den modellen. Nedan ges ett exempel på en parametrisk estimator.

1.2.1 Estimering av AR-parametrar

Betrakta en AR(s)-process på formen

$$Y(n) = -a_1Y(n-1) + \ldots + -a_sY(n-s) + X(n)$$

där $X(n)$ är vitt brus.

AR-parametrarna $\mathbf{a} = [a_1 \cdots a_s]^T$ kan estimeras utifrån realiseringen $\{y(0),\ldots,y(N-1)\}$ som

$$\hat{\mathbf{a}} = -\underbrace{\left[\frac{1}{N}\sum_{n=0}^{N-1}\mathbf{y}(n)\mathbf{y}^T(n)\right]^{-1}}_{\hat{\mathbf{R}}_{\mathbf{Y}}^{-1}} \cdot \underbrace{\frac{1}{N}\sum_{n=0}^{N-1} y(n)\mathbf{y}^T(n)}_{\hat{\mathbf{r}}_{\mathbf{Y}}}$$

där $\mathbf{y}(n) = [y(n-1)\cdots y(n-s)]^T$ med $y(n) = 0$ för $n < 0$ och $\hat{\mathbf{R}}_{\mathbf{Y}}^{-1}$ och $\hat{\mathbf{r}}_{\mathbf{Y}}$ kan ses som skattningar av Yule-Walkerekvationerna.

[1]Variansen går mot 0 då $N \to \infty$.

[2]Estimatorns väntevärde går mot det sanna värdet då $N \to \infty$.

TEORI SAMMANFATTNING, ÖVNING # 7

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1. OPTIMAL FILTRERING

Låt X vara en stokastisk variabel och $\mathbf{Y}$ vara en vektor av N stokastiska variabler,

$$\mathbf{Y} = \begin{pmatrix} Y(0) \\ \vdots \\ Y(N-1) \end{pmatrix}.$$

Vi vill estimera X baserat på en observation $\mathbf{y}$ av $\mathbf{Y}$. Vi skriver det stokastiska estimatet som $\hat{X}(\mathbf{Y})$.

1.1 MMSE-Estimatorn

Ett mått på en estimators noggrannhet är medelkvadratfelet[1],

$$\text{MSE}(\hat{X}(\mathbf{Y})) = \text{E}\left[(\hat{X}(\mathbf{Y}) - X)^2\right].$$

Estimatorn som minimerar medelkvadratfelet, MMSE-estimatorn[2], ges av det betingade väntevärdet

$$\hat{x}_{\text{MMSE}}(\mathbf{y}) = \text{E}\left[X|\mathbf{Y} = \mathbf{y}\right],$$

där $\mathbf{y}$ är en vektor av *kända* observationer. För att beräkna estimatorn $\hat{X}_{\text{MMSE}}(\mathbf{Y})$ krävs att den betingade täthetsfunktionen $f_{X|\mathbf{Y}}(x|\mathbf{y})$ är känd. Ett användbart samband är

$$f_{X|\mathbf{Y}}(x|\mathbf{y}) = \frac{f_{X,\mathbf{Y}}(x,\mathbf{y})}{f_{\mathbf{Y}}(\mathbf{y})}.$$

1.2 LMMSE-Estimatorn

Ofta är det svårt att finna MMSE-estimatorn. Även om MMSE-estimatorn går att räkna ut kan den vara för komplex för t.ex. realtidstillämpningar. Då kan man istället försöka finna den *linjära* estimatorn som minimerar MSE[3].

Vi söker nu alltså en estimator på formen

$$\hat{X}(\mathbf{Y}) = \sum_{l=0}^{N-1} h(l)Y(l) = \mathbf{h}^T\mathbf{Y}, \qquad (1)$$

där N är både antalet observationer och antalet tappar i $\mathbf{h}$.

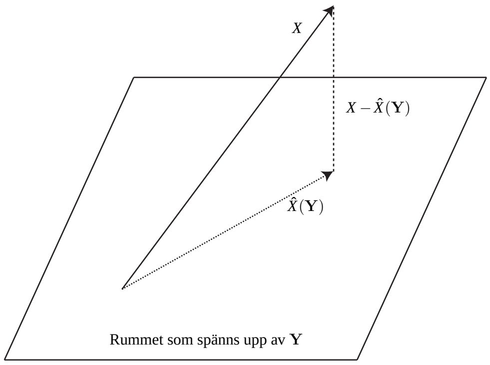
LMMSE-estimatorn kan räknas ut på flera sätt, t.ex. via *ortogonalitetsprincipen* eller via *normalekvationerna*.

1.2.1 *Ortogonalitetsprincipen*

Estimatorn $\mathbf{h}_{\text{LMMSE}}$ ger det minsta MSE av alla *linjära* estimatorer på formen (1) om

$$\text{E}\left[\left(\mathbf{h}_{\text{LMMSE}}^T\mathbf{Y} - X\right)\mathbf{Y}\right] = \mathbf{0}.$$

Notera att $\left(\mathbf{h}_{\text{LMMSE}}^T\mathbf{Y} - X\right)$ är (det stokastiska) estimeringsfelet och att nollvektorn i högerledet har samma dimension som $\mathbf{Y}$. Detta betyder att estimeringsfelet är ortogonalt mot samtliga observationer i $\mathbf{Y}$. Detta illustreras i Figur 1.



Figur 1: Eftersom estimatet $\hat{X}$ är en linjärkombination av alla observationer $\mathbf{Y}$ så måste det ligga i samma rum (i samma plan i figuren) som observationerna. För LMMSE estimatet är estimeringsfelet $X - \hat{X}$ ortogonalt mot alla observationer i $\mathbf{Y}$.

1.2.2 *Normalekvationerna*

Den linjära MMSE-estimatorn ges av

$$\mathbf{h}_{\text{LMMSE}} = \mathbf{R}_{\mathbf{Y}}^{-1}\mathbf{r}_{X\mathbf{Y}}, \qquad (2)$$

där

$$\begin{aligned} \mathbf{R}_{\mathbf{Y}} &= \text{E}\left[\mathbf{Y}\mathbf{Y}^T\right] \\ \mathbf{r}_{X\mathbf{Y}} &= \text{E}\left[X\mathbf{Y}\right] \end{aligned}$$

är $\mathbf{Y}$s korrelationsmatris respektive korskorrelationsvektorn mellan X och $\mathbf{Y}$. Det resulterande MSE fås ur (10.22) i boken.

1.3 Wienerfiltrering

LMMSE estimatorer kallas i praktiken ofta Wienerfilter. De kan delas in i FIR[4]- och IIR[5]-filter. Här antar vi att processen $X(n)$ ska skattas utifrån observationerna $Y(n)$ som

$$\hat{X}(n) = h(n) \star Y(n).$$

[1]MSE: mean square error
[2]MMSE: minimum MSE
[3]LMMSE: linear MMSE
[4]FIR: Finite impulse response
[5]IIR: Infinite impulse response

1.3.1 FIR Wienerfilter

Wienerfilter med ändligt antal tappar kan beräknas utifrån lösningen till normalekvationerna i (2).

1.3.2 IIR Wienerfilter

Det oändligtdimensionella Wienerfiltret kan beräknas utifrån

$$H_{IIR}(\nu) = \frac{R_{XY}(\nu)}{R_Y(\nu)}$$

där $H_{IIR}(\nu)$ är filtrets frekvensfunktion, $R_{XY}(\nu)$ är korsspektrum mellan $X(n)$ och $Y(n)$ och $R_Y(\nu)$ är $Y(n)$s effektspektrum. Det resulterande MSE fås ur (10.46) i boken. Med oändligtdimensionellt menas att filtret har oändligt många tappar och att alltså hela processen $Y(n)$ används vid skattningen av *varje* $X(n)$.

TEORI SAMMANFATTNING, ÖVNING # 9

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1. SAMPLING

Omvandlingen från tidskontinuerliga till tidsdiskreta signaler sker genom sampling.

1.1 Deterministiska signaler

Låt den tidskontinuerliga signalen $x(t)$ samplas med samplingsfrekvensen $f_s = \frac{1}{T_s}$. Den samplade signalen, $y(n)$, ges av

$$y(n) = x(nT_s).$$

Den normerade frekvensen definieras som $\nu = \frac{f}{f_s}$

Det är endast intressant att studera spektrumet för $y(n)$ för $\nu \in [-1/2 \; 1/2]$. Frekvenser i $x(t)$ med högre frekvens än $f_s/2$ kommer att uppträda i $y(n)$s spektrum med synbart lägre frekvens. Denna effekt kallas aliaseffekten eller vikning.

1.1.1 Poissons summationsformel

Den samplade signalens Fouriertransform, $Y(\nu) = \mathscr{F}\{y(n)\}$, ges av

$$Y(\nu) = \frac{1}{T_s} \sum_{m=-\infty}^{\infty} X\left(\frac{\nu - m}{T_s}\right),$$

där $X(f) = \mathscr{F}\{x(t)\}$.

Den samplade signalens Fouriertransform är alltså en periodisk upprepning av den tidskontinuerliga signalens Fouriertransform, skalad med $1/T_s$.

