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Center of gravity height real-time estimation for lightweight vehicles using tire instant effective radius

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ABSTRACT

Center of gravity (CG) height is an important parameter for lightweight vehicles (LWVs). Because of the inherently smaller weight and size, a LWV's CG height is more easily affected by loading conditions compared with conventional vehicles. This paper proposes a novel tire instant effective radius (TIER) method for real-time estimation of the CG height for LWVs. The method utilizes the mathematical correlation between the tire vertical load transfer that is proportional to the CG height, and the TIER variation. A Kalman filter based estimator is designed to simultaneously identify the vehicle CG height as well as the unknown nominal tire effective radius. To verify the performance of the proposed estimator, simulation results are first provided for several vehicles with different CG heights, and then road test results obtained on a lightweight electric ground vehicle (EGV) equipped with an advanced measurement system are given. Both simulation and experimental results show that the developed estimator is capable of providing an accurate estimation of the vehicle CG height in real-time.

1. Introduction

Recently, the lightweight vehicle (LWV) technology is becoming an active topic in automotive industry due to the concerns on energy consumption, environmental issues, and traffic throughput in the transportation sector (Huang & Wang, 2011, 2013; Kidane, Alexander, Rajamani, Starr & Donath, 2008). LWVs feature dramatically-reduced sizes and curb weights compared with the conventional vehicles. Therefore, LWVs' inertial or geometric parameters are prone to changes with payload parametric uncertainties/variations, i.e., passengers aboard and freight loads. On the one hand, these unattended parametric variations may result in deviations of the LWV dynamic responses from the expected ones. These discrepancies, even if sometimes insignificant, can mean life or death especially under some critical maneuvers (Huang & Wang, 2013). On the other hand, in the designs of online control laws for vehicle active safety systems, the knowledge of the actual vehicle parameters can be highly beneficial to executing precise control actuation and shortening the transient response time. Therefore, accurate onboard vehicular parameter estimation is of utmost importance for LWVs.

Among all the important vehicle parameters, center of gravity (CG) height, which cannot be directly measured, is at least as important as the others (Huang & Wang, 2011; Rajamani, Piyabongkarn, Tsourapas & Lew, 2011). It has been investigated that the CG height largely affects the roll dynamics (Marimuthu, Jang & Hong, 2006): a significant decrease in roll time and an increase in maximum roll angle can be observed when there is a 10% increase in the CG height. However, in critical vehicle maneuvers, sometimes one second or one degree is enough to cause a severe accident. This paper therefore aims to develop a method for estimating the CG heights of LWVs.

There has only been scattered research work conducted for the estimation of vehicle CG height. Rajamani et al. (2011) designed a method to estimate the vehicle CG height in real-time based on an accurate observation of the vehicle roll angle. The estimator employed a recursive least squares algorithm (RLS) with a variable forgetting factor, and was suitable for both steady-state and transient cornering maneuvers. In (Huang & Lin, 2008), Huang et al. proposed a model-based estimation approach for the relative CG height utilizing an extended Kalman filter (EKF). A four-state vehicle dynamics model that took into account lateral velocity, yaw rate, roll angle, and roll rate was employed. The CG height parameter was estimated by minimizing the prediction error generated by the EKF. Momiyama, Kitazawa, Miyazaki, Soma and Takahashi (1999) developed a CG height estimation scheme using a least squares method based on the transfer function from the steering angle to the roll angle. Solmaz, Akar, Shorten and Kalkkuhl (2008) proposed a multiple model switching identification strategy, whose beneath principle was to search for the most suitable parameter set that minimized the cost function

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Nomenclature

m	Vehicle mass (kg)
l	Wheel base (m)
l_f	Distance of c.g. from front axle (m)
l_r	Distance of c.g. from rear axle (m)
B_r	Half of the rear track width (m)
h	CG height to be estimated (m)
R	Tire (instant) effective radius (m)
R_u	Unloaded tire radius (m)
R_l	Loaded tire radius (m)
R_0	Nominal tire effective radius under nominal vertical load when vehicle is moving (m)
F_z	Tire vertical force (N)
F_{z0}	Nominal vertical load when vehicle is static or moving at constant speed on flat ground (N)
k_z	Tire vertical stiffness (kN/m)
a_x	Vehicle longitudinal acceleration (m/s^2)
a_y	Vehicle lateral acceleration (m/s^2)
$a_{x,b}$	Vehicle longitudinal acceleration in body frame (m/s^2)
$a_{y,b}$	Vehicle lateral acceleration in body frame (m/s^2)
α	Inclination angle of the vehicle body (rad)
θ	Vehicle pitch angle (rad)
ϕ	Vehicle roll angle (rad)
ϑ	Road grade angle (rad)
φ	Road bank angle (rad)
v_w	Longitudinal velocity at wheel center (m/s)
v_x	Vehicle longitudinal velocity (m/s)
v_y	Vehicle lateral velocity (m/s)
ω_w	Wheel rotational speed (rad/s)
ω_z	Vehicle yaw rate (rad/s)

consisting of the identification error. Moreover, the CG height estimation methods for commercial vehicles are also available in the literature (Cheng & Cebon, 2011; Kober & Hirschberg, 2006).

Even though the aforementioned methods have been supported by simulations or experiments, they may not be favorable in all applications due to the following reasons. First, most of them require the knowledge of a suspension model that is usually not available, and the vehicle roll moment of inertia whose value is directly related to the unknown CG height. Second, many of the existing methods may suffer from the large computational burden involved in the optimization process, such as given in Huang and Lin (2008) and Solmaz et al. (2008). Furthermore, those methods only considered the sprung mass CG height, that is, the CG height only referred to the distance from the sprung mass center to the roll axis. However, in the study of LWV dynamics, the CG height of the entire vehicle that includes both sprung mass and unsprung mass is of more interest because: (1) the ratio of unsprung mass to total vehicle mass is increased for a LWV; and (2) the C.G. height of the entire vehicle is helpful for controller design and load condition monitoring for a LWV (Huang & Wang, 2013).

The objective of this paper is therefore to estimate the CG height of a total LWV unit in real-time. The method is based on the mathematical correlation between a tire's vertical load variation and the change in the tire instant effective radius (TIER). The former can be proven proportional to the vehicle CG height if no suspension dynamic is involved. The estimation problem can ultimately be presented by a regular form that is linear in the parameters. Owing to its structural simplicity and flexibility to tune for multiple parameters (Gustafsson, 1997), a Kalman filter (KF) is selected to estimate the vehicle CG height as well as the unknown nominal tire effective radius simultaneously. The simulation results of sinusoidal steering maneuvers for two different CG height values as well as another simulation of a fishhook steering maneuver prove the effectiveness of the proposed KF based estimator. The experimental results obtained from road tests conducted on a lightweight electric ground vehicle (EGV) are also provided and analyzed.

The main contribution of this work lies in the following aspects. First, the estimation for vehicle parameters using the tire instant effective radius based method is almost nonexistent in the literature, while this paper shows the great potential in this research direction by successful simulation and experimental outcomes. Second, unlike the existing methods, no suspension model is involved, and the vehicle moment of inertia values are not needed in the estimation. Last but not the least, the proposed estimator does not pose much computational burden to the onboard hardware, and is easy to implement, partly because it is independent of the vehicle motion controller designs. In this paper, it is assumed that the vehicle mass and vehicle geometric parameters including the wheelbase and longitudinal position of the CG are known. Also, to facilitate the on-line estimation of CG height, vehicle motion signals such as velocities, accelerations, and angular velocities are all available through an advanced measurement system as described in Chen and Wang (2012) and Wang and Wang (2013).

