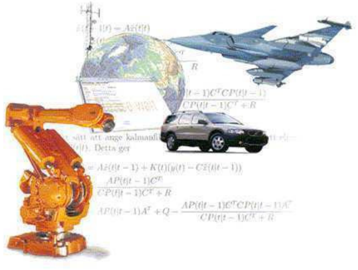
On Extended Target Tracking Using PHD Filters



Karl Granström*,
Christian Lundquist*,
Fredrik Gustafsson*,
Umut Orguner†

* Dept of Electrical Engineering
Linköping University
Linköping, Sweden
† Dept of Electrical and Electronics Engineering
Middle East Technical University
Ankara, Turkey

Multiple target tracking

- Find the location of multiple targets
  - Unknown number.
  - The targets are not always detected.
  - Noisy measurements.
  - Clutter.
  - Difficult data association.

- Early RADAR-airplane-tracking assumed that targets produce at most one measurement.

- Find the location of multiple targets
    - Unknown number.
    - The targets are not always detected.
    - Noisy measurements.
    - Clutter.
    - Difficult data association.

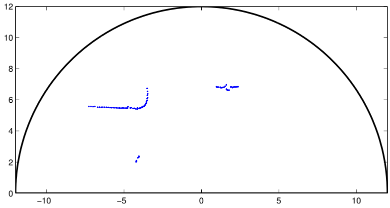
- Early RADAR-airplane-tracking assumed that targets produce at most one measurement.
- Modern sensors have higher resolution, multiple measurements per target.

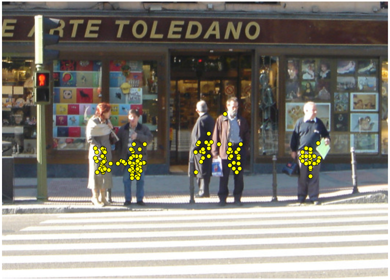
Need framework that handles multiple measurements per target.

Extended Target Tracking – ETT

One measurement per target is often not valid, e.g.,

- laser sensors, camera images, or automotive radar.





Definition:
Extended targets are targets that potentially give rise to more than one measurement per time step.

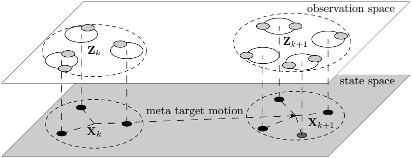
Target tracking setup

- Random finite set (RFS) of targets $\mathbf{X}_k = \left\{ \xi_k^{(i)} \right\}_{i=1}^{N_{x,k}}$

$$\xi_{k+1}^{(i)} = f_k\left(\xi_k^{(i)}, \mathbf{w}_k^{(i)}\right) \qquad \mathbf{w}_k^{(i)} \text{ – process noise.}$$

- RFS of measurements $\mathbf{Z}_k = \left\{ \mathbf{z}_k^{(j)} \right\}_{j=1}^{N_{z,k}}$

$$\mathbf{z}_k^{(j)} = h_k\left(\xi_k^{(i)}, \mathbf{e}_k^{(j)}\right) \qquad \mathbf{e}_k^{(j)} \text{ – measurement noise.}$$



Aim:

Compute target set estimate $\hat{\mathbf{X}}_{k|k}$ using measurement sets $\mathbf{Z}^k$.

- The number of target measurements must be modelled.
- Gilholm *et al.* (2005) suggests using a Poisson model.
- For the i:th target at time step k,

$$N_{z,k}^{(i)} \in \mathcal{POIS}\left(\gamma_k^{(i)}\right).$$

- Effective probability of target detection

$$p_{D,\text{eff}} = \underbrace{\left(1 - e^{-\gamma_k^{(i)}}\right)}_{P\left(N_{z,k}^{(i)} > 0\right)} p_D.$$

- ET-PHD filter under Gilholm's model given by Mahler in 2009.

- Prediction of PHD-intensity performed identically to standard PHD-filter
- $D_{k|k-1}(\xi|\mathbf{Z})$ is predicted PHD-intensity. Corrected PHD-intensity

$$D_{k|k}(\xi|\mathbf{Z}) = L_{\mathbf{Z}_k}(\xi)\, D_{k|k-1}(\xi|\mathbf{Z}),$$

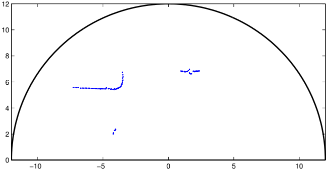
where measurement pseudo-likelihood is given by

$$\begin{aligned} L_{\mathbf{Z}_k}(\xi) =& 1 - \left(1 - e^{-\gamma(\xi)}\right) p_D(\xi) + \\ & e^{-\gamma(\xi)} p_D(\xi) \sum_{\mathrm{p} \angle \mathbf{Z}_k} \omega_{\mathrm{p}} \sum_{W \in \mathrm{p}} \frac{\gamma(\xi)^{|W|}}{d_W} \cdot \prod_{\mathbf{z} \in W} \frac{\phi_{\mathbf{z}}(\xi)}{\lambda_k c_k(\mathbf{z})}. \end{aligned}$$

