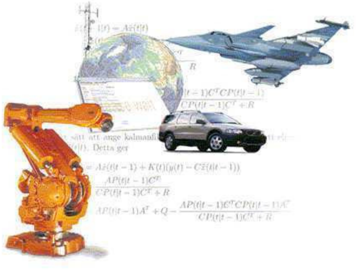
# Tool Position Estimation of a Flexible Industrial Robot using Recursive Bayesian Methods



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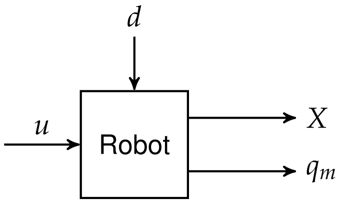
# Outline

1. Introduction
   - Problem Formulation
   - Bayesian Estimation
2. Modelling
   - Robot Model
   - Accelerometer Model
   - Estimation Model
3. Experimental Results
4. Conclusions and Future Work

# Problem Formulation





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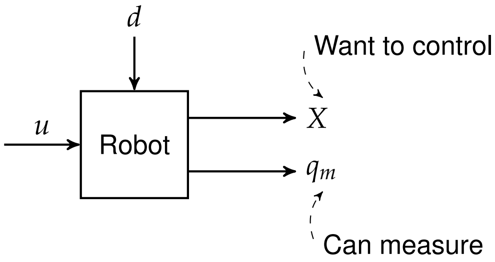


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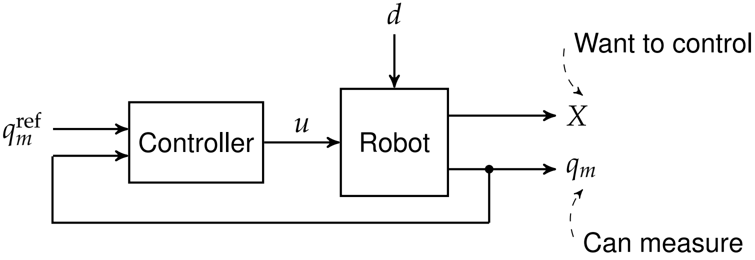


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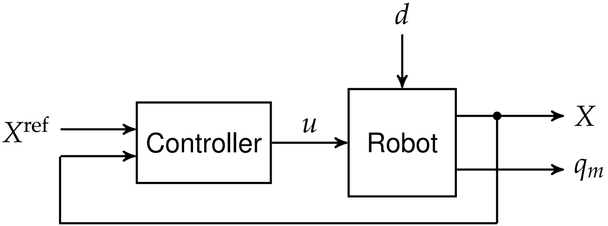


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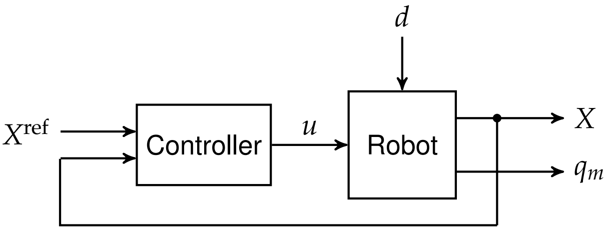


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What can we do instead of measuring the TCP?

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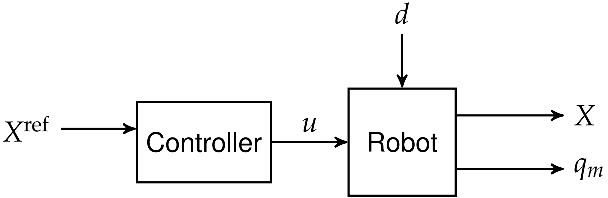
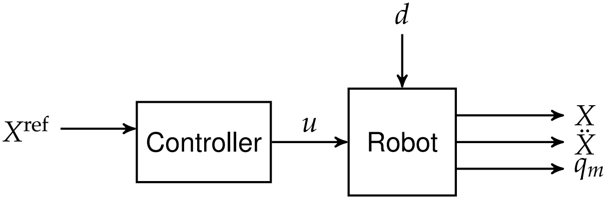
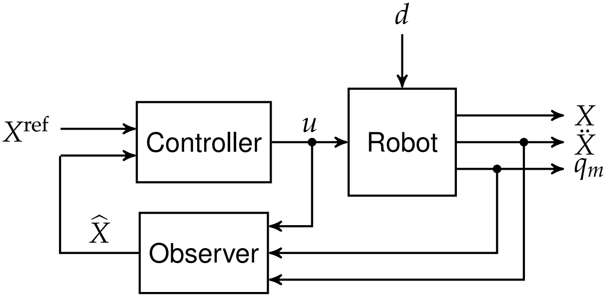
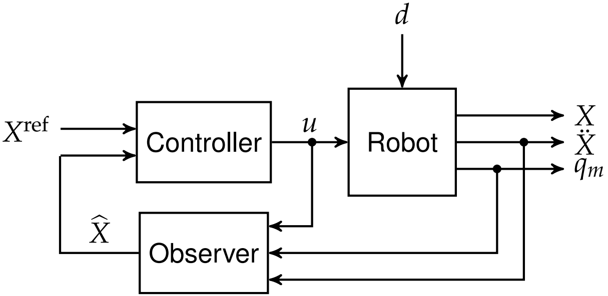