The rest of this paper is organized as follows. Section 2 describes some preliminaries of the proposed CG height estimator including its principle, derivations of tire effective radius and vertical stiffness, the actual CG height measurement and some important assumptions. In Section 3, the real-time parameter estimation method based on the Kalman filter is presented. Simulation studies for different CG height values are given in Section 4, followed by experimental results and analysis obtained from vehicle road tests. Conclusion remarks are given in Section 6.

2. Preliminaries of tire instant effective radius method

The tires are the only components that enable direct contacts between an automobile vehicle and ground. By generating longitudinal, lateral, and vertical elastic forces, tires influence the vehicle performance in almost all aspects such as handling, safety, ride comfort (Jazar, 2008), to name a few. Decades have passed since researchers started to investigate the static and dynamic properties of tires (Carlson & Gerdes, 2005; Vantsevich, Barz, Kubler & Schumacher, 2005). In this paper, the tire's vertical properties are utilized to design a CG height estimator for LWVs. The overview of the proposed estimation method is first introduced in this section. Then, the expression for the tire instant effective radius (TIER) is derived, and an important tire parameter, tire vertical stiffness, is presented. Moreover, the true CG height of the experimental lightweight vehicle is obtained by static measurements. Finally, several essential assumptions are stated to remove the unnecessary complexities of the problem.

2.1. Overview of the TIER-based CG height estimation method

The proposed vehicle CG height estimation method is based on the mathematical correlation between the vertical load exerted on a tire and the TIER (Jazar, 2008), as given in (1), with the TIER defined in (2).

$$R = f_0(F_z, p), \tag{1}$$

$$R = \frac{v_w}{\omega_w}, \tag{2}$$

where, F_z is the tire vertical load; p is the inflation pressure of the tire; v_w is the wheel center velocity in the wheel plane; and ω_w is the wheel angular speed. It is important to point out that the TIER is defined only when the vehicle is moving.

Remark 1. An essential assumption of this work is that the effects on the tire radius by other factors including tire temperature, vehicle speed, wheel slip angle, and tire aging are all assumed negligible compared with the effects brought by the inflation pressure and the vertical load transfer on the tire.

By introducing the tire vertical stiffness, k_z, the variation in the instant effective radius can be related to the normal load transfer, which will be shown to be proportional to the CG height. Therefore, the basic idea of the CG height estimation applies the simple algebraic Eq. (3).

$$\Delta R = f(\Delta F_z(h), k_z(p)),$$
$$\Delta R = R - R_0; \quad \Delta F_z = F_z - F_{z0}. \tag{3}$$

In Eq. (3), ΔF_z and ΔR are normal load transfer and the corresponding instant effective radius variation around their nominal values F_{z0} and R_0, respectively. Note that the nominal TIER R_0 is determined by the loading condition and the tire inflation pressure when the vehicle operates under steady-state, where the normal load transfer is zero.

Two requirements must be met for the proposed estimator to function correctly and accurately. First, the vehicle must take a front-wheel-drive (FWD) pattern, thus either of the free-rolling non-steerable rear wheels can be used to estimate the CG height. The reasons are that: (1) to eliminate the undesirable effects of traction/braking force on the effective radius (Vantsevich et al., 2005); and (2) to avoid possible steering angle errors embedded in the front wheels when calculating the wheel center velocity in (2). Another requirement is that the vehicle motions must be sufficiently "excited" in order to generate a large enough normal load transfer, which ensures the accuracy of the estimation. Similar to what has been proposed in Huang and Lin (2008), Kidane et al. (2008) and Momiyama et al. (1999), the "excitation" here can be provided by the vehicle lateral or yaw motion, which is unfavorable in terms of vehicle roll and yaw stability. Fortunately, the observed TIER itself can help to detect rollover and prevent it: the larger TIER, the smaller the normal force on it. When the normal force tends to zero, TIER approaches the unloaded radius and then a wheel lift occurs. In this paper, the vehicle stability was always guaranteed by carefully selecting the maneuvers. Estimating the vehicle CG height using the TIER with minimum degradation of vehicle driving safety will be further studied in the future work.

2.2. Tire effective radius

According to Jazar (2008), the TIER is not equal to the loaded radius R_l, which is the distance from the wheel center to the tire-ground contact point when the vehicle is stationary. In fact, TIER is related to the loaded tire radius R_l and the unloaded radius R_u by the following expression, as shown in Fig. 1.

$$R = \frac{2}{3}R_u + \frac{1}{3}R_l. \tag{4}$$

The derivation of (4) utilizes the definition of the TIER (2) and the small angle approximation technique towards the tire contact angle β (Jazar, 2008). Interested readers can turn to section 3.4 of reference Jazar (2008) for more details.

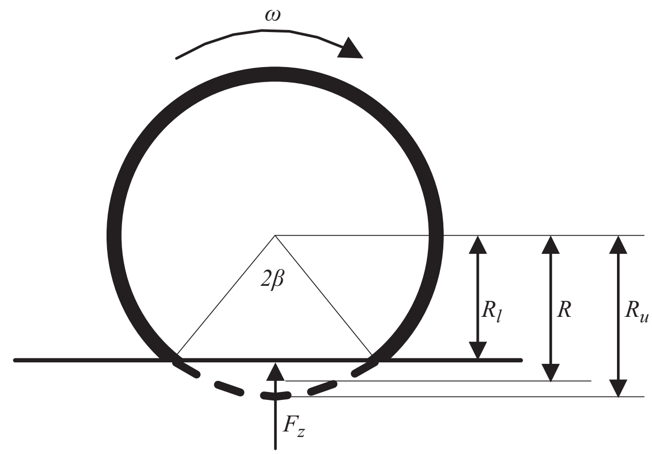

Fig. 1. Tire effective radius R, loaded radius R_l and unloaded radius R_u.

Since the loaded tire radius is a function of the tire vertical load F_z, a factor k_z, namely the tire vertical stiffness, is introduced. This coefficient largely depends on the tire inflation pressure. The loaded tire radius can thus be calculated as

$$R_l = R_u - \frac{F_z}{k_z(p)}. \tag{5}$$

Then, by combining (4) and (5), the TIER can be related to the vertical force as

$$R = R_u - \frac{F_z}{3k_z(p)}. \tag{6}$$

This equation defines f_0 in (1). Whenever a load transfer occurs, either as a result of acceleration or road condition change, the TIER will vary accordingly. Therefore, Eq. (6) can be revised as

$$\Delta R = \frac{\Delta F_z}{3k_z(p)}. \tag{7}$$

This is the mathematical expression where the proposed CG height estimator is grounded in. Note that (7) is the linearized version of (6), which means the unloaded tire radius R_u and the vertical stiffness k_z are fixed under a given loading condition and inflation pressure. The load transfer ΔF_z will be shown to be proportional to the vehicle CG height in the next section.