[Mahler, 2009]

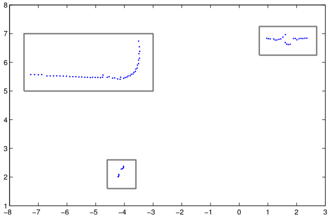
We will focus on the following aspects of ET-PHD:

- Measurement set partitioning.
- Target measurement rate.
- Probablity of detection.
- Two PHD intensity approximations:
    1. Gaussian mixture.
    2. Gaussian inverse Wishart mixture.

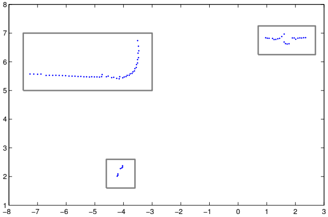


We will focus on the following aspects of ET-PHD:

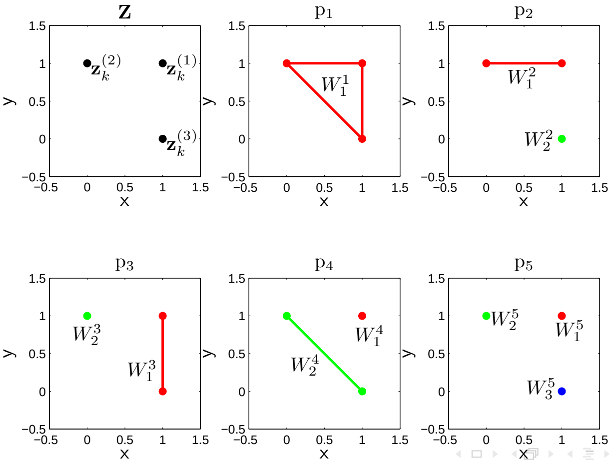
- Measurement set partitioning.
- Target measurement rate.
- Probablity of detection.
- Two PHD intensity approximations:
    1. Gaussian mixture.
    2. Gaussian inverse Wishart mixture.



- In each time step $\mathbf{Z}_k$ must be partitioned.
- A partition p is a division of the set $\mathbf{Z}_k$ into non-empty subsets, called cells W.
- Important because more than one measurement can stem from the same target.



Partition the measurement set $\mathbf{Z}_k = \left\{ \mathbf{z}_k^{(1)}, \mathbf{z}_k^{(2)}, \mathbf{z}_k^{(3)} \right\}$



- Number of possible partitions for n measurements given by n:th Bell number B_n.
- The Bell numbers B_n increase very fast when n increases, e.g. $B_3 = 5$, $B_5 = 52$ and $B_{10} = 115975$.

- Number of possible partitions for n measurements given by n:th Bell number B_n.
- The Bell numbers B_n increase very fast when n increases, e.g. $B_3 = 5$, $B_5 = 52$ and $B_{10} = 115975$.
- Necessary to approximate the full set of partitions with a subset of partitions.
- Intuition: Measurements are from same source if they are close, with respect to some measure or distance.

AUTOMATIC CONTROL
REGLERTEKNIK
LINKÖPINGS UNIVERSITET

- Method: Measurements are in same cell W if distance is “small”.
- Partions p_i where cells contain measurements where distance to nearest measurement is $< d_i$.
- Limit to partitions for thresholds d_i that satisfy

$$d_{\text{min}} \leq d_i < d_{\text{max}}$$

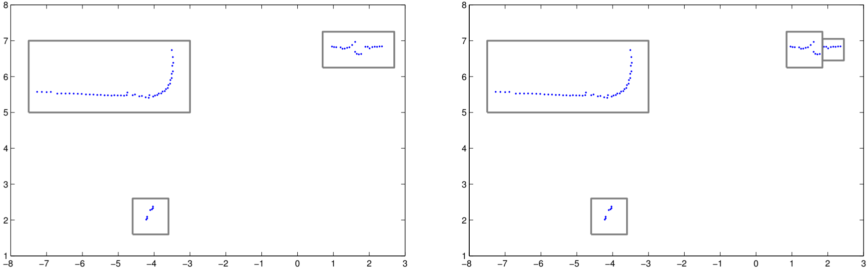
Partitioning method

- Method: Measurements are in same cell W if distance is "small".
- Partions p_i where cells contain measurements where distance to nearest measurement is $< d_i$.
- Limit to partitions for thresholds d_i that satisfy

$$d_{\text{min}} \leq d_i < d_{\text{max}}$$

- If possible, use scenario knowledge to choose distance measure and to determine bounds.
- Important to choose d_{min} and d_{max} conservatively.