1.2 Stokastiska signaler

Låt den tidskontinuerliga svagt stationära signalen $X(t)$ samplas med samplingsfrekvensen $f_s = \frac{1}{T_s}$. Den samplade signalen, $Y(n)$, som också är svagt stationär, ges av

$$Y(n) = X(nT_s).$$

Den samplade signalens väntevärde, akf och effektspektrum ges av

$$\begin{aligned} m_Y &= m_X \\ r_Y(k) &= r_X(kT_s) \\ R_Y(\nu) &= \frac{1}{T_s} \sum_{m=-\infty}^{\infty} R_X\left(\frac{\nu - m}{T_s}\right), \end{aligned}$$

där m_X, $r_X(\tau)$ och $R_X(f)$ är den tidskontinuerliga processens väntevärde, akf respektive effektspektrum. Notera att frekvenser i $R_X(f)$ som ligger utanför Nyquistfrekvensen $f_s = [-f_s/2 \; f_s/2]$ kommer att ge upphov till vikningsdistorsion i den samplade signalens spektrum.

2. PULSAMPLITUDMODULERING

Pulsamplitudmodulering omvandlar en tidsdiskret signal till en tidskontinuerlig genom att modulera varje värde i den tidsdiskreta signalen med en puls.

2.1 Deterministiska signaler

Den tidsdiskreta signalen $y(n)$ omvandlas till den tidskontinuerliga signalen $z(n)$ med hjälp av den tidskontinuerliga pulsen $p(t)$ enligt

$$z(t) = \sum_{n=-\infty}^{\infty} y(n) p(t - nT_s).$$

Signalernas Fouriertransformer är relaterade enligt

$$Z(f) = P(f) Y(fT_s).$$

2.2 Stokastiska signaler

Den tidsdiskreta svagt stationära signalen $Y(n)$ omvandlas till den tidskontinuerliga svagt stationära signalen $Z(n)$ med hjälp av den tidskontinuerliga pulsen $p(t - \Theta)$ enligt

$$Z(t) = \sum_{n=-\infty}^{\infty} Y(n) p(t - nT_s - \Theta),$$

där Θ är en likformigt fördelad stokastisk variabel på intervallet $[0 \; T_s]$. Anledningen till att variabeln Θ införs är att utan den blir $Z(t)$ inte svagt stationär. Den tidskontinuerliga signalens väntevärde och effektspektrum ges av

$$\begin{aligned} m_Z &= \frac{m_Y P(0)}{T_s} \\ R_Z(f) &= \frac{1}{T_s} |P(f)|^2 R_Y(fT_s), \quad \text{(Bennetts formel)} \end{aligned}$$

där m_Y och $R_Y(\nu)$ är den tidsdiskreta processens väntevärde respektive effektspektrum och $P(f) = \mathscr{F}\{p(t)\}$. Den pulsamplitudmodulerade signalens akf ges av (11.19) i boken.

TEORI SAMMANFATTNING, ÖVNING # 10

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1. REKONSTRUKTION

Med rekonstruktion menar vi omvandling från tidskontinuerlig till tidsdiskret signal och sen tillbaks till tidskontinuerlig. Vi kan också mena omvandling från tidsdiskret signal, via tidskontinuerlig tillbaks till tidsdiskret. Transformationerna sker med sampling och pulsamplitudmodulering (PAM). I detta teoriblad studerar vi rekonstruktion av **deterministiska** signaler. Principerna för rekonstruktion av stokastiska processer är analoga.

1.1 Samplingsteoremet

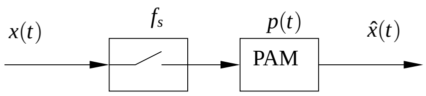
Den deterministisk signal $x(t)$ (med ändlig energi) med bandbredden B samplas med samplingsfrekvensen $f_s = \frac{1}{T_s}$. Den ursprungliga signalen $x(t)$ kan rekonstrueras felfritt från den samplade signalen $x(nT_s)$ om

$$f_s > 2B$$

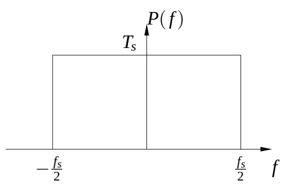
och den ideala pulsen

$$p(t) = \frac{\sin(\pi f_s t)}{\pi f_s t}$$

används. Rekonstruktionens delar visas i figur 1. Pulsens Fouriertransform visas i figur 2.



Figur 1: Den tidskontinuerliga signalen $x(t)$ samplas med samplingsfrekvensen f_s och pulsamplitudmoduleras med pulsen $p(t)$.



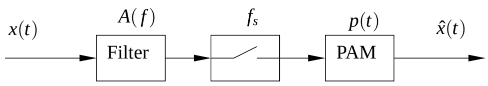
Figur 2: Den ideala pulsens Fouriertransform.

1.2 Rekonstruktionsfel

Eftersom samplingsteoremets villkor ofta är svåra att uppfylla kommer man att få fel vid rekonstruktionen. Ett sätt att minska rekonstruktionsfelet är att använda ett s.k. antivikningsfilter, som i figur 3. Det ideala antivikningsfiltret ges av

$$A(f) = \begin{cases} 1 & |f| \leq f_s/2 \\ 0 & |f| > f_s/2 \end{cases}$$

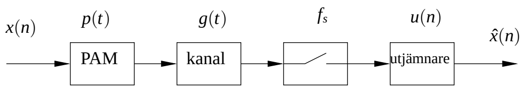
Istället för att få rekonstruktion av $x(t)$ med fel fås perfekt rekonstruktion av den lågpassfiltrerade signalen. Detta innebär dock fortfarande vissa rekonstruktionsfel, eftersom de högfrekventa delarna i $x(t)$ togs bort.



Figur 3: Ett sätt att minska vikningsdistortionen är att lågpassfiltera $x(t)$ innan samplingen.

1.3 Rekonstruktion av tidsdiskreta signaler

I digital kommunikation överför man en digital signal över en kontinuerlig kommunikationskanal (t.ex. telefonledning eller luften). Detta visas förenklat i figur 4, där den tidskontinuerliga pulsamplitudmodulerade signalen påverkas av det linjära filtret $g(t)$ innan det når mottagaren och samplas. För att minska felet vid rekonstruktionen pga kanalens inverkan kan den samplade signalen *utjämnas* med filtret $u(n)$.



Figur 4: En modell av ett kommunikationssystem med PAM och kanal $g(t)$.

A.3 Conference paper on Elektroprojekt del II, EH1020

DEVELOPING AND IMPLEMENTING A PROGRAM INTERFACING PROJECT COURSE IN ELECTRICAL ENGINEERING

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ABSTRACT

In this paper, we describe the ideas behind a second-year Design-Build course in Electrical Engineering. Electrical Engineering is a theoretical subject, and in such it is difficult to maintain the theoretical level in project courses introduced too early in the program, especially when core subjects like electromagnetic field theory are involved. This issue is addressed and we also describe our approach for the assessment of the students. We also discuss the different goals that were set up prior to the course from a program perspective; how we reasoned when designing the course, the assessment structure, and the output once the course was implemented.

KEYWORDS

Project course, Design-Build, Electrical Engineering

INTRODUCTION

At KTH, one of the originators of the CDIO approach, there has recently been a general requirement to strengthen CDIO activities on all five-year programs. The focus was less on Conceive, Design, Implement and Operate (CDIO) activities and Curriculum Design [1], instead the focus were on integrating general engineering skills such as teamwork, written communication, oral presentation, project management, peer-review and innovation into the programme courses. For these areas, KTH provided support functions.

One of KTH programmes is the five-year Electrical Engineering (EE) program. It is a traditional, course-oriented program with a solid theoretical foundation. It also has many of the general engineering skills already built in [2]. However, Design-Build activities are too few, and there is a need for activities tying the different core courses together. While we believe that the general applicability of the content taught in the courses is a strength of the programme, we have seen that students tend to be drawn towards programmes with application-specific names when they cannot visualize the work role of an EE engineer. In fact, there is a general uncertainty in society of what an Electrical Engineer really does. Given that, and the emerging innovation-based structure of industry, the programme would clearly benefit from more design-build activities.

Another recent change at KTH is the evolvement from five-year standalone programmes into five-year programmes with a 3+2 bachelor-master structure. It is formally implemented, but there are still many challenges to address, like e.g. the implementation of CDIO activities and how to profile the bachelor level part of the programme. Since the program was previously a five-year standalone programme, many of the profiling courses were given during the 4th and 5th years. With the change to the bachelor + master structure, we find it important to strengthen the electrical engineering profile already at the bachelor's level. There is however a challenge to maintain the theoretical level and still be able to offer a broad program that prepares the students for the variety of masters programs that are offered to the students during study years 4 and 5.

Enhancing the programs profile is perhaps the most important retention activity we can do for the new generation of students. The students constantly evaluate if their choice of education was the right one [3]. They need to know on much earlier stage where their investment in education will take them. Most often they lack hands-on as well as industrial experience and find many of the theoretical subjects difficult to grasp. This is also related to a lack of understanding of the role of the theoretically heavy subjects such as math and field theory in the applied subjects later in the programme. Thus, an important conclusion is that if we bring together and apply the content from some of the traditionally taught courses of the programme through a Design-Build course in which the students can apply their EE knowledge in a real engineering task, we could strengthen the EE-profile and also help the students (already when doing their bachelor) visualize the work of an electrical engineer. The introduction of a project course with the right kind of project, would not only strengthen the student's general engineering skills, but also bring to life and thereby strengthen the theoretical content of the other courses connected to the project course [4]. The main courses targeted in our case were: Electromagnetic Field Theory, Wave Propagation and Antennas, Signals and Systems part I & II, and Electrical Measurements.