2.3. Tire vertical stiffness

As already mentioned in (5), tire vertical stiffness k_z is a coefficient representing the ratio of tire vertical load over tire normal deflection. Among all factors that influence the vertical stiffness, the most significant one is the tire inflation pressure (Jazar, 2008). Fig. 2 illustrates three tire force–deflection curves under different inflation pressures measured from the experimental electric ground vehicle (EGV). The forces were decreased by lifting one wheel gradually using a car lift jack, and the measurement was achieved by a digital scale (Fig. 4(a)) underneath this wheel. Detailed descriptions of the EGV can be found in the following section.

As can be seen from Fig. 2, the vertical stiffness increases with the tire pressure. Similar to Remark 1, it is considered that the tire vertical stiffness is only affected by the inflation pressure and the loading condition. Notice that the curves in Fig. 2 exhibit more nonlinearities as the vertical load becomes larger. In this paper, under a given tire inflation pressure, the vertical stiffness is supposed to be a constant within the entire normal force range, as the dashed line in Fig. 2 shows, i.e., k_z is equal to the slope of this straight line. This assumption can significantly simplify the problem though certain inaccuracy will be caused in the CG height estimates if the normal load transfer is large. A more precise representation of the tire vertical stiffness will be incorporated in the future. The above assumption also helps to obtain the vertical stiffness in real-time because some advanced sensors, such as those used in the tire pressure monitoring systems (TPMS) that can provide information on the tire pressure, are already available (Lee & Park, 2011; Umeno et al., 2001).

2.4. Static measurement of the vehicle CG height

The knowledge of a LWV's true CG height is necessary to show the effectiveness of the proposed estimator. Though direct measurement of the vehicle CG height is not possible using existing sensors, the authors applied an indirect measurement method (Jazar, 2008; Momiyama et al., 1999) for the experimental EGV. The approach is illustrated in Fig. 3.

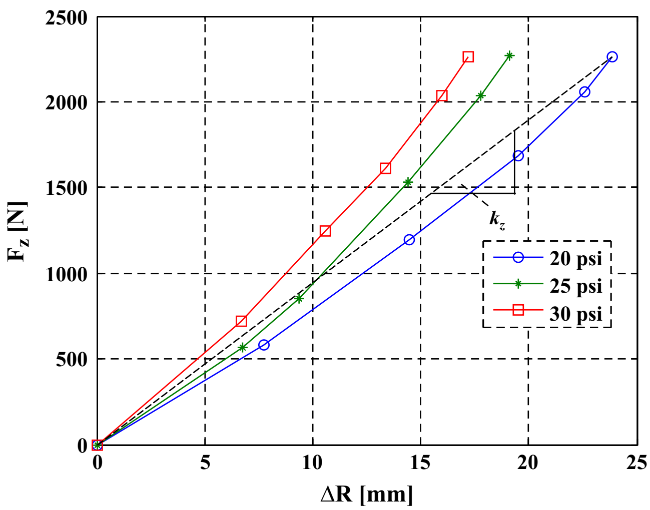

Fig. 2. Vertical load vs. tire radius change.

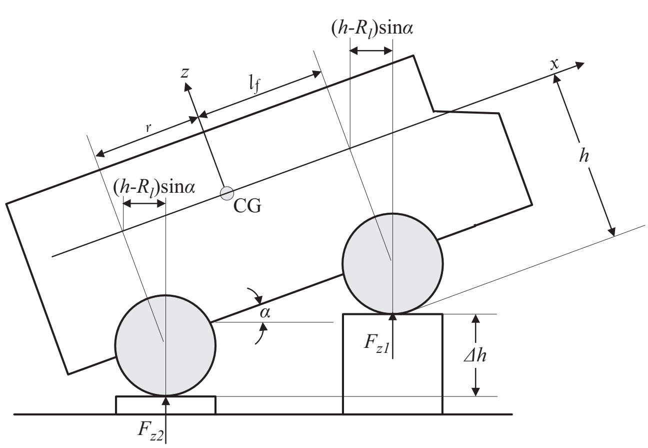

Fig. 3. Scheme of CG height measurement.

The vehicle was first placed on a flat ground, and the static normal forces of the front and rear wheels, F_{z10} and F_{z20}, were measured by a set of large digital scales (Fig. 4(a)). Note that the wheels on the left and right sides were lumped as a single one. Then, after the front wheels were jacked up with an inclination angle α, the rear normal force was measured again as F_{z2}. Since the tire deflections and vehicle pitch angle were small enough (Huang & Wang, 2011), it can therefore be derived that

$$F_{z1}+F_{z2}=F_{z10}+F_{z20}=mg, \tag{8}$$

$$F_{z1}[l_f\cos\alpha+(h-R_l)\sin\alpha]=F_{z2}[l_r\cos\alpha-(h-R_l)\sin\alpha], \tag{9}$$

where, l_f and l_r can be calculated as long as the vehicle wheelbase l, which can be measured, is known.

$$l_f=\frac{F_{z20}}{F_{z10}+F_{z20}}l, \quad l_r=\frac{F_{z10}}{F_{z10}+F_{z20}}l. \tag{10}$$

After some arrangements, we have

$$h=R_l+\left(\frac{F_{z2}l}{mg}-l_f\right)\cot\alpha=R_l+(F_{z2}-F_{z20})\frac{l\cot(\arcsin(\Delta h/l))}{F_{z10}+F_{z20}}. \tag{11}$$

During the test, it is important to ensure that the rear wheels are able to turn freely to eliminate the effect of horizontal friction force. For the safe considerations, front wheels can be locked to prevent sliding, as Fig. 4(b) shows.

The measurements are listed in Table 1, and the result gives an approximate value of $h\approx 0.60$ m. This means that the CG of the experimental EGV is located about 24 cm above the floor and a bit underneath the seats, which is reasonable considering that all the battery packs are directly settled on the floor under the seats, and the unsprung mass (including the in-wheel motor sets) itself contributes a large portion of the total vehicle weight. Therefore, 0.6 m will be treated as the true CG height of the experimental EGV for the rest of this paper.

2.5. Assumptions

Apart from the one mentioned in Remark 1 and that the tire vertical stiffness is constant, several other important assumptions

Table 1
Measurements of CG height test.

Measurements	Value (unit)
F_{z10}	3315 (N)
F_{z20}	4655 (N)
F_{z2}	4743 (N)
l	1.89 (m)
Δh	0.149 (m)
α	4.5 (deg)
R_l	0.312 (m)

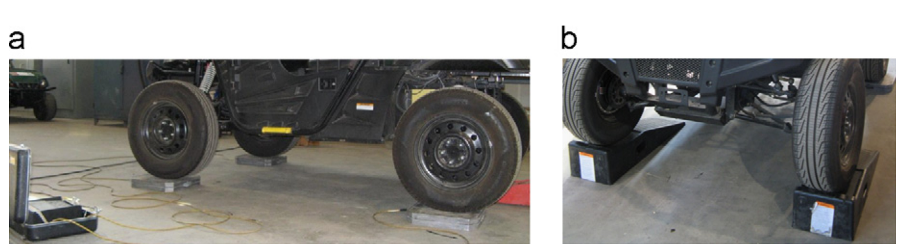

Fig. 4. Normal force measurement: (a) on flat ground and (b) front wheels lifted.

have to be made to ensure the success of the proposed CG height estimator.