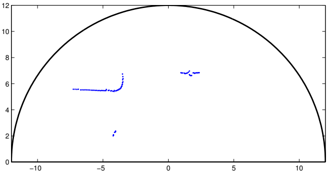
$$p_1 = \{W_1^1, W_2^1, W_3^1\} \qquad p_2 = \{W_1^2, W_2^2, W_3^2, W_4^2\}$$



- Reasonable to discard most partitions as highly unlikely.
- Additional methods given in paper.

We will focus on the following aspects of ET-PHD:

- Measurement set partitioning.
- Target measurement rate.
- Probability of detection.
- Two PHD intensity approximations:
  1. Gaussian mixture.
  2. Gaussian inverse Wishart mixture.



- The measurement rate is a function of the target state, $\gamma(\xi)$.
- Approximated as function of the target estimate

$$\gamma(\xi_k) \approx \gamma\left(\hat{\xi}_{k|k}\right)$$

- Important to have reasonable estimates of the true rates.

- The measurement rate is a function of the target state, $\gamma\left(\xi\right)$.
- Approximated as function of the target estimate

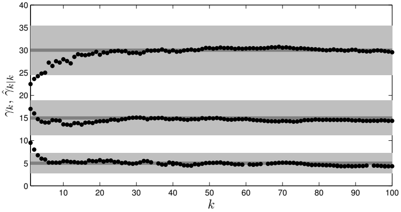
$$\gamma\left(\xi_k\right) \approx \gamma\left(\hat{\xi}_{k|k}\right)$$

- Important to have reasonable estimates of the true rates.
- Sometimes possible to design a model for $\gamma\left(\hat{\xi}_{k|k}\right)$
- Can be modelled as Gamma distributed and estimated online

$$p\left(\gamma_k \middle| \mathbf{Z}^k\right) = \mathcal{GAM}\left(\gamma_k\,;\,\alpha_{k|k}, \beta_{k|k}\right)$$

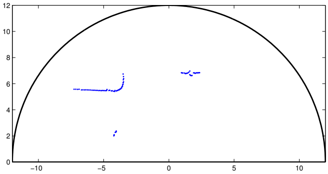
Measurement rate

- Gamma distribution is conjugate prior for Poisson measurements.
- Simple exponential forgetting with effective window length w_e is used for prediction.
- Possible to estimate multiple rates simultaneously



We will focus on the following aspects of ET-PHD:

- Measurement set partitioning.
- Target measurement rate.
- Probability of detection.
- Two PHD intensity approximations:
    1. Gaussian mixture.
    2. Gaussian inverse Wishart mixture.

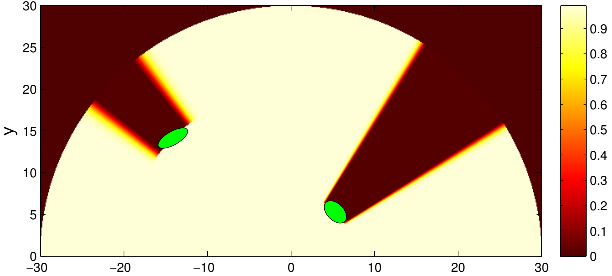


Probability of detection

- Probability of detection is function of target state, $p_{\mathrm{D}}(\xi)$.
- Approximated as function of the target estimate

$$p_{\mathrm{D}}(\xi_k) \approx p_{\mathrm{D}}(\hat{\xi}_{k|k})$$

- This allows for a non-homogeneous probability of detection
  $\Rightarrow$ possible to handle target occlusion.
  $\Rightarrow$ also brings problems: bias in the number of the targets



We will focus on the following aspects of ET-PHD:

- Measurement set partitioning.
- Target measurement rate.
- Probablity of detection.
- Two PHD intensity approximations:
    1. Gaussian mixture.
    2. Gaussian inverse Wishart mixture.

PHD intensity approximations

- At time step k the PHD-intensity is approximated as a weighted mixture of distributions,

$$D_{k|k}\left(\xi|\mathbf{Z}\right) = \sum_{i=1}^{J_{k|k}} w_{k|k}^{(i)} p\left(\xi_k;\ \zeta_{k|k}^{(i)}\right).$$

- ξ_k is the extended target state.
  Kinematical states, states that govern shape and size, etc.
- $\zeta_{k|k}^{(i)}$ is the distribution parameter for the i:th mixture component.

$$D_{k|k}\left(\mathbf{x}|\mathbf{Z}\right) = \sum_{i=1}^{J_{k|k}} w_{k|k}^{(i)} \mathcal{N}\left(\mathbf{x}_k\,;\,m_{k|k}^{(i)}, P_{k|k}^{(i)}\right)$$

- $\xi_k = \mathbf{x}_k$ and $\zeta_{k|k}^{(i)} = \left(m_{k|k}^{(i)}\,,\,P_{k|k}^{(i)}\right)$.
- The extended target state is a vector $\mathbf{x}$ that contains all states, e.g. kinematical states and parameters for shape and size.
- Measurement model

$$p\left(\mathbf{z}_k|\mathbf{x}_k\right) = \mathcal{N}\left(\mathbf{z}_k\,;\,h_k\left(\mathbf{x}_k\right), R_k\right)$$