However, it is difficult to create a real and valuable design and build experience by applying relevant theory in a project course that appears at an early stage in the EE program. This is especially true when highly theoretical core subjects like electromagnetic field theory and signal analysis are involved.

To address some of these issues, a group of teachers responsible for courses at the bachelor level were assigned the task to develop a new project course – a Design-Build course. As a starting point for the discussion, the following list of suggested programme and learning outcome objectives were given to the group.

- Enhance the profile in Electrical engineering by strengthening core courses
- Apply knowledge from the core courses. Enhanced knowledge in basic modelling techniques, like the ability to make estimations and simplifications that reduce the problem to an elementary calculable complexity.
- Be able to formulate, evaluate, and choose a technical solution to a given problem.
- Be able to identify deficient areas in one's knowledge and awareness of the personal responsibility for one's own development of knowledge.
- Training in the ability to explain the theories and technical systems treated in the course for a person who is not an engineer.
- Further training in teamwork.

- Training in giving and receiving feedback in a constructive way.

Next we describe how the course was designed from these learning outcomes. The paper is structured as follows. We begin by describing how the course was designed, what kind of decisions that were made and which alternatives were evaluated. We then continue with how the course was implemented, followed by the outcome of the first round. We analyse the results and discuss areas for improving the course design. Finally, we conclude with our experiences of placing this kind of project course in the second year of the programme.

COURSE DESIGN

Since most assignments in the programme are clearly specified or catered for success, we wanted this assignment to be very open. We decided that the students should be introduced to the requirements, work with them for a while, present a prototype or model, get feedback and then develop their solution.

Although we saw the benefits of giving the whole class a system development task, with different student teams being responsible for different parts, to promote understanding of task interfaces, it was decided at a fairly early stage to let all students solve the same task. The reason was partly because it would have been difficult to find a sufficient number of suitable sub-tasks and partly because it treats all students equally in terms of workload and grading. It was also decided early that students should be randomly assigned into teams of 3 to 4 by the teachers at the start of the course [5]. Assigning the students was a logical continuation since the students were allowed to create their own teams in the first year project course.

With all teams having the same task we had to consider the information flow between the teams. We wanted an open information exchange but we did not want the teams to copy from each other but rather be inspired by each other. For this reason we introduced an element of competition between the teams, cf. the section Assessment.

For the supervision we considered two main alternatives. Either we could give the teams a list of teachers and their special competences and allow the students to book a time when they needed help, or we could create a reference group of teachers and ask the students to present their progress at regular meetings with the reference group. To give the teachers' good and common understanding of the pedagogical process we decided to go for the later alternative with a teacher reference group. Once a week, the students have the opportunity to book a time for supervision by the reference group, and there are also a few compulsory supervision meetings. In a few years when we have gained some experience with the course, this could be reconsidered.

Based on previous positive experience with a multiple-submission assessment such a system was adopted, cf. the section Assessment.

Project task

A significant development time was spent on finding a task fulfilling the following requirements.

- The students should be able to build the device more or less from scratch.
- The design should require theoretical knowledge in electromagnetics, including circuit theory, as well as signal processing and preferably also from other courses that the students have passed so far, e.g. classical mechanics.
- The students should be able to make and present an analytical model to give some guidance for the main parts of the design.
- It should be possible to measure the performance of the final prototype and relate the results to the design.
- The full workload, including project planning, modelling, implementation, measurements and reporting should not exceed 120 hours per student (4.5 ECTS credits).

One idea that was investigated in detail was the wire recorder – a predecessor of tape recorders, where the signal was stored by magnetizing a thin steel wire. We found a kit for building a prototype of such a wire recorder [6, 7, 8], but it did not produce sufficient sound quality to be a convincing example, even though a commercial recording head was used. Since we furthermore wanted the students to design and fabricate their own coils based on the analysis, the idea was abandoned. Among other ideas discussed were metal detectors, a field measuring device, RFID and simple radio communication. The project task that was eventually chosen to be implemented for the first run was that student teams should build their own loudspeakers.

The only prefabricated part that was allowed was the permanent magnet assembly, which could be taken from an existing loudspeaker. All other parts should be designed and fabricated by the students. A budget was also defined not exceeding the equivalent of 50 USD per person, thus about 200 USD per student team. Since this budget corresponds to the typical cost for the reading material in a normal course, we chose to let the students pay the material themselves.

A theoretical modelling of a loudspeaker involves an electrical part, a mechanical part – the membrane, which could be described using the concept of acoustic impedance – and the moving coil, connecting the two. The combined system can be described by a total transfer function. Design parameters that can be determined from such an analysis include dimensioning the coil and the membrane and relating these to the input impedance and frequency band.

Once the prototype has been built, the students should make measurements to judge how well their design fulfilled their design specifications and theoretically predicted performance. We also try to illustrate the benefits of digital signal processing, by asking the students to design an equalizing filter that compensates for the frequency response of the loudspeakers.

Implementation of the course

For practical reasons, it was desirable that the course extended over the full second year. This also has the added advantage that it gives time to reflect and digest. We wanted the activities to be fairly evenly distributed over the fall and spring semesters, so a natural division was to let the students do the planning, analysis and design during the fall term and the building and evaluation during the spring term. A complication was that several of the prerequisite courses are offered during the second study year, i.e., in parallel with the project course, cf. table 1.

We deliberately did not specify the project task in too large detail, but let the students themselves define more specific design goals, such as bandwidth, sound power and as well as budget within the constraint given. Such planning activity is not only intended to train project planning but also to trigger the students to start thinking about the technical details at an early stage.

In an attempt to train different forms of presentation, we decided to have an oral presentation of the theoretic modelling and design in the fall, but do the final reporting in the spring in the form of a written technical report. The oral presentation was organized on a single day with the different student teams attending each other's presentations, thereby providing a possibility for questions and comments both from other teams and from the teachers.

Table 1:
Timeline of the project course and related technical courses.

Year	Quarter	Project course activity	Theoretic courses & relevant concepts
1			Electrical Circuits • jω • Impedance Electro Project I • Basic project management & technical writing
2	1	Planning	Electromagnetic Field Theory • Electromagnetic induction, EMF Mechanics • Newton's 2nd law
2	2	Theoretic design Presentation	Signals & Systems I • Laplace, Fourier • Transfer function
2	3	Implementation Demonstration	Physic • Wave physics, acoustic impedance
2	4	Measurement Digital pre-processing Written reporting	Signals & Systems II • System properties • (Digital) filtering
	Year 3: Q1–Q2		Measurement technology

At KTH, the academic year is divided into four quarters (two per semester, Q1-Q4, see Table 1). The course thus begins a few weeks into the 1th quarter with an introductory seminar that provides practical information related to the course; presentation of the technical problems that the students will work on – designing and building a loudspeaker; an overview about working and thinking in terms of models when applying theory to practical problems. During this quarter, the students take their first course in electromagnetic theory, which provides some of the theory to be used during the second quarter while designing the speaker. Towards the end of quarter 1, the project teams submit preliminary work plans [9], for which the teachers give feedback. After some time to address the feedback, the project teams submit their work plans, which are graded by the teachers.

During the 2nd quarter, the students spend a substantial amount of time on modelling, analysing and designing their loudspeakers. It is however obvious that the students do not have the full theoretical background even in the 2nd quarter and need to search for some additional information, especially related to the concept of acoustic impedance.

The design parameters are to be simple enough so that the students can manufacture all of the parts except the magnet assembly themselves. Towards the end of quarter 2, the project teams present their designs to the class and teachers; and the presentations are graded. Once the design is approved, the students are allowed into a lab to start building their loudspeaker.

During the 3rd quarter, the project teams build their loudspeakers and evaluate their designs. The activity mainly takes place in the student workshop. At the end of the period there is an exhibition arranged, at which the teams demonstrate the performances of their designs and give a poster presentation. The performances and posters are graded.

After the speakers are built and presented, the task during the final 4th quarter is to improve the sound quality of the loudspeaker without changing the construction. The only possible method is by invoking digital filtering, which is a part of the Signal & Systems course. After this improvement of their loudspeakers, the teams work on their project reports; preliminary reports are submitted for peer reviews that are performed individually. Using the feedback from the peer reviews, the reports are finalized and submitted for grading.

Assessment

The assessment of the student is based on five different submissions/presentations (Project plan, Construction and calculation, Exhibition/Demonstration, Technical report, Peer-Review). The grading is a combination of individual and group assessments, thus the students will end up with individual grades. It is based on a structure we have used for many years at the program [2, 10]. The teacher reference group has the responsibility of the assessment process, but at some submissions other teachers are invited to participate as well as are the students (selecting for instance the best speaker at a fair where the speakers are demonstrated).

For each submission, there are criteria for plus and minus credits. For instance, submission 1, i.e., the work plan, has the following criteria.

+1, Preliminary work plan submitted in time before its deadline (otherwise no feedback from teachers).
+3 The work plan (final version) complies with the requirements and is understandable.
+1 The plan is judged to be supportive for the team when carrying out the project work.
+1 The plan is easy to grasp and is presented in a well-organized way.
–2 The plan is submitted after the deadline.

