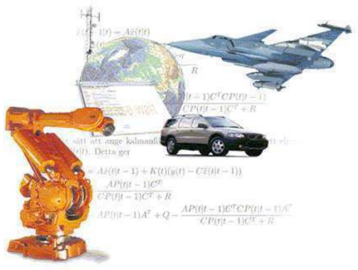
A Gaussian Mixture PHD filter for Extended Target Tracking



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Definition: Extended Target Tracking

- Extended targets often modelled as having extension or shape.
- All targets have extensions and shapes.
- Estimate target size and shape depending on...
  1. ...sensor/target setup.
  2. ...application.

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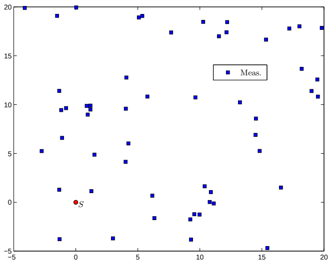
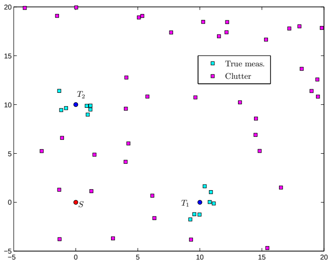
Definition:
Extended targets are targets that potentially give rise to more than one measurement per time step.

- Multiple extended targets, cluttered measurements ⇒ difficult DA.



Contribution

- Multiple extended targets, cluttered measurements $\Rightarrow$ difficult DA.
- GM-PHD-filter works well for cluttered measurements [Vo and Ma, IEEE TSP 2006].
- PHD filter for extended targets, Poisson assumption [Mahler, FUSION 2009].



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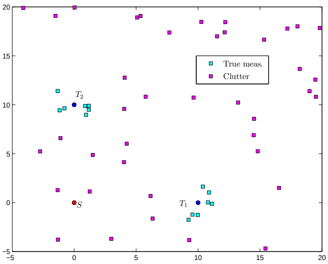

Main contribution:
Implementation of GM-PHD-filter for extended targets.
Minor contribution:
Simple method for measurement set partitioning.

Target tracking setup

- RFS of targets $\mathbf{X}_k = \{\mathbf{x}_k^{(i)}\}_{i=1}^{N_{x,k}}$

$$\mathbf{x}_{k+1}^{(i)} = F_k \mathbf{x}_k^{(i)} + G_k \mathbf{w}_k^{(i)}, \qquad \mathbf{w}_k^{(i)} \in \mathcal{N}\left(\mathbf{0}, Q_k\right).$$

- RFS of measurements $\mathbf{Z}_k = \{\mathbf{z}_k^{(j)}\}_{j=1}^{N_{z,k}}$

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- Generated measurements per target per time step

$$N_{m,k}^{(i)} \in \mathcal{POIS}\left(\beta_D\right).$$

- Effective probability of target detection

$$p_{D,\text{eff}} = \left(1 - e^{-\beta_D}\right) p_D.$$

Measurement Pseudo Likelihood

- Prediction of PHD-intensity performed identically to [Vo and Ma, 2006].
- $v_{k|k-1}(\mathbf{x}|\mathbf{Z})$ is predicted PHD-intensity. Corrected PHD-intensity

$$v_{k|k}(\mathbf{x}|\mathbf{Z}) = L_{\mathbf{Z}_k}(\mathbf{x})\, v_{k|k-1}(\mathbf{x}|\mathbf{Z}),$$

where measurement pseudo-likelihood is given by

$$L_{\mathbf{Z}_k}(\mathbf{x}) = 1 - \left(1 - e^{-\gamma(\mathbf{x})}\right) p_D(\mathbf{x}) + e^{-\gamma(\mathbf{x})} p_D(\mathbf{x}) \sum_{\mathrm{p} \angle \mathbf{Z}_k} \omega_{\mathrm{p}} \sum_{W \in \mathrm{p}} \frac{\gamma(\mathbf{x})^{|W|}}{d_W} \cdot \prod_{\mathbf{z} \in W} \frac{\phi_{\mathbf{z}}(\mathbf{x})}{\lambda_k c_k(\mathbf{z})}.$$

[Mahler, 2009]

- Six assumptions made in derivation of GM-PHD [Vo and Ma, 2006].
- **Additional assumption:**
  The expected number of generated measurements $\gamma\left(\mathbf{x}\right)$ can be approximated as functions of the means of the individual Gaussian components

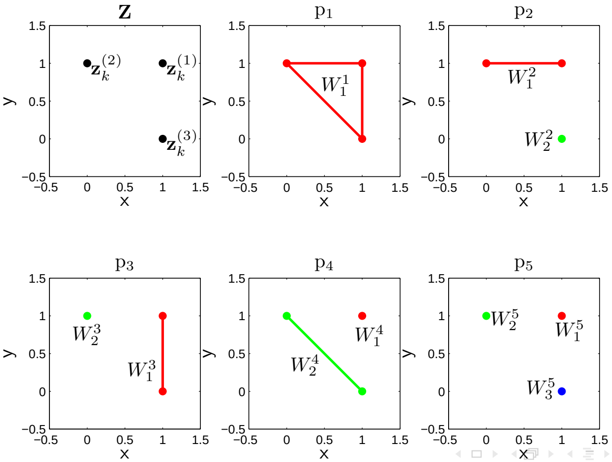
$$\gamma^{(j)} \triangleq \gamma\left(m_{k|k-1}^{(j)}\right).$$

Partitioning the measurements

- In each time step $\mathbf{Z}_k$ must be partitioned.
- A partition p is a division of $\mathbf{Z}_k$ into cells W.
- Important since more than one measurement can stem from the same target.

