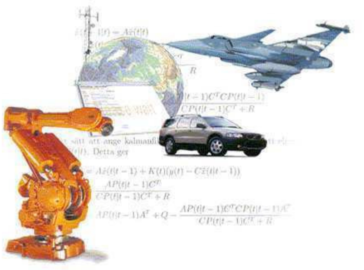
# Estimation and Maintenance of Measurement Rates for Multiple Extended Target Tracking



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# Introduction

- Extended targets $\xi_k$ give rise to sets of measurements

$$\mathbf{Z}_k = \left\{ \mathbf{z}_k^{(j)} \right\}_{j=1}^{N_{z,k}}$$

- $N_{z,k}$ is a random number.
- With multiple targets, clutter and association uncertainty, it is of interest to be able to predict/estimate $N_{z,k}$ for each target.
- Gilholm *et al.* (2005) models the number of measurements as Poisson distributed with rate $\gamma_k$

$$p\left(N_{z,k} | \gamma_k\right) = \mathcal{PS}\left(N_{z,k};\ \gamma_k\right) = \frac{\gamma_k^{N_{z,k}} e^{-\gamma_k}}{N_{z,k}!}.$$

- How can a single rate $\gamma_k$ per target be estimated?

**Contribution:**

- Recursive Bayesian estimator for a single rate $\gamma_k$ per target.

# Problem formulation and contribution

- How can a single rate $\gamma_k$ per target be estimated?

**Contribution:**

- Recursive Bayesian estimator for a single rate $\gamma_k$ per target.

- With multiple targets, distribution mixtures arise naturally. How can a mixture of $\gamma_k$ estimates be reduced?

**Contribution:**

- Merging of weighted sum of $N$ estimates into one estimate.
- Criterion to determine which estimates should be merged.

Conjugate prior is known to be the Gamma distribution,

$$p\left(\gamma_k \middle| \mathbf{Z}^{k-1}\right) = \mathcal{GAM}\left(\gamma_k\,;\,\alpha_{k|k-1},\beta_{k|k-1}\right) = \frac{\beta_{k|k-1}^{\alpha_{k|k-1}}}{\Gamma\left(\alpha_{k|k-1}\right)}\gamma_k^{\alpha_{k|k-1}-1}e^{-\beta_{k|k-1}\gamma_k}.$$

Measurement update known to be

$$p\left(\gamma_k \middle| \mathbf{Z}^{k}\right) = \mathcal{GAM}\left(\gamma_k\,;\,\alpha_{k|k-1},\beta_{k|k-1}\right)\mathcal{PS}\left(N_{z,k};\,\gamma_k\right) = \mathcal{GAM}\left(\gamma_k\,;\,\alpha_{k|k},\beta_{k|k}\right)\mathcal{L}_\gamma\left(\alpha_{k|k-1},\beta_{k|k-1},N_{z,k}\right)$$

where

$$\alpha_{k|k} = \alpha_{k|k-1} + N_{z,k} \qquad \beta_{k|k} = \beta_{k|k-1} + 1$$

and likelihood $\mathcal{L}_\gamma$ is negative-binomial

**Suggested prediction:** Exponential forgetting prediction

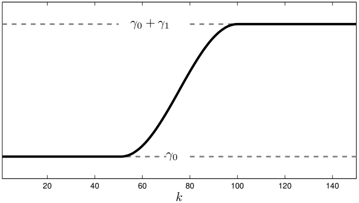
$$\alpha_{k+1|k} = \frac{\alpha_{k|k}}{\eta_k}, \qquad \beta_{k+1|k} = \frac{\beta_{k|k}}{\eta_k}, \qquad \eta_k > 1.$$

Expected value constant, and variance increased by factor $\eta_k$.

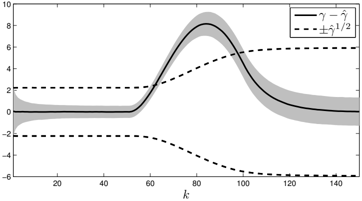
The effective window length is

$$w_e = \frac{\eta_k}{\eta_k - 1},$$

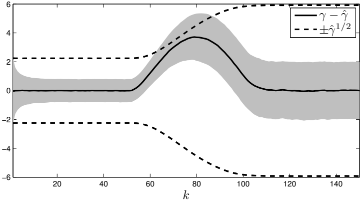
i.e. we only “trust” measurements from last $w_e$ time steps.