- Example state vector

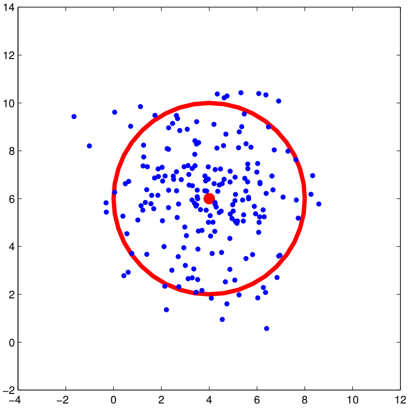
$$\mathbf{x} = \begin{bmatrix} \mathsf{x} & \mathsf{y} & \mathsf{v_x} & \mathsf{v_y} \end{bmatrix}^{\mathrm{T}}$$

- Measurement model

$$p\left(\mathbf{z}_k \middle| \mathbf{x}_k\right) = \mathcal{N}\left(\mathbf{z}_k\,;\, H_k\mathbf{x}_k, R_k\right)$$

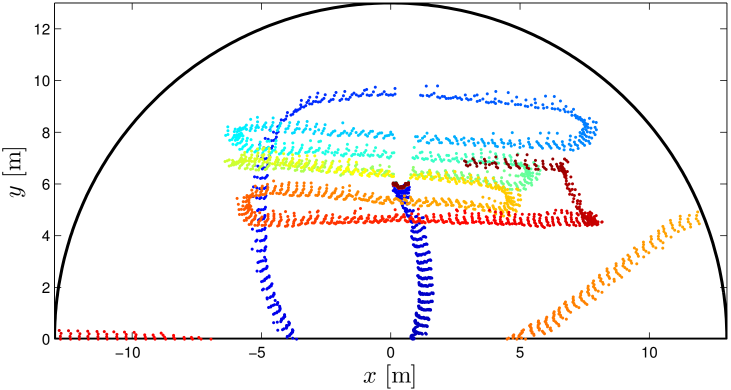
$$H_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

- Extension implicitly assumed circular with constant radius.

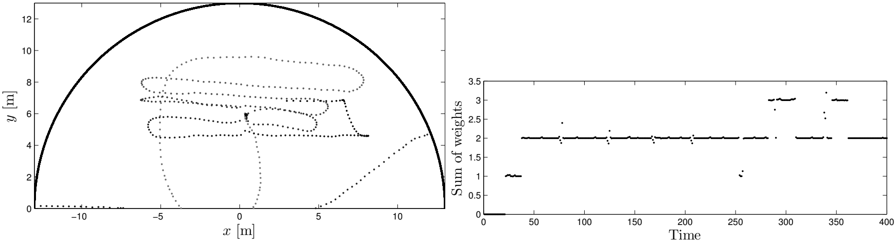


Experiment data

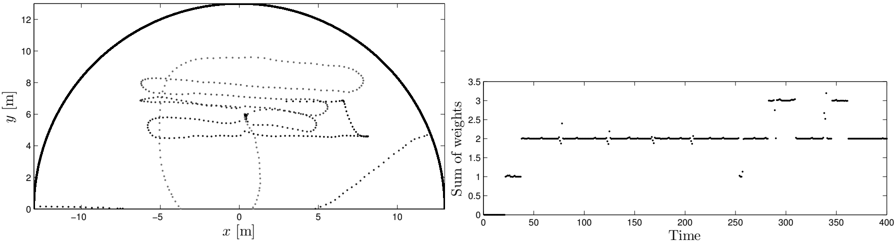
- SICK laser range sensor used to collect data.
- Multiple human targets, at most 3 at the same time.
- Measurements of stationary objects removed beforehand.



Gaussian mixture – results



- Difficult to handle occlusion when targets are close.
- "Multiplicatively biased" cardinality estimate on the edge of low probability of detection areas.



- Difficult to handle occlusion when targets are close.
- “Multiplicatively biased” cardinality estimate on the edge of low probability of detection areas.
- $\Rightarrow$ Estimate the shape and size for each target.

$$D_{k|k}\left(\xi|\mathbf{Z}\right) = \sum_{i=1}^{J_{k|k}} w_{k|k}^{(i)} \mathcal{N}\left(\mathbf{x}_k\,;\, m_{k|k}^{(i)}, P_{k|k}^{(i)} \otimes X_k\right) \mathcal{IW}\left(X_k\,;\, v_{k|k}^{(i)}, V_{k|k}^{(i)}\right)$$

