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Homework set # 1 for the course
"Opinion Dynamics on Social Networs"

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1. **PageRank.** This is a computational exercise about the PageRank algorithm. In the file `PageRank.mat` you will find an adjacency matrix representing hyperlinks of a section of the WWW.

   (a) Compute the PageRank order of the nodes implementing the power iteration method mentioned in class, and verify that indeed it represents the P.F. eigenvector.

   (b) How many interactions of the power method do you need to get to an error tollerance of 10^{-4} to the "true P.F. eigenvector" (the one computed using `eig( )` function of Matlab)?

   (c) Is the order of PageRank nodes invariant to changes of m? In particular what happens when $m = 0$?

   (d) in the paper

   > H. Ishii, R. Tempo. "The PageRank problem, multiagent consensus and web aggregation", IEEE Control Systems Magazine, June 2014.

   at page 39-40, a distributed randomized method for PageRank computation is described. You should implement this method and compare its accuracy and speed of convergence to the (non-distributed) power method above.

2. **Leontief input-output model**. Consider a group of n industries each of which is producing only one commodity, and to do so requires to use commodities from the other industries (for example to produce books you need paper and ink produced by other industries, etc.). Assume that a part of the produced commodities is for satisfying external demand. Denote $d \in \mathbb{R}_+^n$, $d \geqslant 0$ the demand vector. Call x_i the amount of product of the industry i and $t_{ij} \geqslant 0$ the units of product j needed per unit of product i. The Leontief model is given by the static relationship

$$x_i = \sum_j t_{ij} x_j + d_i \qquad i = 1, \dots, n$$

or in matrix form

$$x = Tx + d$$

(a) Under what conditions a production vector x exist?

(b) Is is unique?

(c) What kind of properties do T and d have to obey in order for x to be economically meaningful?

(d) What can happen when $d = 0$?

(e) Show that if a Leontief model with matrix T is feasible, then the sum of the entries of T is less than one in at least one column.

(f) OPTIONAL: If you were to replace the matrix T with a nonlinear dependence from x (i.e., $T(x)$) what kind assumption on $T(x)$ (if any) would you impose to guarantee existence and uniqueness of x?