Above all, vehicle loading conditions are assumed unchanged during the estimation process. This is typical for ground vehicles because it is unusual for a running vehicle to lose or gain any weight in a single maneuver. Second, the roll angle and the pitch angle are small (less than 5°) such that the CG height can be treated as constant. Third, the camber angle and the toe angle of the tire are not considered, therefore, the vertical load can be seen exerted in the wheel plane. Moreover, road roughness and wind forces are only taken as noises. Note that no assumption is claimed that the vehicle must be running on the flat ground, as both the road grade angle and bank angle can be integrated into the formulation of the estimator. This will be discussed in the following section.

3. Center of gravity height real-time estimation

The relationship between the vertical load change and the corresponding variation of a tire's instant effective radius (7) is utilized as the foundation of the proposed CG height estimator. The definition in Eq. (2) is used for real-time calculation of the TIER, while the tire vertical load transfer ΔF_z can be shown proportional to the CG height. In this section, the expression for the calculation of tire normal force is derived. Then, all sensor signals needed in Eq. (7) are described and the problem is formulated into a regular form. Thereafter, a Kalman filter based estimator is presented to simultaneously estimate both the nominal TIER and the CG height.

3.1. Normal force calculation

In this paper, the CG height estimator utilizes the information on the free-rolling rear-left wheel, whose normal force can be derived as (Cho, Yoon, Yim, Koo & Yi, 2010; Ray, 1997)

$$F_{zRl} = mgl_f/2l + mh[a_x B_r - a_y l_f - g(\theta - \vartheta)B_r - g(\phi - \varphi)l_f]/2B_r l, \tag{12}$$

where, all angles (Fig. 5) are assumed small and the approximation technique is adopted. The acceleration terms are calculated as

$$a_x = \dot{v}_x - v_y\omega_z,$$
$$a_y = \dot{v}_y + v_x\omega_z. \tag{13}$$

The first part of (12) is the static or nominal vertical load F_0 on the rear-left wheel. As long as the vehicle geometric parameters and mass are known, it can be easily obtained. The second part in (12) is the normal load transfer

$$\Delta F_{zRl} = mh[a_x B_r - a_y l_f - g(\theta - \vartheta)B_r - g(\phi - \varphi)l_f]/2B_r l. \tag{14}$$

The road-surface coordinate system (xyz) and the vehicle body-fixed coordinate system ($x_b y_b z_b$) are shown in Fig. 5. Both road grade angle ϑ and vehicle pitch angle θ (Fig. 5(a)) affect the longitudinal component of the load transfer, so do the road bank angle φ and the vehicle roll angle ϕ (Fig. 5(b)) to the lateral component of the load transfer. Those need to be taken into consideration for more accurate calculation of ΔF_{zRl}, and have been reflected in (14). Note that a positive road grade angle always refers to the one that elevates the front part of the vehicle, and road bank angle is positive when the right wheels are elevated if looking from behind the vehicle. As long as the pitch angle and the roll angle are small enough (less than 5°), which is usually true as can be verified by the vehicle road tests, the desired CG height can be viewed as unchanged (Solmaz et al., 2008).

Remark 2. A great advantage of the proposed approach over most existing CG height estimation methods is that no suspension model is involved in this TIER based method. In fact, suspensions only affect the transient behaviors of the vehicle pitch and roll motions, rather than the steady-state ones.

3.2. Signal description

When calculating the TIER, the longitudinal velocity of vehicle CG needs to be transformed to the wheel center velocity in the wheel plane. Particularly, for the rear-left wheel under investigation,

$$v_w = v_x - B_r\omega_z. \tag{15}$$

Together with (15), all the signals needed for the CG height estimation (7) can now be summarized as follows.

For TIER, vehicle longitudinal velocity v_x, yaw rate ω_z, and wheel angular speed ω_w are necessary. For normal load transfer ΔF_z, longitudinal acceleration a_x and lateral acceleration a_y, pitch angle θ and roll angle ϕ, road grade angle ϑ and bank angle φ are indispensable.

Fortunately, ω_z and ω_w are readily available by a gyroscope (Baffet, Charara & Lechner, 2009; Kim, 2009) and vehicle wheel speed sensors (Carlson & Gerdes, 2005), respectively. Observation of v_x has long been a research topic. In the experiments it was given by an advanced global positioning system (GPS). As to the pitch angle and the roll angle, since the inertia measurement unit used in the experiments was based on a strapdown system (Oxford Technical Systems, 2006), if the effects of the roll angle and the pitch angle are assumed uncoupled, they, together with the two road angles, can all be incorporated into vehicle body accelerations that are directly measurable (Hsu & Chen, 2009; Phanomchoeng, Rajamani & Piyabongkarn, 2011).

$$a_{x,b} = a_x - g(\theta - \vartheta), \tag{16}$$

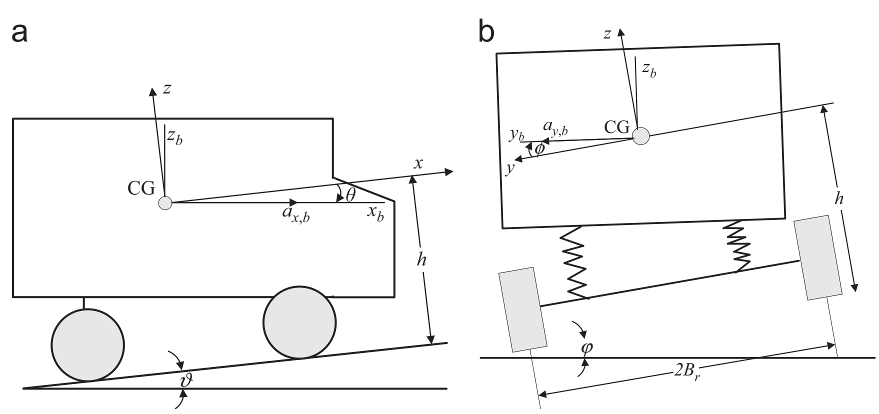

Fig. 5. Vehicle pitch and roll: (a) side view and (b) rear view.

$$a_{y,b} = a_y + g(\phi - \varphi). \tag{17}$$

Therefore, the normal load transfer for the rear-left wheel can be rewritten as

$$\Delta F_{zRl} = mh(a_{x,b}B_r - a_{y,b}l_f)/2B_r l. \tag{18}$$

3.3. Kalman filter based parameter estimation

A parameter estimation problem can be specifically formulated by combing (2), (3), (7), (15), and (18). For the rear-left wheel,

$$R_{0,Rl} - \frac{v_x - \omega_z B_r}{\omega_{Rl}} = \frac{\Delta F_{zRl}(h)}{3k_{zRl}(p)} = \frac{mh(a_{x,b}B_r - a_{y,b}l_f)}{6B_r l k_{zRl}(p)}, \tag{19}$$

where the subscript *Rl* denotes "rear-left". The above equation can be rewritten as

$$\frac{v_x - \omega_z B_r}{\omega_{Rl}} = R_{0,Rl} + \frac{m(a_{y,b}l_f - a_{x,b}B_r)h}{6B_r l k_{zRl}}. \tag{20}$$

An obstacle in implementing the TIER method lies in finding the nominal TIER R_0. This value cannot be directly measured, yet can be calculated using (2) when the vehicle is running at constant speed along a straight line on a flat ground, implying the tire normal load transfer is zero. If this value is not given accurately, the estimation results will be largely affected, as can be seen from Eq. (20). One possible solution is to conduct an additional constant speed straight-line maneuver and to estimate the R_0 in steady state using Eq. (2). But this complicates the estimation process. Instead, a Kalman filter with the flexibility to tune for multiple parameters was used to estimate the R_0 and the desired CG height h simultaneously.