- $\xi_k = (\mathbf{x}_k, X_k)$ and $\zeta_{k|k}^{(i)} = \left(m_{k|k}^{(i)}\,,\, P_{k|k}^{(i)}\,,\, v_{k|k}^{(i)}\,,\, V_{k|k}^{(i)}\right)$.
- The extended target state decomposes to the kinematical state vector $\mathbf{x}$ and the extension state matrix X.
- Extension shape is modelled as elliptic.
- Measurement model

$$p\left(\mathbf{z}_k|\xi_k\right) = \mathcal{N}\left(\mathbf{z}_k\,;\, h_k\left(\mathbf{x}_k\right), X_k\right)$$

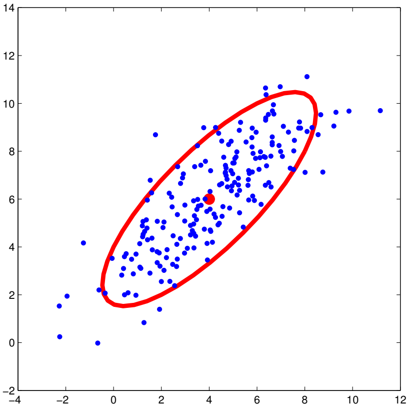
- Example state vector

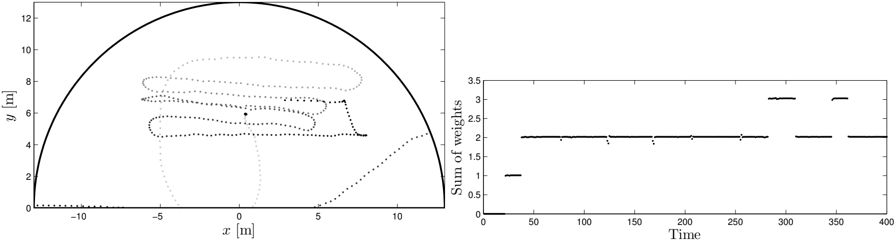
$$\mathbf{x} = \begin{bmatrix} \mathrm{x} & \mathrm{y} & \mathrm{v_x} & \mathrm{v_y} \end{bmatrix}^{\mathrm{T}}$$

- Measurement model

$$p\left(\mathbf{z}_k | \mathbf{x}_k\right) = \mathcal{N}\left(\mathbf{z}_k\, ;\, H_k \mathbf{x}_k, X_k\right)$$

$$H_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

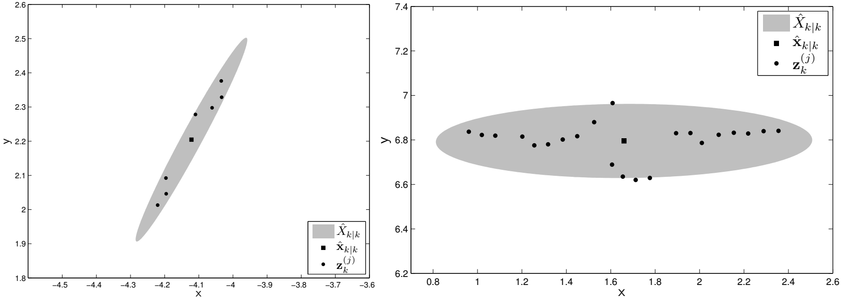




- Occlusion when targets are close is no longer difficult.
- Cardinality estimate on the edge of low probability of detection areas still “biased”, but less so.
- Overall the performance is improved.

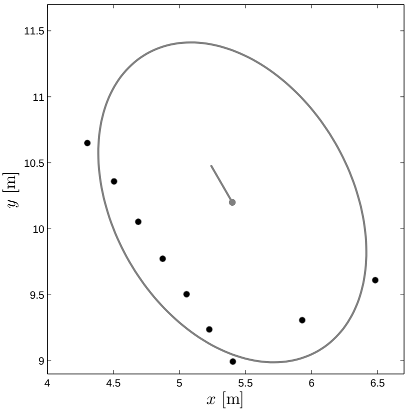
Estimating the extension

- The GIW model applied to human and bike in laser data. Reasonable approximation of shape.

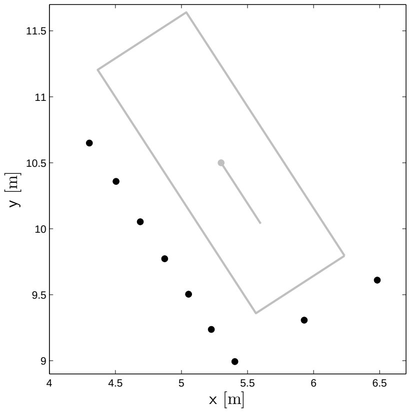


- Less suitable shape model for cars measured with laser sensor. Need for models that do not assume a certain shape.