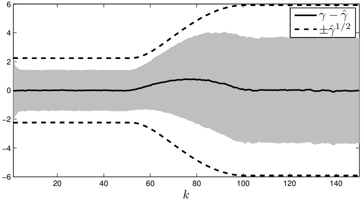

True $\gamma_k$




$\eta_k = 1.10$, $w_e = 11$




$\eta_k = 1.25$, $w_e = 5$




$\eta_k = 2.25$, $w_e = 1.8$

Let $\xi_k$ denote the augmented extended target state,

$$\xi_k = (\gamma_k, \mathbf{x}_k),$$

where $\mathbf{x}$ contains states for position, velocity, size, shape etc.

Given a set of measurements $\mathbf{Z}_k$ and a prior distribution $p\left(\xi_k|\mathbf{Z}^{k-1}\right)$, the posterior distribution is

$$\begin{aligned} p\left(\xi_k|\mathbf{Z}^k\right) =& p\left(\mathbf{Z}_k|\xi_k\right) p\left(\xi_k|\mathbf{Z}^{k-1}\right) \\ =& p\left(N_{z,k}|\gamma_k\right) p\left(\mathbf{Z}_k|\mathbf{x}_k\right) p\left(\gamma_k|\mathbf{Z}^{k-1}\right) p\left(\mathbf{x}_k|\mathbf{Z}^{k-1}\right). \end{aligned}$$

The posterior distribution and predicted likelihood is

$$\underbrace{p\left(\gamma_k|\mathbf{Z}^k\right) p\left(\mathbf{x}_k|\mathbf{Z}^k\right)}_{\text{posterior}} \underbrace{\mathcal{L}_\gamma\left(\alpha_{k|k-1}, \beta_{k|k-1}, N_{z,k}\right) \mathcal{L}_{\mathbf{x}}\left(\check{\mathbf{x}}_{k|k-1}, \mathbf{Z}_k\right)}_{\text{predicted likelihood}},$$

where $\check{\mathbf{x}}_{k|k-1}$ denotes the sufficient statistics of $\mathbf{x}_k$.

Any framework that estimates multiple states $\mathbf{x}_k$ can be augmented to also estimate of the measurement rates $\gamma_k$.

- e.g. the ET-PHD filter.

# Multiple extended targets

Distribution mixtures often used in multi target tracking, e.g. MHT, PHD

$$p\left(\xi_k\right) = \sum_{j=1}^{J_{k|k}} w_j \mathcal{GAM}\left(\gamma_k\,;\, \alpha_{k|k}^{(j)}, \beta_{k|k}^{(j)}\right) p\left(\mathbf{x}_k; \check{\mathbf{x}}_{k|k}^{(j)}\right).$$

- For large $J_{k|k}$, mixture reduction necessary, e.g. merging.
- Merging for Gaussian distributed $\mathbf{x}$ well known.
- We show merging for Gamma distributed Poisson rates, using minimization of the Kullback-Leibler divergence.

# Kullback-Leibler divergence



- KL-div is a measure of how similar $q$ and $p$ are

$$\mathrm{KL}\left(p||q\right) = \int p(x) \log\left(\frac{p(x)}{q(x)}\right) \mathrm{d}x,$$

- Positive, $\mathrm{KL}\left(p||q\right) \geq 0$.
- In general asymmetric, $\mathrm{KL}\left(p||q\right) \neq \mathrm{KL}\left(q||p\right)$.
- Well known moment matching characteristcs.
- Considered optimal difference measure in max likelihood sense.
- Minimizing KL-div can be rewritten as maximization problem

$$\min_q \mathrm{KL}\left(p||q\right) = \max_q \int p(x) \log\left(q(x)\right) \mathrm{d}x.$$

Let $p(\,\cdot\,)$ be a weighted sum of Gamma components,

$$p\left(\gamma\right) = \sum_{i=1}^{N} w_i \mathcal{GAM}\left(\gamma\,;\,\alpha_i,\beta_i\right) = \sum_{i=1}^{N} w_i p_i\left(\gamma\right),$$

where $\bar{w} = \sum_{i=1}^{N} w_i$. Let

$$q\left(\gamma\right) = \bar{w}\mathcal{GAM}\left(\gamma\,;\,\alpha,\beta\right)$$

be the minimizer of the KL-div between $p\left(\gamma\right)$ and $q\left(\gamma\right)$ among all Gamma distributions, i.e.