A further organization of (20) gives the regular form

$$z = \boldsymbol{\theta}^T \boldsymbol{\varphi}, \tag{21}$$

where,

$$z = \frac{v_x - \omega_z B_r}{\omega_{Rl}};$$

$$\boldsymbol{\theta}^T = \begin{bmatrix} R_{0,Rl} & h \end{bmatrix};$$

$$\boldsymbol{\varphi}^T = \begin{bmatrix} 1 & \dfrac{m(a_{y,b}l_f - a_{x,b}B_r)}{6B_r l k_{zRl}} \end{bmatrix}. \tag{22}$$

For the problem as (21), either adaptive approach (You, Hahn & Lee, 2009), least squares, or recursive least squares (Rajamani et al., 2011; Vahidi, Stefanopoulou & Peng, 2005) methods can be applied to tackle this parameter identification problem. In Vahidi et al. (2005), a vector-type forgetting factor was developed to simultaneously identify two parameters. Real-time applications on the vehicle systems may require that this estimator poses as small computational burden as possiple on the hardware. Therefore, a simple Kalman filter based estimator (Gustafsson, 1997) is proposed as following:

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{x}(k) + \mathbf{v}(k), \\ y(k) &= \mathbf{H}(k)\mathbf{x}(k) + e(k), \end{aligned} \tag{23}$$

where, the estimated parameter vector $\mathbf{x}^T = \boldsymbol{\theta}^T = [R_{0,Rl} \quad h]$ is treated as a random walk, the output $y = z$ and $\boldsymbol{H} = \boldsymbol{\varphi}^T$. It is also assumed that the process noise $\boldsymbol{v}$ and measurement noise e are white noises and are independent, with covariance matrices $\boldsymbol{Q}$ and $\boldsymbol{S}$, respectively. More details of applying a Kalman filter for parameter estimation are omitted here and can be found in Best (2010), Gustafsson (1997) and Kim (2009) and the references therein.

4. Simulation results and analysis

Due to limitations of the experimental equipment, large CG height variations are not yet achievable in the vehicle road tests. Therefore, simulation results for both large and small CG heights were first used to illustrate the effectiveness of the proposed estimation method. A full-vehicle LWV model was constructed in CarSim®. Most parameters of this model were the same as those measured from the prototype EGV (Table 2). In the first two simulations, the CG height values were set to be 0.8 m and 0.4 m under a sinusoidal steering maneuver. Then, another more common fishhook steering maneuver, in which the CG height was 0.6 m, was also conducted. It is important to point out that since CarSim® did not take into account the tire instant effective radius, the authors proposed to affiliate an external Magic Formula (Pacejka & Bakker, 1993) tire model that included the vertical characteristics in the Matlab/Simulink environment for the purpose of simulation study. The structure of the simulation is depicted in Fig. 6.

4.1. Large CG height case

The first simulation was conducted for the LWV model with a large CG height. In this study, to achieve rich information from the vehicle lateral motions, the steering input was selected as a sinusoidal signal with the frequency of 0.13 Hz. The road surface of the testing ground was selected to be flat with a tire-road friction coefficient (TRFC) equal to 1. A reference generator (Fig. 6) helped to coordinate the magnitudes of the steering angle and the desired vehicle longitudinal velocity. The main objective of this generator was to prevent a dangerous rollover by limiting the steering angle or decreasing the vehicle speed. A proportional and integral (PI) vehicle speed controller was also added to the model for the purpose of determining the front wheel driving/braking torque, which together with the steering angle, were the two inputs to the entire LWV model.

The steering angle input and the vehicle longitudinal velocity profiles are given in Fig. 7. The magnitude of the steering angle is 3° and the velocity also exhibits small fluctuations with a tendency of increasing. The reference generator prevented vehicle from rolling over by simply limiting the steering angle. Note that

Table 2
Parameters of EGV and estimator.

Parameter	Value (unit)
m	880 (kg)
l_f	1.082 (m)
l_r	0.808 (m)
B_r	0.673 (m)
p	20 (psi)
R_u	0.340 (m)
R_l	0.312 (m)
k_z	210 (kN/m)
$\mathbf{Q}$	[1 0; 0 1e^{-6}]
S	[1]

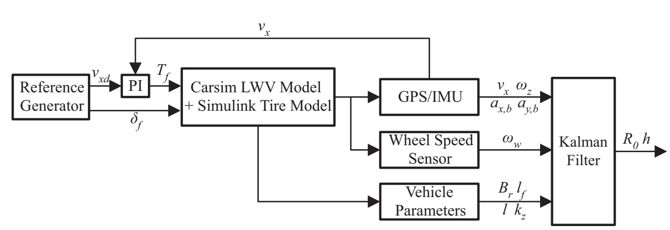

Fig. 6. Simulation structure.

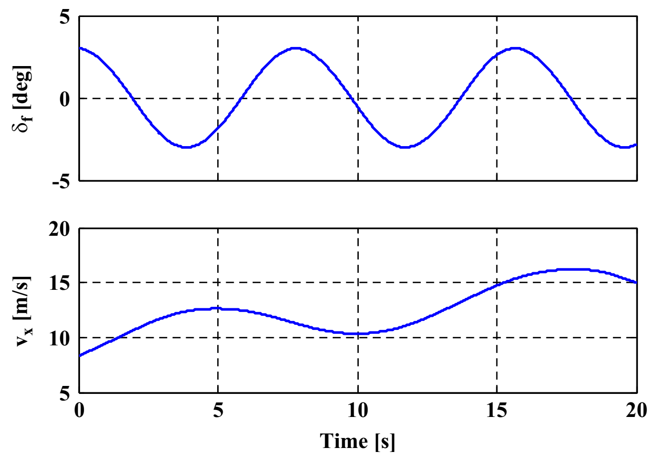

Fig. 7. Front steering angle and vehicle longitudinal velocity.

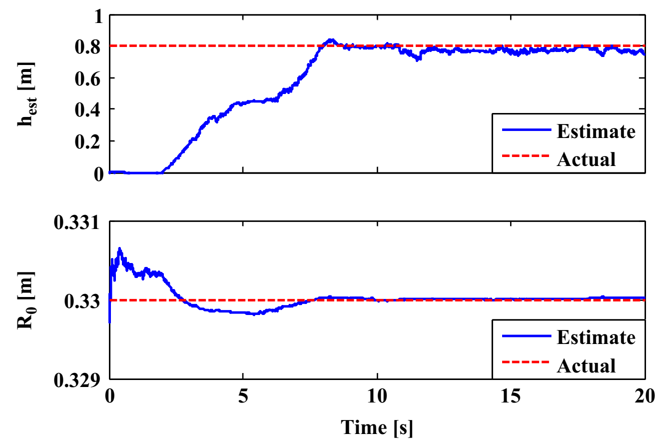

Fig. 9. The estimated CG height and estimated nominal TIER.