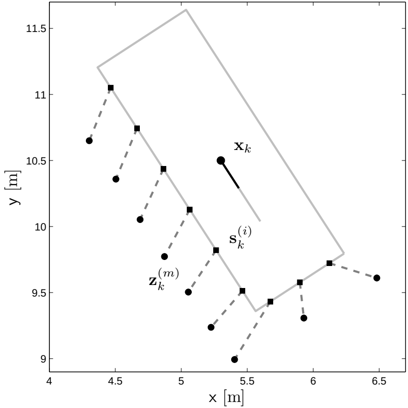
- Consider a number of measurements
- Target can be estimated as an extended target, without considering shape.



- Consider a number of measurements
- Target can be estimated as an extended target, without considering shape.
- Considering shape – How to describe relation between point measurements and shape?



- Consider a number of measurements
- Target can be estimated as an extended target, without considering shape.
- Considering shape – How to describe relation between point measurements and shape?
- Reflection point – Data association problem



Measurements

$$\mathbf{Z}_k = \left\{ \mathbf{z}_k^{(1)}, \ldots, \mathbf{z}_k^{(\mathfrak{z}_k)} \right\}$$

- Point observations $\mathbf{z}_k^{(m)}$
- number of measurements $\mathfrak{z}_k$ not fixed, due to:
  - detection uncertainty
  - spurious measurements
  - unknown number of reflection points

Measurement generating points

$$\mathbf{S}_k = \left\{ \mathbf{s}_k^{(1)}, \ldots, \mathbf{s}_k^{(\mathfrak{s}_k)} \right\}.$$

- A MGP $\mathbf{s} \in \mathbb{R}$ on the boundary
- number of MGPs $\mathfrak{s}_k$ is not fixed
- A MGP is defined on a one dimensional coordinate axis
- restricted to MGP-space $[\mathbf{s}_{\min}, \mathbf{s}_{\max}]$

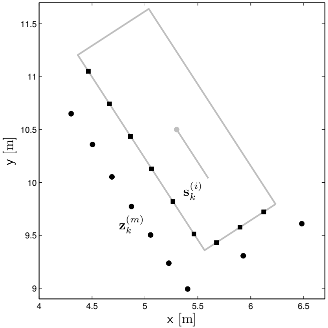
Modeling MGP

- $\mathbf{S}$ – RFS of all MGP on boundary
- $\mathbf{S}_k \subset \mathbf{S}$ detectable MGP
- as target moves new MGP appear $\mathbf{B}_k(\mathbf{x}_k) \subset \mathbf{S}$

$$\mathbf{S}_k = \underbrace{\mathbf{F}_k(\mathbf{S}_{k-1})}_{\text{surviving MGP}} \cup \mathbf{B}_k(\mathbf{x}_k)$$

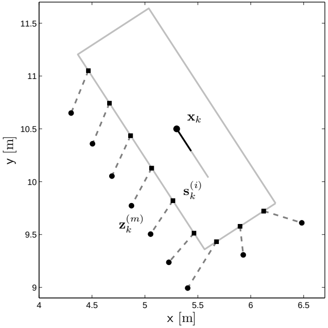
- RFS of measurements $\mathbf{Z}_k$

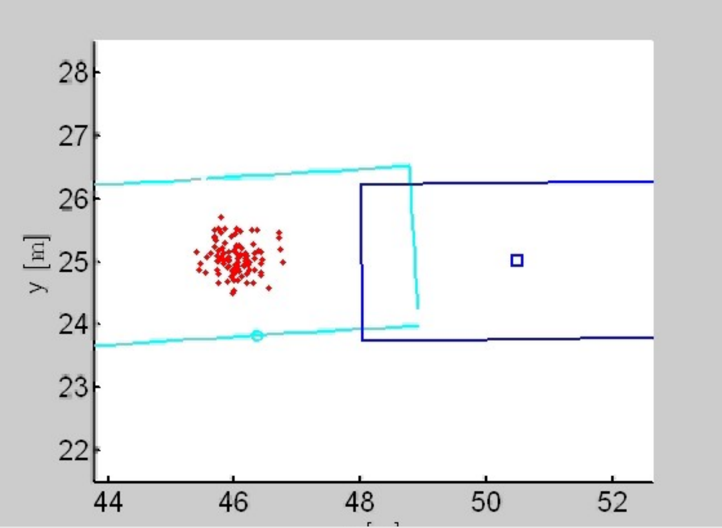
$$\mathbf{Z}_k = \bigcup_{\mathbf{s} \in \mathbf{S}_k} \underbrace{\mathbf{H}_k(\mathbf{s}, \mathbf{x}_k)}_{\text{target generated}} \cup \mathbf{C}_k$$



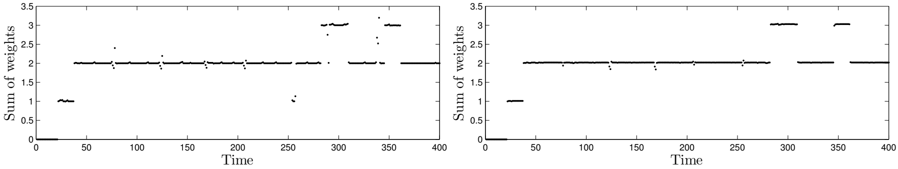
Realization

- The target state is $\mathbf{x}$ that contains e.g., kinematical states and parameters for shape and size.
- spline representation of target shape
- First order moment of RFS $\mathbf{S}_k$ approximated by a PHD D as a means to estimate $\mathbf{x}_k$
- Rao-Blackwellized particle filter implementation for state $\mathbf{x}_k$
  - particles for nonlinear states
  - KF for linear states




28
27
26
25
24
23
22
y [m]
44
46
48
50
52

- The PHD filter is sensitive to low $p_{\mathrm{D}}(\,\cdot\,)$.