Similar structures are used for all five submissions. In the exhibition, one additional credit was also given for the "best loudspeaker" as voted by the teacher and student groups respectively. The final grade is then based on the sum of credits for each student.

OUTCOME

At the time of writing, the course is still running, but most of the assessment submissions have taken place and there has been one course evaluation meting with student representatives and teachers as well as a partial course evaluation enquiry.

The students understood more or less what was required of them regarding the task, but they found the project plan of limited value in their work, because some parts of the task took much more time than they had estimated.

They were generally able to use the knowledge from the courses in electrical circuits and electromagnetic field theory, but felt that the mechanics course had not provided them with enough of tools for the mechanical design. When we write this, the signals and systems course has not yet been applied in the project.

A somewhat discouraging view from several students was that the theory and modelling part of the project did not really help them to make a better loudspeaker. The practical constraints and application of guidelines from literature reduced the design freedom too much. The exhibition event when the students were to demonstrate the loudspeakers was also criticised by the students. Apparently it was a very noisy event.

The student felt that the course is overall "medium" (average 3 on a scale from 5 (very good) to 0 (very bad). They appreciated the goals presented from the start, but felt that the information given was too vague and that the course took to much time related to the 4.5 ECTS credits it gives.

The students were also asked how much time they had spent on theory/modelling and actual building. The average was around 50 h on theory/modelling and 70 h on building. The signals and systems task and the report writing is not yet completed at the time of writing, but can perhaps be assumed to correspond to another 30 h, giving a projected time of 150 h. With the course design goal of 120 h of work, the subjective experience of too much work agrees with the reported time usage.

The presented grading scale was by many students considered unfair, because of the voting by teachers and students for best loudspeakers. This feeling was reinforced by the fact that only those with very good results in terms of bonus credits could get an A grade.

From the teacher's perspective, we saw that the students were strongly engaged in the project. We also saw that the modelling and the design were a bit separated from each other. In fact the modelling was quite difficult for the students and they were in many cases happy to achieve some kind of model that could be reported. All teams made classical electrodynamic cone-type loudspeakers even though there are other possibilities.

We also note that the possibility for the students to book supervision meetings with the reference group was very little used, although we felt that this would have helped the students.

One important side effect the course has had from a program perspective is that it ties the faculty together, since a group of core teachers active at the bachelor's level both designed the project course and formed teacher reference group, which supervised and assessed the students. Situations like this, where teachers meet and together help the students creates awareness and sets in motion a discussion of what should be dealt with by teachers of other courses.

ANALYSIS AND SUGGESTED IMPROVEMENTS

A preliminary analysis suggests some changes for next round of the course.

The fact that the student did not feel that the time plan supported their work was perhaps partly because of the difficulties in estimating the time for various parts of the work. The feeling of not seeing the value of the plan were probably enhanced by the fact that there were no requirements for following up on the plan nor evaluating lessons learned in a final report. A bit more information and hints on time usage and perhaps also changes in the required form of the time plan could give a more realistic time plan resulting in a more structured work.

A little bit more guidance in the modelling task may be needed. One could think of more lectures content on modelling combined with a compulsory supervisor meeting to discuss the model at an early stage. This would also probably help to avoid some confusion and reduce the total time usage. On the other hand it is important to leave the implementation completely open.

In fact we would like to see some more unorthodox solutions such as attempts to make a piezo-based or electrostatic loudspeaker. Hinting at such possibilities in the introductory presentations and changing the teacher prize from "the best loudspeaker" to the "most innovative loudspeaker", may also promote this. However, we realize that this is perhaps the drawback of choosing a task based on a relatively well-known device: "everyone knows how a loudspeaker works."

Already at the present course round we plan to adjust the grading, so that also students in teams that have not received any of the distinctions for "best loudspeaker" have a fair chance of getting an A. The competition should be an extra encouragement, not the way to achieve an A grade.

CONCLUSION

The project course has achieved the main goal set at the program level, i.e., to strengthen the profile of the electrical engineering program. This by visualizing for the students what kind of work an electrical engineer actually does and how the theory taught in the courses on electrical magnetic field theory, circuit analysis, mechanics, etc., relates to the functionality of an everyday used device.

From an intended learning outcome perspective, the course has partly helped the students to reach the goals that were set. The student have learned to approach an engineering problem from a system perspective and to take the theories taught in the different courses and bring them together to model the functionality of a system composed of several parts. However, the course has not fully reached the goal that students should learn to apply their theoretical results when designing the hardware of a system. To achieve this, the system specifications in the project specification most set at a level so that the students have to apply the theory in the hardware design to meet the system specifications. Further, learning activities that helps the students reflect and analyze the outcome of the hardware designed to their theoretical model of the system should be introduced.

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A.4 Laboratory instructions for Elektroprojekt, del II, EH1020

EH1020 ELEKTROPROJEKT, Del II
Mätning

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Mars 2014

1 Allmänt

Nu när er högtalare börjar låta, är det dags att testa hur bra den är. Resultaten av dessa aktiviteter ska redovisas i er tekniska rapport. I det här PMet kommer en beskrivning av hur mätningarna ska göras.

2 Uppgifter

2.1 Frekvensområde

Ni har i er projektplan angett ett mål för i vilket frekvensområde er högtalare ska fungera. Er första mätuppgift är därför att mäta upp vilket frekvensområde er högtalare klarar av.

Vi vill att ni löser denna uppgift med hjälp av en tongenerator (som genererar sinustoner), en mikrofon och ett oscilloskop. Fundera på hur denna utrustning kan användas för att mäta högtalarens bandbredd. Fundera även på vad som är en lämplig definition av bandbredden.

2.2 Linearitet

Försök avgöra hur linjär er högtalare är, med hjälp av samma utrustning som i den förra uppgiften. Ni får själva föreslå hur mätningen ska gå till och hur resultaten ska tolkas.

2.3 Frekvenssvar

Som ett komplement till mätningen ni gör med signalgenerator och oscilloskop, har vi även gjort ett litet Matlab-program som genererar en brussignal, spelar in hur er högtalare, återger denna samt analyserar resultatet. För detta ändamål har vi gjort specialsladdar som fördelar utsignalen från datorn, dels till er högtalare, dels till vänsterkanalen på datorns ingång (med en impedansanpassning för att få lämplig signalstyrka). Signalen från mikrofonen kopplas till högerkanalen på datorns ingång. På så sätt har vi tillgång till både högtalar- och mikrofonsignalen och kan därifrån beräkna en uppskattning av högtalarsystemets frekvenssvar. Notera att denna specialsladd bara ska användas tillsammans med Matlab-programmet `characterizeLoudspeaker`.

`[in,out,Fs]=characterizeLoudspeaker` Spelar upp en brussignal (`in`) under 10 sekunder och spelar in motsvarande ljud som det uppfattas av mikrofonen (`ut`) samt samplingsfrekvensen `Fs`. Dessutom plottas på skärmen en uppskattning av Bode-diagrammet för högtalaren.

Rent praktiskt får ni alltså göra följande steg:

1. Koppla in både högtalarsignal och mikrofon via vår specialsladd, till mikrofoningången på datorn.
2. Kör `characterizeLoudspeaker`, vänta på att kommandoprompten kommer tillbaka (ca 10s).

Glöm inte spara alla användbara figurer på ett USB-minne (eller skicka dem via web-mail) för att ha till hands när ni skriver slutrapporten.

3 Utrustning

Ni behöver ha med er:

- Er högtalare.
- Egen förstärkare till högtalaren.

Sladdar kommer att finnas på plats för att ansluta de vanligast förekommande ljudkontakterna. Logga in på datorerna med ert vanliga KTH-id.

Observera

- Tänk på att ert mätresultat kommer att vara beroende på avstånd och riktning mellan högtalaren och mikrofonen. Se till att ni har med denna information när ni presenterar mätningarna i er tekniska rapport.
- Koppla in mikrofonen i datorn innan ni startar Matlab, annars verkar inte Matlab hitta den "device" som motsvarar mikrofonsignalen.
- Ni kan behöva ändra volyminställningen på datorn (både högtalar och mikrofonsignal) samt på er egen förstärkare, för att både undvika överstyrning och undvika att någon av signalerna blir för svaga. Matlab-funktionen `characterizeLoudspeaker` kollar att både in- och utsignal har vettig amplitud och skriver ut en varning annars.
- Stäng av mikrofonen (ta ur batteriet) när ni har labbat klart, annars tar batteriet slut väldigt snabbt!
- Innan ni kopplar in signalgeneratorn till er förstärkare, se till att den är inställd på amplitud 0.136V (eller åtminstone inte högre än 1V), så att ni inte riskerar att bränna er förstärkare.
- Matlab-funktionen bör finnas inlagd på alla datorerna, men om den inte gör det, finns en kopia i Bilda, under `Kursmaterial läsåret 2013/2014`.

A.5 Concept test question for Sig & Sys II, EQ1100

KTH Electrical Engineering

SNABBA FRÅGOR

1. Om utsignalen från ett tidsdiskret LTI system alltid är identiskt med systemets insignal, då är systemets pulsvar

A $h[n] = 0, \quad \forall n$

B $h[n] = 1, \quad \forall n$

C $h[n] = \delta[n]$

D Inget av dessa alternativ.