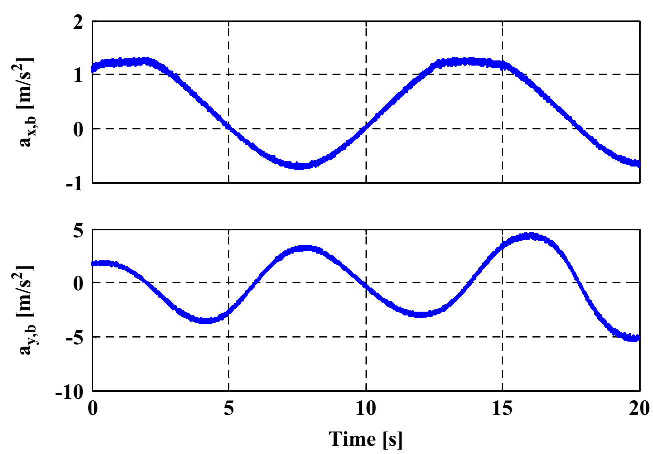

Fig. 8. Vehicle body longitudinal acceleration and vehicle body lateral acceleration.

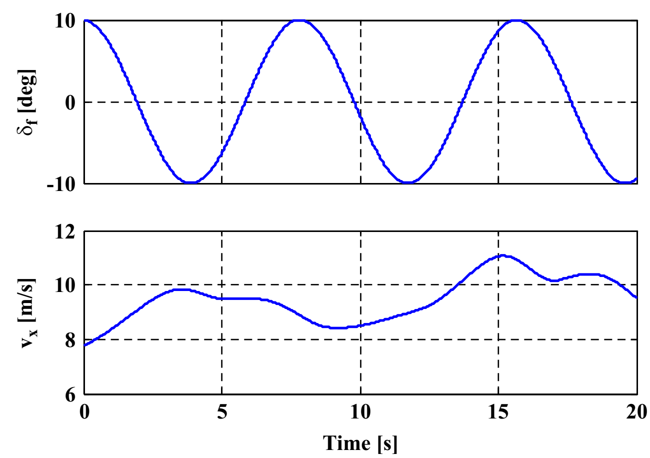

Fig. 10. Front steering angle and vehicle longitudinal velocity.

no controller for vehicle roll motion was involved, and the roll stability was guaranteed through monitoring of the vehicle lateral acceleration and TIER, as already stated in Section 2.1.

The excitation of the vehicle motions, that is, vehicle body longitudinal acceleration and lateral acceleration, are both illustrated in Fig. 8. As can be observed, the two accelerations were limited to be under 1.2 m/s^2 and 5 m/s^2, respectively. White noise was added to both signals to simulate the actual sensor noises.

Fig. 9 shows the estimation results obtained by the proposed Kalman filter based estimator. The Kalman filter was tuned by a trial-and-error process, mainly owing to the tradeoff between convergence rate and smoothness of the estimates. Finally, $\boldsymbol{Q}$ was set to be [1 0; 0 1e^{-7}] and $\boldsymbol{S}$ to be 10, which were used as guidelines for tuning in the road tests. As the curves in Fig. 9 show, the CG height estimates converge to 0.78 m in about 8 s. Meanwhile, the estimated nominal TIER converges to 0.33 m. Considering that the unloaded and loaded tire radii are around 0.34 m and 0.31 m, respectively, it can be concluded that both parameters have been accurately estimated.

4.2. Small CG height case

The second simulation was conducted for the LWV with a small CG height, i.e., 0.4 m. In this case, a larger front wheel steering angle was generated by the reference generator, because more severe lateral motion can be tolerated by this vehicle. The steering angle input and the vehicle longitudinal velocity are

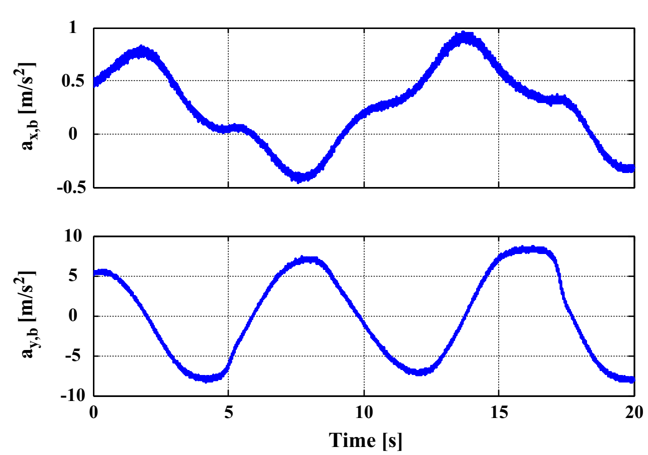

Fig. 11. Vehicle body longitudinal acceleration and vehicle body lateral acceleration.

shown in Fig. 10. The magnitude of the steering angle is 10° and the velocity does not exceed 11 m/s.

The vehicle body longitudinal acceleration and lateral acceleration are depicted in Fig. 11. Their maximum magnitudes are about 0.9 m/s^2 and 8 m/s^2. Again, white noise was added to both signals to simulate the actual sensor noises.

The estimation results obtained by the proposed Kalman filter based estimator are shown in Fig. 12. The covariance matrices

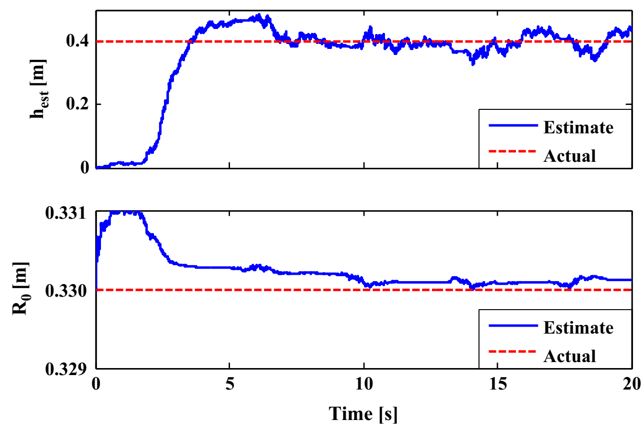

Fig. 12. The estimated CG height and estimated nominal TIER.

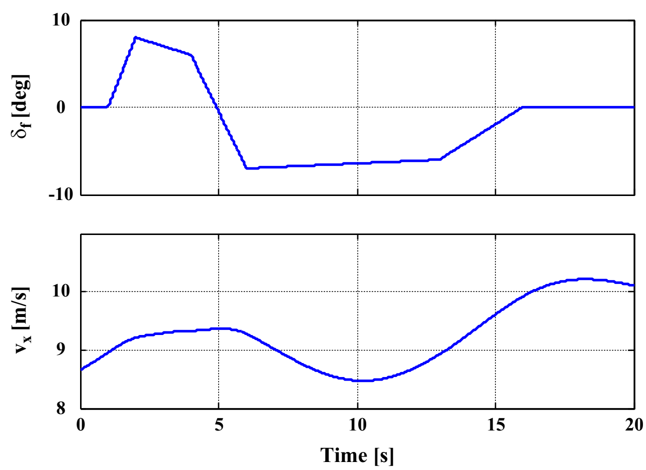

Fig. 13. The front steering angle and longitudinal velocity.