- The Cardinalized PHD (CPHD) filter solves this by propagating the full probability mass function of the target cardinality.

PHD	CPHD

PHD

Z_k^T, Z_k^{FA} have non-homog. Poisson pdfs

$$f(Z_k^T|x) = e^{-N_k^z} \prod_{z_k \in Z_k^T} \gamma(\xi) p_z(z_k|\xi)$$

$$f(Z_{FA}) = e^{-N_z^{FA}} \prod_{z_k \in Z_k^{FA}} \lambda p_{FA}(z_k)$$

Prior $f(\mathbf{X}_k|\mathbf{Z}_{0:k-1})$ is assumed Poisson

$$e^{-N_{k|k-1}} \prod_{\xi_k \in \mathbf{X}_k} N_{k|k-1} p_{k+1|k}(\xi_k)$$

Aim:

Obtain the updated PHD $D_{k|k}(\xi_k)$ using the measurement set Z_k.

CPHD

Z_k^T, Z_k^{FA} have pdfs (i.i.d. cluster)

$$f(Z_k^T|x) = N_k^z! P_z(N_k^z|\xi) \prod_{z_k \in Z_k^T} p_z(z_k|\xi)$$

$$f(Z_{FA}) = N_z^{FA}! P_{FA}(N_k^{FA}) \prod_{z_k \in Z_k^{FA}} p_{FA}(z_k)$$

Prior $f(\mathbf{Z}_k|\mathbf{Z}_{0:k-1})$ is assumed cluster process

$$N_{k|k-1}! P_{k+1|k}(N_{k+1|k}) \prod_{\xi_k \in \mathbf{X}_k} p_{k+1|k}(\xi_k)$$

Aim:

Obtain the updated PHD $D_{k|k}(\xi_k)$ and updated posterior cardinality pmf $P_{k|k}(N_k)$ using the measurement set Z_k.

- PHD update

$$D_{k|k}(x) = \begin{pmatrix} \kappa(1 - P_D(x) + P_D(x)G_z(0)) \\ +P_D(x)\frac{\sum_{\mathcal{P}\angle Z}\sum_{W\in\mathcal{P}}\sigma_{\mathcal{P},W}\prod_{z'\in W}\frac{p_Z(z'|x)}{p_{FA}(z')}}{\sum_{\mathcal{P}\angle Z}\sum_{W\in\mathcal{P}}\alpha_{\mathcal{P},W}\beta_{\mathcal{P},W}} \end{pmatrix} D_{k|k-1}(x)$$

- Cardinality update

$$P_{k|k}(n) = \frac{\sum_{\mathcal{P}\angle Z}\sum_{W\in\mathcal{P}}\alpha_{\mathcal{P},W}G^{(n)}_{k|k-1}(0)\begin{pmatrix} G_{FA}(0)\frac{\eta_W[0,1]}{|\mathcal{P}|}\frac{\rho[1]^{n-|\mathcal{P}|}}{(n-|\mathcal{P}|)!}\delta_{n\geq|\mathcal{P}|} \\ +G^{(|W|)}_{FA}(0)\frac{\rho[1]^{n-|\mathcal{P}|+1}}{(n-|\mathcal{P}|+1)!}\delta_{n\geq|\mathcal{P}|-1} \end{pmatrix}}{\sum_{\mathcal{P}\angle Z}\sum_{W\in\mathcal{P}}\alpha_{\mathcal{P},W}\beta_{\mathcal{P},W}}$$