$$q\left(\gamma\right) \triangleq \underset{q(\gamma)\in\mathcal{GAM}}{\arg\min}\ \mathrm{KL}\left(p\left(\gamma\right)||q\left(\gamma\right)\right).$$

The parameter $\beta$ is given by

$$\beta = \frac{\alpha}{\frac{1}{\bar{w}} \sum_{i=1}^{N} w_i \frac{\alpha_i}{\beta_i}},$$

and the parameter $\alpha$ is the solution to

$$0 = \log \alpha - \psi_0(\alpha) + \frac{1}{\bar{w}} \sum_{i=1}^{N} w_i \left( \psi_0(\alpha_i) - \log \beta_i \right) - \log \left( \frac{1}{\bar{w}} \sum_{i=1}^{N} w_i \frac{\alpha_i}{\beta_i} \right).$$

Corresponds to matching the expected values of $\gamma$ and $\log \gamma$,

$$\bar{w}\, \mathrm{E}_q[\gamma] = \sum_{i=1}^{N} w_i\, \mathrm{E}_{p_i}[\gamma]$$

$$\bar{w}\, \mathrm{E}_q[\log \gamma] = \sum_{i=1}^{N} w_i\, \mathrm{E}_{p_i}[\log \gamma].$$

# Theorem 1 – proof

- The proof is simple.
- Requires basic knowledge of calculus.
- Full details given in the paper.

- Kullback-Leibler *difference* for components $i$ and $j$ defined as

$$D_{\text{KL}}\left(p_i, p_j\right) = \text{KL}\left(p_i || p_j\right) + \text{KL}\left(p_j || p_i\right)$$

- Due to assumed conditional independence it becomes

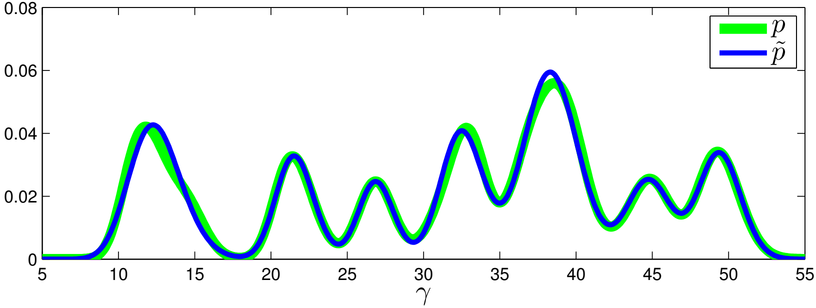
$$\begin{aligned} D_{\text{KL}}\left(p_i(\xi), p_j(\xi)\right) =& D_{\text{KL}}\left(p_i(\mathbf{x}), p_j(\mathbf{x})\right) + D_{\text{KL}}\left(p_i(\gamma), p_j(\gamma)\right) \\ =& D_{\text{KL}}^{\mathbf{x}} + D_{\text{KL}}^{\gamma}, \end{aligned}$$

- Merge components $i$ and $j$ if

$$D_{\text{KL}} < U \quad \text{or if} \quad \left(D_{\text{KL}}^{\mathbf{x}} < U_{\mathbf{x}}\right) \,\&\, \left(D_{\text{KL}}^{\gamma} < U_{\gamma}\right)$$

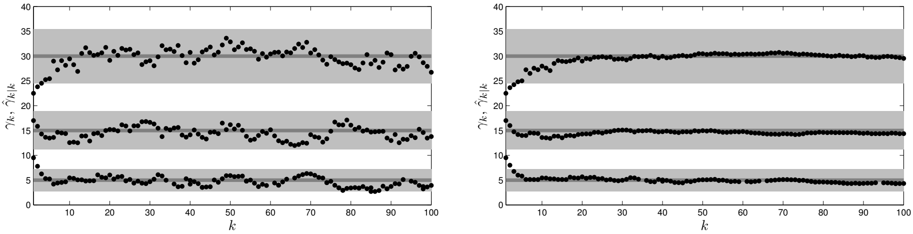
- Further information about criterion in the paper.

20 gamma components reduced to 7 by merging.



# Multiple targets – results

Three extended targets, with true rates $5$, $15$ and $30$.
Extended target PHD filter used for multiple target tracking.




$\eta_k = 1.25,\ w_e = 5$ $\qquad$ $\eta_k = 1.01,\ w_e = 101$

**Main contributions:**

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**Future work:**

- Use further in implementations of PHD filter for extended targets.
- Include position, velocity, shape and size of target into prediction of rate.

# Thank you for listening!

# Any questions?