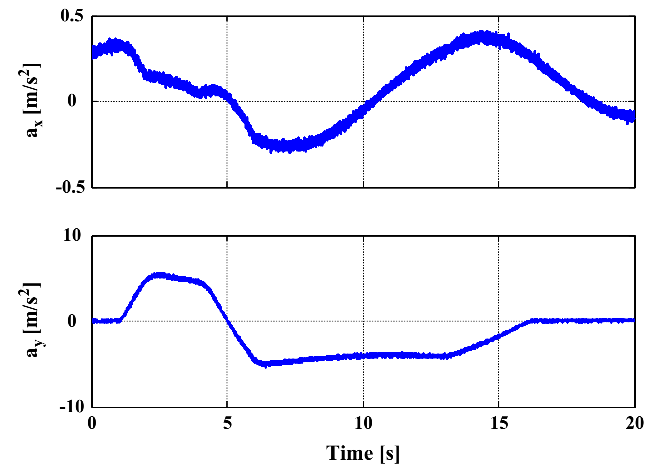

Fig. 14. The vehicle body longitudinal acceleration and vehicle body lateral acceleration.

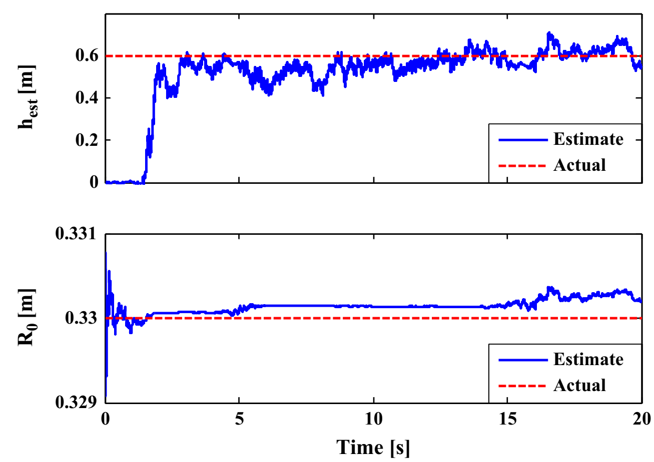

Fig. 15. The estimated CG height and estimated nominal TIER.

Q and ***S*** were the same as the ones in the previous simulation. The CG height estimate converges to 0.4 m within 4 s. Meanwhile, the estimated nominal TIER converges to 0.33 m with a small over-estimation. It can again be concluded that the proposed Kalman filter based estimator is able to provide accurate estimations on the CG height as well as the nominal TIER.

4.3. *Fishhook-like steering maneuver*

The third simulation was a fishhook-like steering maneuver, which is more common in the real-world vehicle operations, and the actual CG height was set as 0.6 m. The pre-filtered steering angle input and the vehicle longitudinal velocity are shown in Fig. 13. As can be indicated, this maneuver is not as intense as the previous one. The magnitude of the steering angle is no more than 8° and the velocity does not exceed 11 m/s.

The vehicle body longitudinal acceleration and lateral acceleration are shown in Fig. 14. Their maximum magnitudes are about 0.4 m/s^2 and 5 m/s^2. White noise was added to both signals to simulate the actual sensor noises.

The estimation results obtained by the proposed Kalman filter based estimator are shown in Fig. 15. The measurement noise covariance matrix ***S*** was changed from 10 to 1. As can be observed, the CG height estimate approaches to the true CG value immediately after there is a steering action, though the CG height is still a bit underestimated at the beginning. Meanwhile, the estimated nominal TIER converges to 0.33 m with small over-estimates. Compared with the previous two maneuvers, the faster response in this simulation is mainly attributed to the retuned design parameters of the Kalman filter. As a compromise, the CG height estimate became more noisy.

Remark 3. The simulation results in the above three cases supported that the proposed estimation method had good performance regardless of whether the CG height was large or small. A further conclusion can be made towards the excitation level by comparing Figs. 9 and 12: the more the vehicle motion is excited, the faster the convergence rate will be, though this relationship still needs to be further studied. Moreover, the design parameters of the Kalman filter had a significant impact on the estimation performance. It was a little tricky to tune the two covariance matrices, specifically, the process noise covariance matrix ***Q***. One approach is to utilize all available information, such as sensor noise characteristics and the maneuver undergoing, to achieve a best "guess" of the covariance matrices as the researchers in Gustafsson (1997) did. Another more systematic solution would be to design an adaptive Kalman filter as in Karasalo and Hu (2011), which automatically determines its process covariance matrix using an optimization tool after obtaining a short window of data. In this paper, the first approach was adopted, and the covariance matrices were selected to be time-invariant. This simplified the implementation of the estimator and relieved the computational burden. A more insightful study on how to tune the Kalman filter based estimator will be left for the future work.

5. Experimental results and analysis

Some road tests were also conducted for the proposed CG height estimation method. The prototype LWV is a four-in-wheel-motor-driven electric ground vehicle that has already been developed by the Vehicle Systems and Control Laboratory (VSCL) at The Ohio State University (Huang & Wang, 2011; Wang, Chen, Feng, Huang & Wang, 2011; Chen & Wang, 2012), as shown in Fig. 16. Some measured or calibrated values of the EGV parameters are listed in Table 2. Note that the loaded tire radius R_l and the tire vertical stiffness k_z are typically difficult to measure accurately. In this paper, they were measured following the procedure in Fig. 2. In other words, the loaded tire radius was calculated by substracting the tire radius change under a nominal load from the known unloaded tire radius R_u, and the tire vertical stiffness was assumed as a constant equal to the slope of the dashed straight line shown in Fig. 2.

The experimental setup for the CG height real-time estimation will be first described in this section. Then a road test under a sinusoidal-type steering is studied to show the performance of the proposed estimator.

5.1. Experimental setup

As has been stated in Section 2.1, the vehicle had to take the FWD pattern (and the rear wheels are free-rolling) to eliminate the undesirable effects brought by the traction/braking and steering. This was easily realized on a four wheel independently driven vehicle. The tires under study were of the type P215/70R15 with the rated unloaded radius of 0.34 m. The vehicle motion signals were measured by a high-end RT3003 Navigation System, which contained an integrated dual-antenna differential global positioning system (DGPS) and an integrated inertial measurement unit (IMU). Data acquisition was accomplished by a Micro-AutoBox from dSPACE with a sampling time of 0.01 s. Note that all the measurements were with regard to an inside point of the IMU (the red box shown in Fig. 16) (Oxford Technical Systems, 2006), which was mounted to the middle bar right behind the seats. The desired signals for CG height estimation included wheel speed, longitudinal velocity, yaw rate, and vehicle body accelerations.

The accuracy of the estimation is highly related to the quality of the data. Hence, before being input to the estimator, some of the raw signals, including wheel angular speeds and vehicle body accelerations, were pre-filtered by some carefully tuned Kalman filters.

Fig. 16. Electric ground vehicle with in-wheel motors.

5.2. Sinusoidal steering test

The sinusoidal steering test was conducted for the estimation of the vehicle CG height. Rear-left wheel was selected with the formulation given in (22). The road wheel steering angle and the vehicle longitudinal velocity are shown in Fig. 17. The magnitude of the steering angle was no more than 7° and the vehicle velocity was kept moderate for safety considerations.