- PHD update

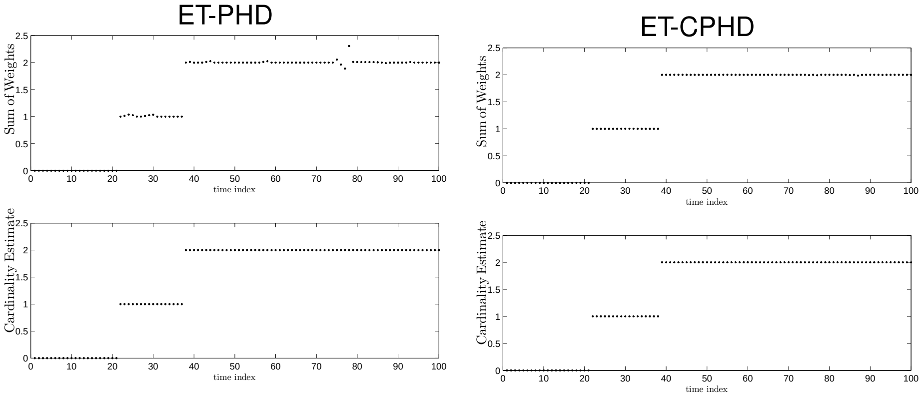
$$D_{k|k}(x) = \left(\begin{array}{c} \kappa(1 - P_D(x) + P_D(x)G_z(0)) \\ +P_D(x)\frac{\sum_{\mathcal{P}\angle Z}\sum_{W\in\mathcal{P}}\sigma_{\mathcal{P},W}\prod_{z'\in W}\frac{p_Z(z'|x)}{p_{FA}(z')}}{\sum_{\mathcal{P}\angle Z}\sum_{W\in\mathcal{P}}\alpha_{\mathcal{P},W}\beta_{\mathcal{P},W}} \end{array} \right) D_{k|k-1}(x)$$

- Cardinality update

$$P_{k|k}(n) = \frac{\sum_{\mathcal{P}\angle Z}\sum_{W\in\mathcal{P}}\alpha_{\mathcal{P},W}G^{(n)}_{k|k-1}(0)\left(\begin{array}{c} G_{FA}(0)\frac{\eta_W[0,1]}{|\mathcal{P}|}\frac{\rho[1]^{n-|\mathcal{P}|}}{(n-|\mathcal{P}|)!}\delta_{n\geq|\mathcal{P}|} \\ +G^{(|W|)}_{FA}(0)\frac{\rho[1]^{n-|\mathcal{P}|+1}}{(n-|\mathcal{P}|+1)!}\delta_{n\geq|\mathcal{P}|-1} \end{array} \right)}{\sum_{\mathcal{P}\angle Z}\sum_{W\in\mathcal{P}}\alpha_{\mathcal{P},W}\beta_{\mathcal{P},W}}$$

- Complexity is same order as ETT-PHD (single level of partitioning)
  - Same partitioning method used.
- Gaussian mixture implementation assuming that
  - The prior PHD is a Gaussian mixture
  - The measurement model is linear and Gaussian.

Comparison of Gaussian mixture implementations.

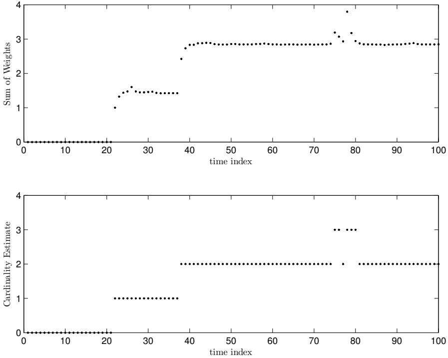


$$P_D^0 = 0.99$$

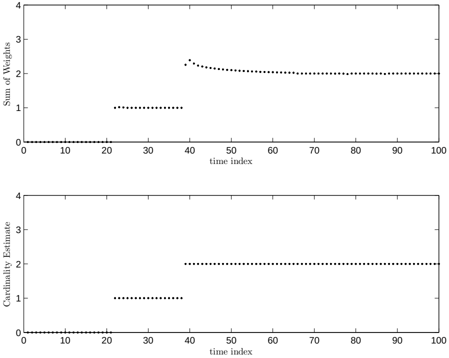
ET-PHD vs. ET-CPHD: Results

Comparison of Gaussian mixture implementations.

ET-PHD



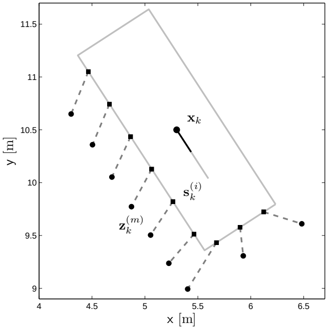
ET-CPHD



$$P_D^0 = 0.70$$

- The optimal CPHD filter is more or less of the same infeasible complexity as the PHD filter.
- Early simulation results show that the more robust characteristics of CPHD type algorithms apply also in the extended target case.
- More experiments are necessary for testing the advantages and the applicability of these algorithms.

- The sensor moves, the landmarks are stationary.
- Target tracking is similar to SLAM.
  - In case each landmark gives more than one measurement, the landmarks could be defined as extended objects, or
  - MGP could be interpreted as landmarks
- The extended target PHD and CPHD filters could be generalized to the SLAM problem.



Summary

We have presented

- Implementational aspects of the ET-PHD filter,
  - Measurement set partitioning.
  - Target measurement rate.
  - Probablity of detection.
  - Two PHD intensity approximations.
- A model for estimating the shape and size of extended targets.
- A CPHD filter for extended targets.

Thank you for listening!

Any questions?