2. Om ett LTI system har ett tidsbegränsat impulssvar och man skickar in en tidsbegränsad signal, då kommer utsignalen $y[n]$ att vara

A $y[n] = 0, \quad \forall n$

B $y[n] = \alpha, \quad \forall n$

C $h[n] = \begin{cases} \alpha_n & n = K, K+1, \ldots, K+M \quad M < \infty \\ 0 & \text{annars} \end{cases}$

D Inget av dessa alternativ.

Signaler & System II 4 Föreläsning 3

KTH Electrical Engineering

3. Om $h[n] = 0$, förutmom $h[0] = h[1] = h[2] = 1/3$, vad är då den bästa beskrivningen av systemets filteregenskap?

A Ett lågpassfilter

B Ett högpassfilter

C Ett bandpassfilter

D Ett allpassfilter.

A.6 Project assignment for Optimal Filtering EM3200

Optimal Filtering, 2014–09–19
Due, 2012–10–27
Isaac Skog

Project Assignment

In this project assignment you will work with the design of a motion tracking filter for a global positioning system (GPS) receiver. The GPS-receiver is going to be used to track the motion of car driving around a race track. On the course homepage you can download a matlab file `GPSdata.mat` with the true (position) trajectory of the car and the pseudo-range measurements recorded by the GPS-receiver. Your job will be to design and tune a Kalman filter that tracks the motion of the car.

Today the GPS consists of 30 satellites that orbits the earth at an altitude of 22 000 km. These satellites constantly transmits time encoded signals that may be received by a GPS-receiver at earth. The GPS-receiver uses the received signals to calculated the distances between the GPS-receiver and the satellites. These distance estimates are generally referred to as pseudo-range measurements since they also include an range offset caused by the offset in the GPS-receiver clock. The pseudo range measurement to the i:th satellite at time k can be modelled as

$$y_k^{(i)} = \|\mathbf{p}_k^{(i)} - \mathbf{p}_k^{(rec)}\| + c\,\Delta t_k + v_k^{(i)}, \tag{1}$$

where $\mathbf{p}_k^{(i)}$ and $\mathbf{p}_k^{(rec)}$ is the position of the satellite[1] and the receiver, respectively. Further, c is the speed of light and $v_k^{(i)}$ is the measurement noise, which can be assumed zero-mean and have the variance $\mathbb{E}\{v_k^{(i)} v_l^{(j)}\} = \sigma_r^2\,\delta_{i-j,k-l}$. Moreover, the clock offset in the receiver can be modelled as

$$\begin{bmatrix}\Delta t_{k+1}\\ \dot{\Delta} t_{k+1}\end{bmatrix} = \begin{bmatrix}1 & T_s\\ 0 & 1\end{bmatrix}\begin{bmatrix}\Delta t_k\\ \dot{\Delta} t_k,\end{bmatrix} + \mathbf{w}_k^{clk}, \tag{2}$$

where T_s is the sampling period and $\mathbf{w}_k^{clk}$ is white noise with the covariance

$$\mathbf{Q}_k^{clk} = \begin{bmatrix} S_\phi T_s + T_s^3/3S_f & T_s^2 S_f\\ T_s^2 S_f & S_f T_s\end{bmatrix}. \tag{3}$$

Here S_ϕ and S_f is the power spectral density of the phase and frequency process noise.

If the signals from four or more satellites are received then its possible to, without introducing a model for the motion of the GPS-receiver or the clock-offset, calculate a position estimate using a nonlinear least squares algorithm. The matlab function `NonLinearLeastSquares.m` that you can download from the homepage is an implementation of such an algorithm. The function also gives an illustration of how to interpret the data content in the `GPSdata.mat` file.

[1]The position of the satellite $\mathbf{p}_k^{(i)}$ is known by the receiver, but the satellites from which the GPS-receiver can receive signals varies with time and the position of the receiver.

1

Download the material from the homepage and do the following:

i) Plot the trajectory of the car and the position estimates calculated by the GPS-receiver if the nonlinear least squares algorithm is used.

ii) A survey of motion dynamic models for target tracking is presented in [1]. Combine the constant velocity or constant acceleration model described on pages 1333-1337 in the paper with the clock model in (2) and implement a Kalman filter that tracks the motion of the car.

  a) Specify the state-space and observation equations that you use in your filter.
  b) Is the stochastic process described by the state-space equations a stationary process? Does it matter?
  c) What happens if the GPS-receiver stops receiving signals from the satellites? How will that affect the state covariance?

iii) Plot the position estimation errors (each coordinate axis in a separate plot) and clock offset and clock drift estimation errors. Include also the 3σ confidince intervals based on the state covariance outputted by the filters. Further, in the plots with the position estimation errors, also included the results of the non-linear least squares algorithm.

  a) What happends when the covariance of the process noise of the motion dynamic model becomes very small?
  b) What happends when the covariance of the process noise of the motion dynamic model becomes very large?
  c) Discuss how the innovation and the 3σ confidence intervals can be used to tune the filter?
  d) Throughout the course we have studied linear estimators (filters) that are optimal in the mean square error sense. Name at least two reasons why the implemented filter may not be optimal?

You may work 1 or 2 people in each group. Please write a technical report (not a thesis!) per group illustrating your examples and document your conclusions. Write the report in the format of a conference paper, for example using the style file and templates available at

`http://www.cmsworldwide.com/ICASSP2011/papers/PaperKit.html`.

The report should be submitted by email to `skog@kth.se` and `gerami@kth.se`.

References

[1] Li, X.R.; Jilkov, V.P., "Survey of maneuvering target tracking. Part I. Dynamic models, *IEEE Transactions on Aerospace and Electronic Systems , vol.39, no.4, pp.1333-1364, Oct. 2003,*

A.7 Homework for WASP Autonomous Systems course

SENSING AND PERCEPTION:
LOCALIZATION AND POSITIONING
ISAAC SKOG

Homework: GNSS-positioning

In this homework you will study how global navigation satellite system (GNSS) receiver calculates its position estimates. The GNSS-receiver is going to be used to track the motion of car driving around a race track and your task is to: (i) implement a nonlinear least squares estimator that estimates the position of the car, and (ii) study the effect of range errors and the satellite geometry constellation on the accuarcy of the GNSS-receiver. On the course homepage you can download a matlab file `GPSdata.mat` with the true (position) trajectory of the car and the data recorded by the GNSS-receiver.

Today the global positioning system (GPS), which is one out of four GNSS systems, consists of 30 satellites that orbits the earth at an altitude of 22 000 km. These satellites constantly transmits time encoded signals that may be received by a GPS-receiver at earth. The GPS-receiver uses the received signals to calculated the distances between the GPS-receiver and the satellites. These distance estimates are generally referred to as pseudo-range measurements since they also include an range offset caused by the offset between the GPS-receiver clock and the GPS satellite clocks. The pseudo range measurement to the i:th satellite at time k can be modelled as

$$y_k^{(i)} = h_i(\mathbf{p}_k^{(rec)}, \Delta t_k) + v_k^{(i)}, \tag{1}$$

where

$$h_i(\mathbf{p}, \Delta t_k) = \|\mathbf{p}_k^{(i)} - \mathbf{p}\| + c\,\Delta t_k. \tag{2}$$

Here $\mathbf{p}_k^{(i)}$ and $\mathbf{p}_k^{(rec)}$ denotes the position of the i:th satellite[1] and the receiver, respectively. Further, c denotes the speed of light, Δt_k denotes the offset in the GPS-receivers clock, and $v_k^{(i)}$ denotes the measurement noise, which can be assumed zero-mean with the covariance $\mathbb{E}\{v_k^{(i)} v_l^{(j)}\} = \sigma_{r_i}^2\,\delta_{i,j,k,l}$. Assuming that signals from M satellites are received, then the measurements from these satellites can be modelled as

$$\mathbf{y}_k = \mathbf{h}(\boldsymbol{\theta}_k) + \mathbf{v}_k \tag{3}$$

where

$$\mathbf{y}_k = \begin{bmatrix} y_k^{(1)} & \dots & y_k^{(M)} \end{bmatrix}^\top \quad \boldsymbol{\theta}_k = \begin{bmatrix} (\mathbf{p}_k^{(rec)})^\top & \Delta t_k \end{bmatrix}^\top \quad \mathbf{v}_k = \begin{bmatrix} v_k^{(1)} & \dots & v_k^{(M)} \end{bmatrix}^\top$$

and

$$\mathbf{h}(\boldsymbol{\theta}_k) = \begin{bmatrix} h_1(\mathbf{p}_k^{(rec)}, \Delta t_k) & \dots & h_M(\mathbf{p}_k^{(rec)}, \Delta t_k) \end{bmatrix}^\top.$$

[1] The position of the satellite $\mathbf{p}_k^{(i)}$ is known by the receiver, but the satellites from which the GPS-receiver can receive signals varies with time and the position of the receiver.

Given the signal model in (3) the nonlinear least square estimator for the position and clock offset is given by

$$\widehat{\boldsymbol{\theta}}_k = \underset{\boldsymbol{\theta}}{\operatorname{argmin}}\Big(\|\mathbf{y}_k - \mathbf{h}(\boldsymbol{\theta})\|^2_{\mathbf{Q}^{-1}} \Big) \tag{4}$$

where $\|\mathbf{a}\|^2_{\mathbf{P}} = \mathbf{a}^\top \mathbf{P} \mathbf{a}$ and $\mathbf{Q} = \operatorname{diag}(\sigma^2_{r_1}, \ldots, \sigma^2_{r_M})$.