- Let the acceleration of the tool be a measurement.




- Let the acceleration of the tool be a measurement.
- Use an observer to estimate the TCP.

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**How can we estimate the TCP?**



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Model:

$$
\begin{aligned}
x_{t+1} &= f(x_t, u_t, w_t), \\
y_t &= h(x_t) + e_t.
\end{aligned}
$$

Bayesian inference:

$$
\begin{aligned}
p(x_{t+1}|\mathbb{Y}_t) &= \int_{\mathbb{R}^n} p(x_{t+1}|x_t) p(x_t|\mathbb{Y}_t)\, \mathrm{d}x_t, \\
p(x_t|\mathbb{Y}_t) &= \frac{p(y_t|x_t) p(x_t|\mathbb{Y}_{t-1})}{p(y_t|\mathbb{Y}_{t-1})}.
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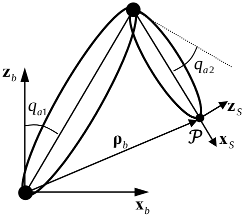
  - Extended Kalman filter (EKF)
    - Approximate the system with a linearisation of the nonlinear equations.
    - Assume additive Gaussian noise.
  - Particle filter (PF)
    - Approximate the posterior distribution with a large number of particles.
    - The optimal proposal distribution approximated using an EKF.

- Serial robot with 2 DOF.
- Linear stiffness.
- No friction.



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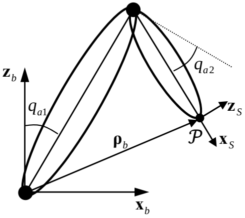
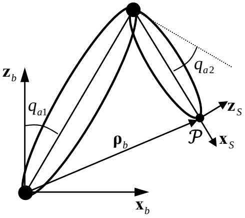
$$q_a = \begin{pmatrix} q_{a1} & q_{a2} \end{pmatrix}^{\mathsf{T}},$$
$$q_m^a = \begin{pmatrix} q_{m1}/\eta_1 & q_{m2}/\eta_2 \end{pmatrix}^{\mathsf{T}},$$
$$\tau_m^a = \begin{pmatrix} \tau_{m1}\eta_1 & \tau_{m2}\eta_2 \end{pmatrix}^{\mathsf{T}},$$




$$M_a(q_a)\ddot{q}_a + C(q_a, \dot{q}_a) + G(q_a) + T(q_a - q_m^a) + D(\dot{q}_a - \dot{q}_m^a) = 0,$$
$$M_m\ddot{q}_m^a - T(q_a - q_m^a) - D(\dot{q}_a - \dot{q}_m^a) = \tau_m^a.$$

# Accelerometer Model

- Acceleration from the motion.
- Acceleration due to the gravity.
- Rotation matrix from $Ox_b z_b$ to $Ox_s z_s$.
- Bias parameter.




$$\ddot{\rho}_s(q_a) = \mathcal{R}_{b/s}(q_a)\left(\ddot{\rho}_b(q_a) + G_b\right) + \mathfrak{b}^{\text{ACC}}$$

- State space vector:

$$x = \begin{pmatrix} x_1^\mathsf{T} & x_2^\mathsf{T} & x_3^\mathsf{T} \end{pmatrix}^\mathsf{T} = \begin{pmatrix} q_a^\mathsf{T} & \dot{q}_a^\mathsf{T} & \ddot{q}_a^\mathsf{T} \end{pmatrix}^\mathsf{T}$$

- Linear dynamic model (Double integrator in discrete time):

$$x_{k+1} = \mathcal{F} x_k + \mathcal{G}_u u_k + \mathcal{G}_v v_k$$

- The input signal is the jerk of the arm angle reference.