Partitioning the measurements — example

Partition the measurement set $\mathbf{Z}_k = \left\{ \mathbf{z}_k^{(1)}, \mathbf{z}_k^{(2)}, \mathbf{z}_k^{(3)} \right\}$



Partitioning method

- Measurements belong to same cell W if distance is "small".
- "Small" measured by Mahalanobis distance

$$(\mathbf{z}_k^{(1)} - \mathbf{z}_k^{(2)})^{\mathrm{T}} R_k^{-1} (\mathbf{z}_k^{(1)} - \mathbf{z}_k^{(2)}) < \delta_{P_G}$$

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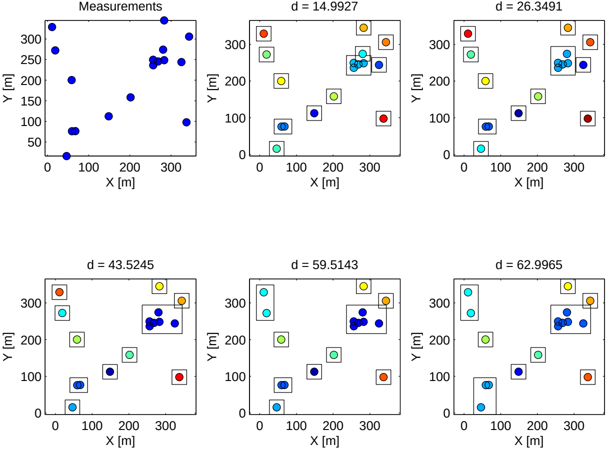
  for some gate probability P_G.
- With $R_k = \sigma_e^2 \mathbf{I}_2$ this reduces to

$$\left\| \mathbf{z}_k^{(1)} - \mathbf{z}_k^{(2)} \right\|_2 < \sigma_e \sqrt{\delta_{P_G}} = d_i.$$

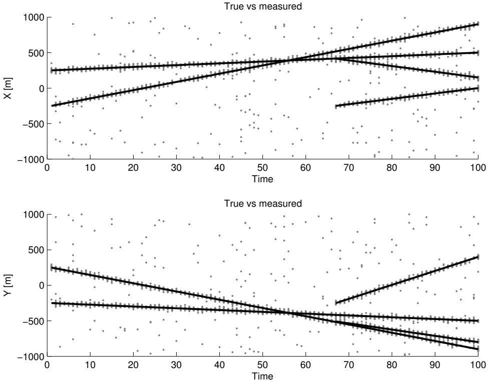
- Let $\{d_i^m\}_{i=1}^{N_d}$ be set of measurement to measurement distances.
- Good partitions for d_i corresponding to

$$\sigma_e \sqrt{\delta_{0.30}} \leq d_i^m < \sigma_e \sqrt{\delta_{0.80}}$$

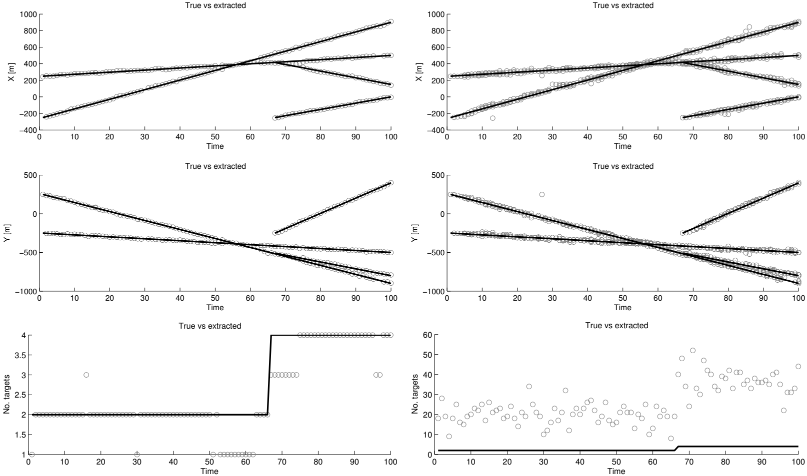
Partitioning method — example



Simulation — Measurements



- True target track crossing at time $k = 56$.
- New target birth and target spawned at time $k = 66$.



Extended target GM-PHD

Standard GM-PHD

Conclusions

Main contribution:
Implementation of GM-PHD-filter for extended targets.
Minor contribution:
Simple method for measurement set partitioning.

The suggested implementation handles...

- ...unknown number of targets.
- ...extended target measurements.
- ...cluttered measurement sets.

Matlab code used for simulations in paper available online

`http://www.control.isy.liu.se/publications/doc?id=2299`

Thank you for listening!

Any questions?

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