TASK #1: Download the material from the home page and implement a nonlinear least square estimator that estimates the positions of the car. You may use Matlab's nonlinear least squares solvers or implement you own solver. Print a plot that shows the estimated trajectory as well as the true trajectory of the car.

TASK #2: What is the minimum number of satellites M that the GNSS-receiver must be able to measure the distance to in order for the nonlinear least square estimation problem in (4) to be well defined?

The accuracy of the nonlinear least square estimator is given by

$$\operatorname{Cov}(\hat{\boldsymbol{\theta}}) \simeq \Big(\mathbf{H}^\top \mathbf{Q}^{-1} \mathbf{H} \Big)^{-1} \tag{5}$$

where

$$\mathbf{H} = \begin{bmatrix} \nabla_{\boldsymbol{\theta}} h_1 \\ \vdots \\ \nabla_{\boldsymbol{\theta}} h_M \end{bmatrix} \tag{6}$$

and $\nabla_{\boldsymbol{\theta}} h_i$ is the gradient of the function $h_i(\mathbf{p}, \Delta t_k)$ defined in (2).

TASK #3: Use (5) to evaluate the theoretical accuracy of the nonlinear least squares solver that you implemented in Task #1. Plot the 3σ bounds for the horizontal and vertical accuracy as well as the horizontal and vertical estimation error as a function of time. How is the agreement between the theoretical and true accuracy? In which direction is the positioning accuracy worst?

TASK #4: Assuming that only five satellites are used in the position calculation, and the satellites positions in the local tangent plane (North,East, Up) in polar coordinates {Azimuth, Inclination} are given by $\{0, \pi/2\}$, $\{0, x\}$, $\{\pi/2, x\}$,$\{\pi, x\}$, and $\{3\pi/4, x\}$. Further, assume that variance of the ranging error are the same for all satellites. Plot how the horizontal and vertical accuracy varies with the inclination $x \in (0, \pi/2)$. By aid of the plots, discuss how an urban environment with a lot of high-rise building effects the positioning accuracy of the GPS receiver. What other reasons are there that may cause a GPS receiver to work poorly in an urban area with a lot of high rise buildings?

Unfortunately, most GPS receivers doesn't output the full covariance matrix $\text{Cov}(\hat{\boldsymbol{\theta}})$, but instead output one, or multiple, so called dilution-of-precision (DOP) figures. These DOP figures indicates the goodness of the satellite geometry from a positioning accuracy perspective, and can be related to the position accuracy as

$$\text{Tr}\{\text{Cov}(\hat{\boldsymbol{\theta}})\} \approx \text{UERE} \cdot \text{DOP}. \tag{7}$$

Here, UERE stands for user equivalent ranging error and is a measure of the size of the typical ranging error.

TASK #5: If $\sigma^2_{r_i} = \sigma^2_{r_j}$ $\forall i, j$, show that the right hand side of (5) can be factorized in two parts, one which only depends on the ranging accuracy and one that only depends on the direction to the satellites from the GNSS-receiver.

TASK #6: If the GPS-receiver position estimates are to be fused with the measurements from another positioning sensor, what is the drawback of only having access to the dilution-of-precision (DOP) figures and not the full covariance matrix $\text{Cov}(\hat{\boldsymbol{\theta}})$?

A.8 Project assignment for WASP Autonomous Systems course

Sensing and Perception:
Localization and positioning
Isaac Skog

Project Assignment: GNSS aided INS

In this project assignment you will work with a type of navigation system referred to as a global navigation satellite system (GNSS) aided inertial navigation system (INS), and evaluate the effects of adding a motion model and additional sensors to the navigation system. The GNSS-aided INS is going to be used to track the motion of a car and your task is to: (i) evaluate the effects of GNSS signal outage on the navigation solution, (ii) implement a vehicle motion model using non-holonomic constraints, and (iii) include the measurements from a speedometer into the navigation fusion filter. On the course homepage you can download a ZIP-folder (`GNSSaidedINS.zip`) with Matlab files that implements a GNSS-aided INS and a data set with real-world GNSS-receiver, speedometer, and inertial measurement unit (IMU) data.

Next a short description of the GNSS-aided INS system implemented in the Matlab code, is presented. To run the GNSS-aided INS execute the file `main.m`. If interested in a more in-depth description of GNSS-aided INS the book [1] provides an excellent introduction to the topic. If you have any questions about the Matlab code or the project in general contact me at `skog@kth.se`.

GNSS aided INS – A short introduction

Inertial navigation is a self-contained navigation technique in which measurements provided by accelerometers and gyroscopes are used to track the position, velocity, and attitude (orientation) of an object relative to a known start position, velocity, and attitude. Since an INS is self-contained, i.e., it does not rely on any external information sources that can be disturbed or jammed, it is an attractive means of navigation for many applications where 100% coverage and a high continuity-of-service is needed. Further, it also provides a full 6 degree-of-freedom navigation solution and generally has a high update rate ($\geq$ 100 Hz); properties of high importance in the design of various autonomous systems. In Fig. 1 a block diagram of an INS for navigation in a geographically limited area using low-cost inertial sensors (accelerometers and gyroscopes), is shown. Mathematically, the navigation process of an INS can be described as follows. Define the navigation state vector and inertial measurement input vector as

$$\mathbf{x}_k \triangleq \begin{bmatrix} \mathbf{p}_k^\top & \mathbf{v}_k^\top & \mathbf{q}_k^\top \end{bmatrix}^\top \tag{1}$$

and

$$\mathbf{u}_k \triangleq \begin{bmatrix} \mathbf{s}_k^\top & \omega_k^\top \end{bmatrix}^\top, \tag{2}$$

respectively. Here $\mathbf{p}_k$ $[m]$, $\mathbf{v_k}$ $[m/s]$, and $\mathbf{q}_k$ $[-]$ denote the position, velocity, and attitude (quaternion representation) of the navigation system at time instant k.

1

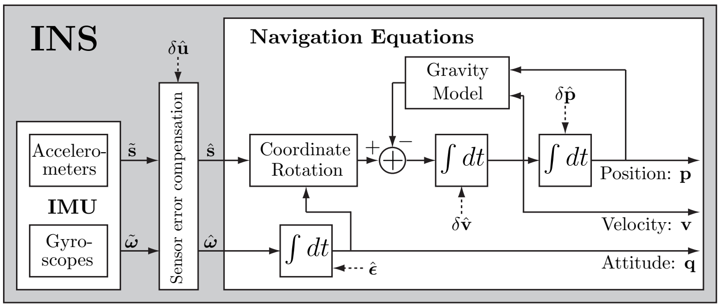

Figure 1: Conceptional sketch of an INS. The dash arrows indicates possible points for insertion of correction (calibration) data.

Further, $\mathbf{s}_k$ $[m/s^2]$ and $\boldsymbol{\omega}_k$ $[rad/s]$ denote the specific force[1] and angular velocity, respectively. A discrete time version of the inertial navigation equations in Fig. 1 is given by the nonlinear differential equation

$$\mathbf{x}_k = f(\mathbf{x}_{k-1}, \mathbf{u}_k), \tag{3}$$

where

$$\mathbf{p}_k = \mathbf{p}_{k-1} + T_s \mathbf{v}_{k-1} + \frac{T_s^2}{2}\Big(\mathbf{R}_b^n(\mathbf{q}_{k-1})\mathbf{s}_k - \mathbf{g}\Big) \tag{4}$$

$$\mathbf{v}_k = \mathbf{v}_{k-1} + T_s\Big(\mathbf{R}_b^n(\mathbf{q}_{k-1})\mathbf{s}_k - \mathbf{g}\Big) \tag{5}$$

$$\mathbf{q}_k = \Big(\cos(0.5\,T_s\,\|\boldsymbol{\omega}_k\|)\mathbf{I}_4 + \frac{1}{\|\boldsymbol{\omega}_k\|}\sin(0.5\,T_s\,\|\boldsymbol{\omega}_k\|)\boldsymbol{\Omega}_k\Big)\mathbf{q}_{k-1} \tag{6}$$

and

$$\boldsymbol{\Omega}_k = \begin{bmatrix} 0 & [\boldsymbol{\omega}_k]_z & -[\boldsymbol{\omega}_k]_y & [\boldsymbol{\omega}_k]_x \\ -[\boldsymbol{\omega}_k]_z & 0 & [\boldsymbol{\omega}_k]_x & [\boldsymbol{\omega}_k]_y \\ [\boldsymbol{\omega}_k]_y & -[\boldsymbol{\omega}_k]_x & 0 & [\boldsymbol{\omega}_k]_z \\ -[\boldsymbol{\omega}_k]_x & -[\boldsymbol{\omega}_k]_y & -[\boldsymbol{\omega}_k]_z & 0 \end{bmatrix}. \tag{7}$$

Here, T_s denotes the sampling period of the data and $\mathbf{R}_b^n(\mathbf{q})$ denotes the directional cosine matrix (rotation matrix) that rotates a vector from the body coordinate frame (b-frame) to the navigation coordinate frame (n-frame). Further, $\mathbf{g}$ denotes the gravity vector expressed in the navigation coordinate system. Noteworthy is that the attitude in the mechanized INS equations is described and propagated using the quaternion vector $\mathbf{q}$. It is possible to use other attitude representations such as Euler angles or directional cosine matrices, but the quaternion representation is generally more numerically stable. The function `Nav_eq.m` in the folder with the Matlab files implements these navigation equations. The folder also includes several functions for converting between different attitude representations such as Euler angles and directional cosine matrices.