The vehicle body longitudinal acceleration and lateral acceleration are shown in Fig. 18. As can be seen, the generated longitudinal acceleration was much smaller than the lateral acceleration, which reached up to 6 m/s^2. This means that the load transfer on the rear-left wheel was mainly caused by the lateral component of the vehicle body accelerations. It is important to notice that the accelerations given in Fig. 18 are defined by (16) and (17), which incorporate the road angles. Due to the existence of an up-hill road inclination angle of the test field, the vehicle body longitudinal acceleration signal remained positive even when the longitudinal velocity started to decrease from the 16th second.

Fig. 19 shows the calculated TIER using (2) and the scaled dynamic part of the normal load, which was obtained by dividing the right-hand side of (14) by h. A direct observation is that the larger the scaled normal load transfer, the smaller the TIER, and the TIER was oscillating around its nominal value.

The estimation results by the proposed estimator are demonstrated in Fig. 20. The upper plot shows that the estimated CG height converges to a value between 0.55 m and 0.6 m after the

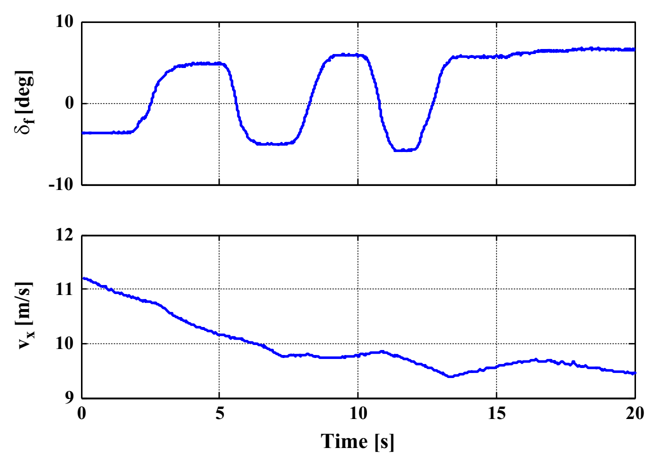

Fig. 17. The front steering angle and longitudinal velocity.

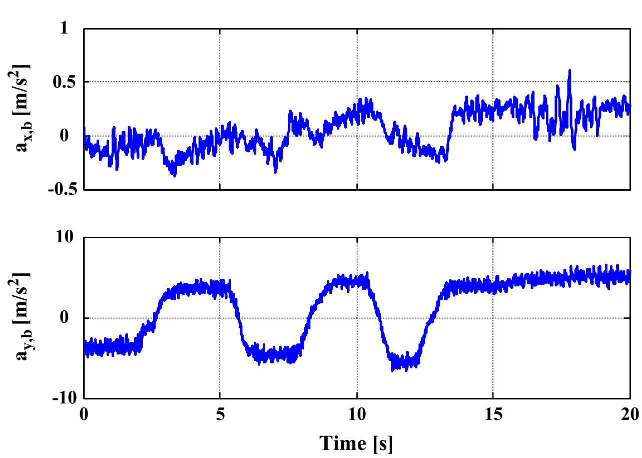

Fig. 18. The vehicle body longitudinal acceleration and vehicle body lateral acceleration.

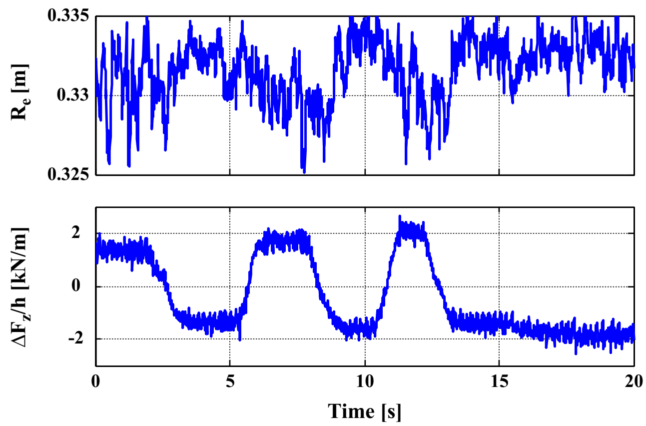

Fig. 19. The TIER and scaled normal load transfer.

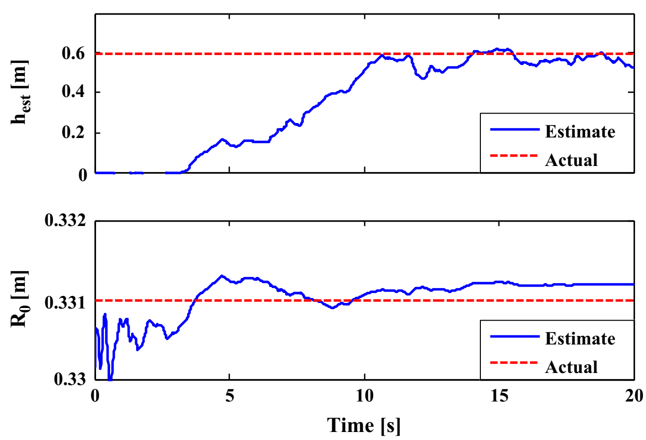

Fig. 20. The estimated CG height and estimated nominal TIER.

10th second. This is very close to the measured value of 0.6 m, which means that the proposed method is capable of providing CG height estimation with an acceptable accuracy. The relatively long convergence time was due to the vehicle motion excitation level as well as the tradeoff between the estimation smoothness and speed for the design parameters of the Kalman filter based estimator. As already mentioned in Remark 3, the design matrices can be made adaptive to the maneuver and signal properties, thus improving the convergence rate. The small underestimation of the CG height might be caused by the assumption that the tire vertical stiffness k_z remained constant within the entire normal force range. A more precise representation of the tire vertical stiffness as a function of tire pressure, normal force, and other factors may be necessary for more accurate estimation. The lower plot shows the estimated $R_{0,Rl}$, which also converges to a value a bit higher than 0.331 m within 5 s. This value is reasonable considering that the unloaded tire radius is 0.34 m and the loaded (under the same nominal load as in the road tests) radius is 0.312 m.

6. Conclusions

In this paper, a novel vehicle CG height estimator is designed. The method is based on the relationship between a tire's instant effective radius and the normal load on it. Due to its structural simplicity and flexibility to tune for multiple parameters, a Kalman filter based estimator is developed to simultaneously estimate the unknown nominal TIER and the CG height. As an improvement to the existing methods, no suspension parameters or vehicle moment of inertia values are necessary in the proposed method. The CarSim® simulation results show that the designed estimator exhibits good performance in estimating both large and small CG height values. The road experimental results obtained on a lightweight electric ground vehicle show that the designed estimator is capable of providing estimation of the CG height and the nominal TIER in real-time with acceptable accuracies. However, the reliance on sufficient excitation of the vehicle lateral motion may be undesirable and the utilization of advanced measurement system may limit the application of this method on commercial vehicles.

Rather than exploring new estimation techniques, this paper provides insight into using TIER to estimate the vehicle CG height, which has never been studied before. It is revealed that this method can succeed under certain valid assumptions and conditions. Therefore, this work sheds light on the possibility of using certain tire properties for estimating some of the vehicle parameters in the future.

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