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- The input signal is the jerk of the arm angle reference.
- Measurement equation:

$$y_k = \begin{pmatrix} x_{1,k} + K^{-1}\left(M_a(x_{1,k}) x_{3,k} + C(x_{1,k}, x_{2,k}) + G(x_{1,k})\right) \\ \mathcal{R}_{b/s}(x_{1,k}) \left( J_{\mathsf{ACC}}(x_{1,k}) x_{3,k} + \left( \frac{d}{dt} J_{\mathsf{ACC}}(x_{1,k}) \right) x_{2,k} + G_b \right) \end{pmatrix} + e_k$$

# Experimental Setup

- Experiments performed on an ABB IRB4600.
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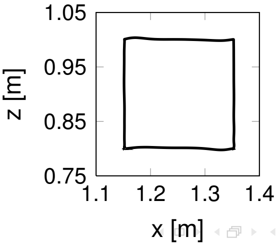

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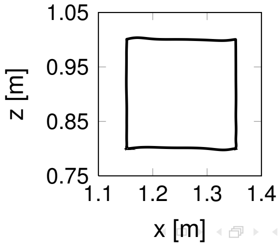
- Synchronization errors and kinematic errors are present.



# Experimental Setup

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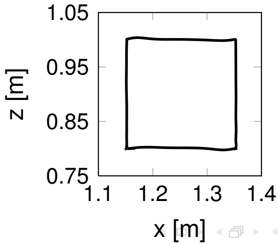
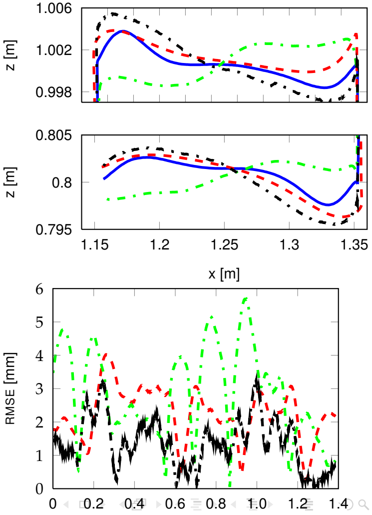
- Only joints 2 and 3 are used.
- The true path is measured by a laser system from Leica.
- Synchronization errors and kinematic errors are present.
- EKF and PF are compared to the forward kinematic using measured motor angles, $\mathcal{T}_{\text{TCP}}(q_m)$.



- Both filters follow the true path.
- The EKF passes in the corners.
- $\mathcal{T}_{\text{TCP}}(q_m)$ out of phase.




● True path ● EKF ● PF ● $\mathcal{T}_{\text{TCP}}(q_m)$

- Both filters follow the true path.
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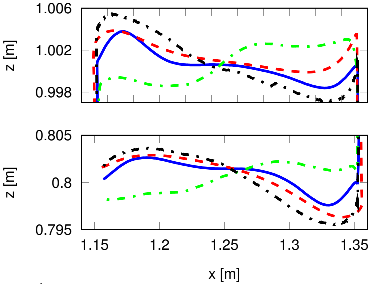
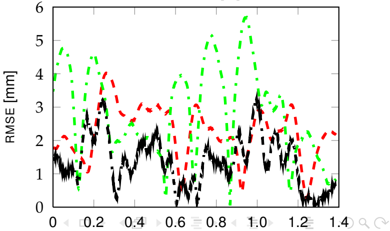
- $\mathcal{T}_{\text{TCP}}(q_m)$ out of phase.
- Bias states for both motor and accelerometer measurements are required to get good results.
- Root Mean Square Error

| | EKF | PF | $\mathcal{T}_{\text{TCP}}(q_m)$ |
|---|---|---|---|
| RMSE | 0.124 | 0.083 | 0.167 |

- The current MATLAB implementation not in real time for the EKF and PF.

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Conclusions

- Estimation of the robot's TCP using Bayesian methods and an accelerometer.
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# Conclusions and Future Work

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Future Work

- Sensitivity analysis of model parameters.
- Time varying covariance matrices for the EKF to handle sharp turns.
- Extend to 6 DOF. More sensors? Where to place the sensors?
- Control using the estimated position.

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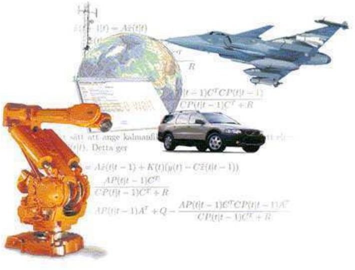

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