[1] The output of an accelerometer is the difference between the inertial and gravitational acceleration, which is referred to as specific force.

Neglecting the discretization and quantization errors introduced in the mechanization of the navigation equations, the inertial navigation system will perfectly track the position, velocity, and attitude of the navigation platform as long as the specific force and angular velocity is measured without errors. In reality there exist no error free measurements and due to the integrative nature of the inertial navigation equations, the measurement errors will be accumulated and cause the position and velocity errors in the navigation solution to grow without bound. As a rule of thumb, for an INS using low-cost sensors, the position error grows cubically with time and is proportional to the magnitude of the bias in the gyroscope measurements.

TASK #1: Use the function `Nav_eq.m` to evaluate how the position error grows with time. Assume that the navigation system is stationary; the initial position and velocity is zero; the initial roll, pitch, and yaw are zero; the accelerometer measurements are error free; and the gyroscope measurements are error free except from a bias in the x-axis gyroscope with a magnitude of $0.01\ ^\circ/s$. Plot the error growth as a function of time. Approximately how long time can you navigate before the position error is 10 meters?

From your simulation it should be clear that an INS that uses low-cost inertial sensors only can be used for stand alone navigation for a very limited time period. To overcome this problem, the navigation information provided by a low-cost INS is frequently fused with the navigation information provided by a GNSS-receiver. If done properly the result is a navigation system that: (1) has an position accuracy as a GNSS-receiver; (2) provides a full 6 degrees-of-freedom navigation solution; (3) has a high update rate; (4) for shorter time periods can provide a navigation solution even during GNSS-receiver outages; and (5) has a higher system integrity as faulty GNSS-measurements can be detected and rejected.

In Fig. 2 one way of fusing the information between an INS and a GNSS-receiver is shown. The displayed information fusion strategy is referred to as a loosely-coupled[2] closed-loop GNSS-aided INS configuration. The main idea is that the INS is used as the backbone in the navigation system and constantly provides a 6 degree-of-freedom navigation solution. Whenever the GNSS-receiver produces a position estimate the difference between the position estimates of the two systems is calculated and used as the input for a filter that tries to estimates the errors of the INS navigation solution, as well as the errors in the IMU sensors. The estimated errors are then used to correct the navigation solution and to calibrate the sensors. A short discussion about various GNSS-aided INS information fusion strategies and their pros and cons can be found in [2].

Many different filter algorithms can be used to estimate the errors in the navigation solution. In the GNSS-aided INS implemented in the Matlab code, a standard Kalman filter algorithm is used. The Kalman filter uses the following linear state space model to describe the error dynamics of the INS as well as

[2] The term *loosely-coupled* refers to that the position solutions provided by the GNSS-receiver, and not the pseudo range measurements, are used as measurements in the information fusion.

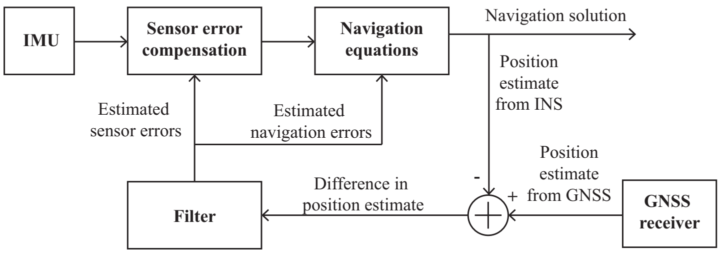

Figure 2: Conceptional sketch of a loosely-coupled closed-loop GNSS-aided INS.

dynamics of the inertial sensor errors. Let the IMU measurements be described by the signal model

$$\tilde{\mathbf{u}}_k = \mathbf{u}_k - \delta\mathbf{u}_k + \mathbf{w}_k^{(1)} \tag{8}$$

where $\mathbf{w}_k^{(1)}$ denotes the measurement noise which is assumed to be additive white noise with covariance matrix $\mathbf{Q}^{(1)}$, and $\delta\mathbf{u}_k$ denotes the slowly varying measurement bias which is modeled as the random walk process

$$\delta\mathbf{u}_k = \delta\mathbf{u}_{k-1} + \mathbf{w}_k^{(2)}. \tag{9}$$

Here, $\mathbf{w}_k^{(2)}$ denotes the random walk process noise which is assumed to be additive white noise with covariance matrix $\mathbf{Q}^{(2)}$; the random walk process noise and the measurement noise are assumed uncorrelated. Next, define the perturbation between the INS estimated $\hat{\mathbf{x}}_k$ navigation state and the true navigation state $\mathbf{x}_k$ as

$$\delta\mathbf{x}_k \triangleq \begin{bmatrix}\delta\mathbf{p}_k^\top & \delta\mathbf{v}_k^\top & \boldsymbol{\epsilon}_k^\top\end{bmatrix}^\top, \tag{10}$$

where the position perturbation (error) $\delta\mathbf{p}_k$ and velocity perturbation (error) $\delta\mathbf{v}_k$ are defined as $\delta\mathbf{p}_k = \mathbf{p}_k - \hat{\mathbf{p}}_k$ and $\delta\mathbf{v}_k = \mathbf{v}_k - \hat{\mathbf{v}}_k$, respectively. The attitude perturbation vector $\boldsymbol{\epsilon}_k$ is defined as the small (Euler) angle sequence that that rotates the attitude vector $\hat{\mathbf{q}}_k$ into $\mathbf{q}_k$. That is, $\mathbf{q}_k = \Gamma(\hat{\mathbf{q}_k}, \boldsymbol{\epsilon}_k)$, where the implicit function Γ is defined as

$$\Gamma(\hat{\mathbf{q}}, \boldsymbol{\epsilon}) \triangleq \{q \in \mathbb{S}^3 | \mathbf{R}_b^n(\mathbf{q}) = (\mathbf{I}_3 - [\boldsymbol{\epsilon}]_\times)\mathbf{R}_b^n(\hat{\mathbf{q}})\}. \tag{11}$$

Here, $[\mathbf{a}]_\times$ denotes the skew-symmetric matrix representation of $\mathbf{a}$, for which $[\mathbf{a}]_\times\mathbf{b} = \mathbf{a} \times \mathbf{b}$. With the perturbation vector defined as in (10), the error propagation in the navigation equations (3) when fed with the measurement vector $\tilde{\mathbf{u}}_k$ can, for small error perturbations, be described by the linear state space model

$$\mathbf{z}_k = \mathbf{F}(\mathbf{x}_k, \mathbf{u}_k)\mathbf{z}_{k-1} + \mathbf{G}_k(\mathbf{x}_k)\mathbf{w}_k, \tag{12}$$

where

$$\mathbf{z}_k \triangleq \begin{bmatrix}\delta\mathbf{x}_k \\ \delta\mathbf{u}_k\end{bmatrix} \quad \mathbf{w}_k \triangleq \begin{bmatrix}\mathbf{w}_k^{(1)} \\ \mathbf{w}_k^{(2)}\end{bmatrix} \tag{13}$$

and the state transition matrix and noise gain matrix are defined as

$$\mathbf{F}(\mathbf{x}_k,\mathbf{u}_k)=\begin{bmatrix}\mathbf{I}_3 & T_s\mathbf{I}_3 & \mathbf{0}_{3,3} & \mathbf{0}_{3,3} & \mathbf{0}_{3,3}\\ \mathbf{0}_{3,3} & \mathbf{I}_3 & T_s[\mathbf{R}_b^n(\mathbf{q}_k)\mathbf{s}_k]_\times & T_s\mathbf{R}_b^n(\mathbf{q}_k) & \mathbf{0}_{3,3}\\ \mathbf{0}_{3,3} & \mathbf{0}_{3,3} & \mathbf{I}_3 & \mathbf{0}_{3,3} & -T_s\mathbf{R}_b^n(\mathbf{q}_k)\\ \mathbf{0}_{3,3} & \mathbf{0}_{3,3} & \mathbf{0}_{3,3} & \mathbf{I}_3 & \mathbf{0}_{3,3}\\ \mathbf{0}_{3,3} & \mathbf{0}_{3,3} & \mathbf{0}_{3,3} & \mathbf{0}_{3,3} & \mathbf{I}_3\end{bmatrix} \tag{14}$$

and

$$\mathbf{G}_k(\mathbf{x}_k)=\begin{bmatrix}\mathbf{0}_{3,3} & \mathbf{0}_{3,3} & \mathbf{0}_{3,3} & \mathbf{0}_{3,3}\\ T_s\mathbf{R}_b^n(\mathbf{q}_k) & \mathbf{0}_{3,3} & \mathbf{0}_{3,3} & \mathbf{0}_{3,3}\\ \mathbf{0}_{3,3} & T_s\mathbf{R}_b^n(\mathbf{q}_k) & \mathbf{0}_{3,3} & \mathbf{0}_{3,3}\\ \mathbf{0}_{3,3} & \mathbf{0}_{3,3} & \mathbf{I}_3 & \mathbf{0}_{3,3}\\ \mathbf{0}_{3,3} & \mathbf{0}_{3,3} & \mathbf{0}_{3,3} & \mathbf{I}_3\end{bmatrix}, \tag{15}$$

respectively. Next, the GNSS-receiver position measurements are modelled as

$$\tilde{\mathbf{p}}_k^{\text{GPS}}=\mathbf{p}_k+\mathbf{e}_k^{(1)} \tag{16}$$

where $\mathbf{e}_k^{(1)}$ is additive white noise with covariance matrix $\mathbf{R}^{(1)}$. Then the observation equation for the INS error propagation state space model can be modelled as

$$\mathbf{y}_k^{(1)}\triangleq\tilde{\mathbf{p}}_k^{\text{GPS}}-\hat{\mathbf{p}}_k=\mathbf{H}^{(1)}\mathbf{z}_k+\mathbf{e}_k^{(1)} \tag{17}$$

where

$$\mathbf{H}^{(1)}\triangleq\begin{bmatrix}\mathbf{I}_3 & \mathbf{0}_{3,12}\end{bmatrix} \tag{18}$$

