Tentamen 2017
Robust Multivariable Control
Duration 3 days (3×24 hrs)

Anders Helmersson
`http://www.control.isy.liu.se/~andersh`
email: `andershelmersson@liu.se`
phone: 013-157 888 (home)
013-482 8419 (RUAG Space)
0734 278 419 (mobile)

1

A system has a time delay of 0.1 seconds and a pole in +1 rad/s.

What is the lowest possible peaks of the sensitivity functions S and T in closed loop.

2

Let $\Delta = \begin{bmatrix} \delta_1 & \delta_2 \\ \delta_2 & \delta_1 \end{bmatrix}$. Determine the set of matrices, D, that commute with Δ: $D\Delta = \Delta D$.

Consider the problem of minimizing $\bar{\sigma}(DMD^{-1})$ with respect to all non-singular such D. Can you be sure that you always find the global minimum?

Can this structure be extended to higher dimensions?

3

Minimize the maximum singular value of

$$\begin{bmatrix} 1 & x \\ 3 & 4 \end{bmatrix}$$

with respect to x.

Next, minimize

$$\begin{bmatrix} 1 & x & y \\ 3 & 4 & z \\ 2 & 5 & 6 \end{bmatrix}$$

with respect to x, y and z.

Are the optimal values of x, y and z unique (in both examples)?

4

The Riccati equation can be solved by finding the eigenvalues and eigenvectors of the Hamiltonian matrix,

$$H = \begin{bmatrix} A & R \\ -Q & -A^T \end{bmatrix}, \quad \text{where} \quad R = R^T,\ Q = Q^T.$$

Show that the eigenvalues are symmetric with respect to the real and imaginary axes.

In order to find a real solution X to the Riccati equation

$$XA + A^T X + Q + XRX = 0,$$

how should you combine the eigenvectors?

5

Consider the three first-order systems with inputs $\begin{bmatrix} w \\ u \end{bmatrix}$ and outputs $\begin{bmatrix} z \\ y \end{bmatrix}$:

(1)

$$G_1(s) = \left[\begin{array}{c|cc} 0 & 1 & 1 \\ \hline 1 & 1 & 1 \\ 1 & 1 & 0 \end{array}\right]$$

(2)

$$G_2(s) = \left[\begin{array}{c|cc} 0 & 1 & 1 \\ \hline 1 & 1 & 1 \\ 1 & 0 & 0 \end{array}\right]$$

(3)

$$G_2(s) = \left[\begin{array}{c|cc} 0 & 1 & 1 \\ \hline 1 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right]$$

Consider the H_∞ controller problem, using y as input and u as output, for these three systems.

What is the smallest achievable gains and the corresponding controllers for these systems?

Can you find a zeroth order controller for each case?

Are the controllers acceptable in all cases?

6

Robin is making a toy robot vehicle consisting of a 0.6-meter vertical rod, on to which the lower end two small electrical motors are attached. Each motor drives a wheel with a diameter of 100 mm. Also, an encoder is attached measuring the rotation of the motor shaft. The motors are controlled by a microprocessor, which uses the encoder signals and gyro data from a sensor mounted on the rod.

Use the following model for the pitch dynamics: θ is the angle of the rod relative to the vertical, ϕ is the mean rotation angle of the motor shafts relative to the robot measured by the encoder.

$x = (\theta + \phi)r$ (location of the center of the wheel), $r = 0.05\ m$.

$$\begin{bmatrix} I + m\ell^2 & m\ell\cos\theta \\ m\ell\cos\theta & m \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} gm\ell\sin\theta \\ 0 \end{bmatrix} + m\ell\dot{\theta}\sin\theta \begin{bmatrix} \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 1 \\ -1/r \end{bmatrix} u$$

where $I = 0.03$ kgm^2, $m = 0.5$ kg, $\ell = 0.2$ m and $g = 9.81$ m/s^2. The control signal, u, is the total motor torque produced by the two motors. The same command is given to both motors.

We have neglected the slipping of the wheels relative to the ground surface.

A linearized model around $\theta = 0$ and $\dot{\theta} = 0$ becomes

$$\begin{bmatrix} 0.05 & 0.1 \\ 0.1 & 0.5 \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0.981 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ x \end{bmatrix} + \begin{bmatrix} 1 \\ -20 \end{bmatrix} u$$

or

$$\frac{d}{dt} \begin{bmatrix} \dot{\theta} \\ \dot{x} \\ \theta \\ x \end{bmatrix} = \begin{bmatrix} 0 & 0 & 32.7 & 0 \\ 0 & 0 & -6.54 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{x} \\ \theta \\ x \end{bmatrix} + \frac{1}{3} \begin{bmatrix} 500 \\ -220 \\ 0 \\ 0 \end{bmatrix} u$$

and

$$\begin{bmatrix} \dot{\theta} \\ \phi \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 20 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{x} \\ \theta \\ x \end{bmatrix}$$

The measurements ϕ from the encoder and $\dot{\theta}$ from the gyro are available for the controller to produce the torque commands, u, to the motors.

Consider the problem of designing a controller with the following requirements:

- The controller should stabilize the vehicle;
- The controller shall be able to hande a delay of 0.02 s in the loop;
- The gain and phase margins should be adequate, aim at 5 dB and 30 deg at the input of the motor;
- The controller shall be able to accept a reference input in x;
- The attitude angle, θ, shall be zero in steady state;
- Try to make the step response in x as fast as possible and with resonable overshoot.