With the state space model as defined by (12) and (17) the Kalman filter based algorithm for realizing the loosely-coupled closed-loop GNSS-aided INS is given in Algorithm 1 (See last page.). Noteworthy is that the Kalman filter time update does not include any propagation of the state vector $\mathbf{z}_k$. The reason for this is that after the estimated navigation state perturbations $\delta\hat{\mathbf{x}}$ and sensor biases $\delta\hat{\mathbf{u}}$ have been fed back into the INS and used to correct the navigation solution the errors are considered to be equal to zero.

TASK #2: Familiarize yourself with the Matlab code that implements the GNSS-aided INS and execute the script `main.m`. Modify the code to simulate a GNSS-receiver outage from 260 seconds and onward. Plot the position error growth as a function of time. You can assume the navigation solution from GNSS-aided INS without GNSS outages to be the true trajectory.

Even though the GNSS-aided INS estimates the sensor biases the position error growth during the GNSS-signal outages is substantial and the navigation solution soon becomes useless. One way to reduce the error growth rate is to include a model of the vehicle's motion dynamics into the information fusion process. There are many different ways to include motion dynamics models into the information process. One way is to define a set of motion constraints and enforces these constraints on the navigation solution using so called constrained filtering. See [4] for an introduction to different constraint filtering techniques.

The motion of a wheeled vehicle on a surface is governed by two non-holonomic constraints. During ideal driving conditions a vehicle experience no side slip and no motion in the direction normal to the road surface, and the velocity in the y-axis and z-axis direction of the vehicle coordinate frame are constrained to be equal to zero. In reality the velocity in the y-axis and z-axis direction of the vehicle coordinate frame will not be perfectly zero and the constraint has to be relaxed. One way to include a set of "relaxed" motion constraints into the information fusion filter is through the relationship [5]

$$\mathbf{A}\mathbf{R}_b^p\mathbf{R}_n^b(\mathbf{q})\mathbf{v}^n = \mathbf{e}^{(2)}, \quad \mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{19}$$

where $\mathbf{R}_b^p$ denotes the directional cosine matrix that rotates a vector from the IMU coordinate frame (b-frame) to the vehicle coordinate frame (p-frame), and $\mathbf{e}^{(2)}$ denotes a small error term that describes the deviation from the constraint. Modeling the error term $\mathbf{e}^{(2)}$ as a white noise term with covariance $\mathbf{R}^{(2)}$, then the relaxed constraint can be included in the Kalman filter framework by introducing the pseudo-observation

$$\mathbf{y}_k^{(2)} \triangleq \mathbf{0}_{2,1} - \mathbf{A}\mathbf{R}_b^p\mathbf{R}_n^b(\hat{\mathbf{q}}_k)\hat{\mathbf{v}}_k^n = \mathbf{H}^{(2)}\mathbf{z}_k + \mathbf{e}_k^{(2)} \tag{20}$$

where

$$\mathbf{H}^{(2)} \triangleq \begin{bmatrix} \mathbf{0}_{2,3} & \mathbf{A}\mathbf{R}_b^p\mathbf{R}_n^b(\hat{\mathbf{q}}_k) & \mathbf{0}_{2,9} \end{bmatrix}. \tag{21}$$

TASK #3: Implement the non-holonomic motion constraints into the GNSS-aided INS framework using the idea of pseudo observations. Tune the filter by changing the magnitude of $\mathbf{Q}^{(2)}$ until you get a decent performance and then plot the position error growth during the GNSS outage. The matrix $\mathbf{R}_b^p$ can be found by calling the function `get_Rp2b.m`.

Another way to improve the navigation performance during GNSS signal outages is to include additional sensors. A commonly used sensor in vehicle applications is a speedometer. If a speedometer is available, the measurements $\tilde{s}_k$ from this sensor can be included in the Kalman filter framework by introducing the observation equation

$$y_k^{(3)} \triangleq \tilde{s}_k - \mathbf{B}\mathbf{R}_b^p\mathbf{R}_n^b(\hat{\mathbf{q}}_k)\hat{\mathbf{v}}_k^n = \mathbf{H}^{(3)}\mathbf{z}_k + e_k^{(3)}, \tag{22}$$

where

$$\mathbf{H}^{(3)} \triangleq \begin{bmatrix} \mathbf{0}_{1,3} & \mathbf{B}\mathbf{R}_b^p\mathbf{R}_n^b(\hat{\mathbf{q}}_k) & \mathbf{0}_{1,9} \end{bmatrix}, \quad \mathbf{B} \triangleq \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \tag{23}$$

and $e_k^{(3)}$ denotes the measurement noise.

TASK #4: The data included in the `GNSSaidedINS.zip` folder also includes measurements from a speedometer. Modify the GNSS-aided INS information fusion filter to also include these measurements, and then plot the position error growth during the GNSS outage.

Project report

Describe the project and the results of the four tasks in a 2 page double column report. E-mail the report (in PDF format) to `skog@kth.se`. Name the file with your first and last name like `FirstName_LastName_Proj2.pdf`. Do not submit any Matlab code.

References

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[4] D. Simon, "Kalman filtering with state constraints: a survey of linear and nonlinear algorithms", *IET Control Theory & Applications*, vol. 4, no. 8, pp. 1303-1318, Aug. 2010.

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Algorithm 1 GNSS-aided INS algorithm.

$k = 1$
$\hat{\mathbf{x}} \leftarrow$ **Process**{ Initial navigation state }
$\delta\hat{\mathbf{u}} \leftarrow$ **Process**{ Initial sensor bias state estimate }
$\mathbf{P} \leftarrow$ **Process**{ Initial Kalman filter state covariance matrix }
loop
  `% Calibrate the sensor measurements using current sensor bias estimate.`
  $\hat{\mathbf{u}} \leftarrow \tilde{\mathbf{u}}_k + \delta\hat{\mathbf{u}}$

  `% Update the INS navigation state.`
  $\hat{\mathbf{x}} \leftarrow f(\hat{\mathbf{x}}, \hat{\mathbf{u}})$

  `% Time update of the Kalman filter state covariance.`
  $\mathbf{P} \leftarrow \mathbf{F}(\hat{\mathbf{x}}, \hat{\mathbf{u}})\mathbf{P}\,\mathbf{F}^\top(\hat{\mathbf{x}}, \hat{\mathbf{u}}) + \mathbf{G}(\hat{\mathbf{x}}) \begin{bmatrix} \mathbf{Q}^{(1)} & \mathbf{0}_{3,3} \\ \mathbf{0}_{3,3} & \mathbf{Q}^{(2)} \end{bmatrix} \mathbf{G}^\top(\hat{\mathbf{x}})$

  if GNSS measurement available **then**

    `% Calculate the Kalman filter gain.`
    $\mathbf{K} \leftarrow \mathbf{P}(\mathbf{H}^{(1)})^\top \left(\mathbf{H}^{(1)}\mathbf{P}(\mathbf{H}^{(1)})^\top + \mathbf{R}^{(1)}\right)^{-1}$

    `% Calculate the measurement vector.`
    $\mathbf{y}^{(1)} \leftarrow \tilde{\mathbf{p}}_k^{\text{GPS}} - \hat{\mathbf{p}}$

    `% Update the perturbation state estimate.`
    $\begin{bmatrix} \delta\hat{\mathbf{x}} \\ \delta\hat{\mathbf{u}} \end{bmatrix} \leftarrow \begin{bmatrix} \mathbf{0}_{9,1} \\ \delta\hat{\mathbf{u}} \end{bmatrix} + \mathbf{K}\left(\mathbf{y}^{(1)} - \mathbf{0}_{3,1}\right)$

    `% Update the Kalman filter state covariance.`
    $\mathbf{P} \leftarrow \left(\mathbf{I}_{15} - \mathbf{K}\mathbf{H}^{(1)}\right)\mathbf{P}$

    `% Correct the navigation states using current perturbation estimates.`
    $\hat{\mathbf{p}} \leftarrow \hat{\mathbf{p}} + \delta\hat{\mathbf{p}}$
    $\hat{\mathbf{v}} \leftarrow \hat{\mathbf{v}} + \delta\hat{\mathbf{v}}$
    $\hat{\mathbf{q}} \leftarrow \Gamma(\hat{\mathbf{q}}, \hat{\boldsymbol{\epsilon}})$
  end if
  $k \leftarrow k + 1$
end loop