On Fusion of Sensor Measurements and Observation with Uncertain Timestamp for Target Tracking

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Background - Rhino Tracking

Courtesy of Martin Stenmarck

- Not always observable from sensors
- Leave other traces, such as
  - Droppings
  - Foot prints
  - Damage
- How to use this information?

Background - Orienteering

- Sometimes poor GPS
- Known position of controls
- Time not always available

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Illustrative Example - Model

Consider a simple but illustrative model,

$$x_k = x_{k-1} + v_k, \qquad v_k \sim \mathcal{N}(0, Q),$$
$$y_j = x_j + e_j^y, \qquad e_j^y \sim \mathcal{N}(0, R^y)$$

for $k \in \{1, \ldots, N\}$ and two measurements y_1 and y_N.

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for $k \in \{1, \ldots, N\}$ and two measurements y_1 and y_N.

In addition there is one more observation

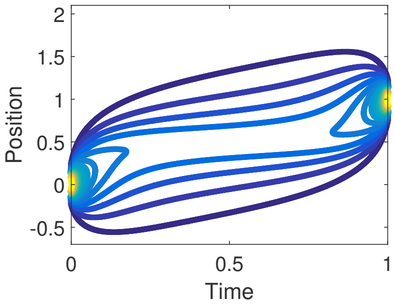
$$z = x_\tau + e^z, \qquad e^z \sim \mathcal{N}(0, R^z)$$

where $\tau \in \{1, \ldots, N\}$ is uncertain with a prior $p(\tau)$.

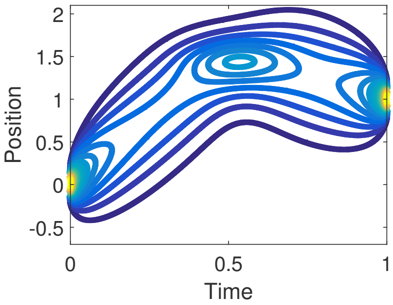
Illustrative Example - Posterior Distribution

Measurements are $y_1 = 0$ and $y_N = 1$ and observation is

$z = 0.5$ with flat prior.



$z = 1.5$ with narrow prior.



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General Model

Consider a linear Gaussian state space model,

$$\begin{aligned} &\mathbf{x}_k = \mathbf{F}_k \mathbf{x}_{k-1} + \mathbf{v}_k, && \mathbf{v}_k \sim \mathcal{N}(0, \mathbf{Q}_k), && k \in \mathcal{K} = \{1, \dots, N\} \\ &\mathbf{y}_j = \mathbf{H}_j^y \mathbf{x}_j + \mathbf{e}_j^y, && \mathbf{e}_j^y \sim \mathcal{N}(0, \mathbf{R}_j^y), && j \in \mathcal{J} \subseteq \mathcal{K} \\ &\mathbf{x}_0 \sim \mathcal{N}(\bar{\mathbf{x}}_0, \mathbf{P}_0) \end{aligned}$$

with notations

$$\mathcal{X} = \{\mathbf{x}_k\}_{k \in \mathcal{K}} \qquad \text{and} \qquad \mathcal{Y} = \{\mathbf{y}_j\}_{j \in \mathcal{J}}.$$

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Extend the model with

$$\mathbf{z} = \mathbf{H}^z \mathbf{x}_\tau + \mathbf{e}^z, \qquad \mathbf{e}^z \sim \mathcal{N}(0, \mathbf{R}^z), \qquad \tau \sim p(\tau).$$

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Posterior Distributions - Time

The posterior distribution of the uncertain time is

$$
\begin{aligned}
w_\tau &\triangleq p(\tau|\mathcal{Y}, \mathbf{z}) \\
&\propto p(\tau)p(\mathbf{z}|\tau, \mathcal{Y}) \\
&= p(\tau)\mathcal{N}(\mathbf{z}|\, \hat{\mathbf{z}}_\tau,\, \mathbf{S}_\tau),
\end{aligned}
$$

which is obtained using a Rauch-Tung-Striebel smoother.

Posterior Distributions - States

The full posterior distribution is

$$\begin{aligned} p(\mathcal{X}|\mathcal{Y}, \mathbf{z}) &= \sum_{\tau \in \mathcal{K}} p(\tau|\mathcal{Y}, \mathbf{z}) \cdot p(\mathcal{X}|\mathcal{Y}, \mathbf{z}, \tau) \\ &= \sum_{\tau \in \mathcal{K}} w_\tau \cdot \mathcal{N}(\mathcal{X}|\hat{\mathcal{X}}^\tau, \mathbf{P}^\tau). \end{aligned}$$

Posterior Distributions - States

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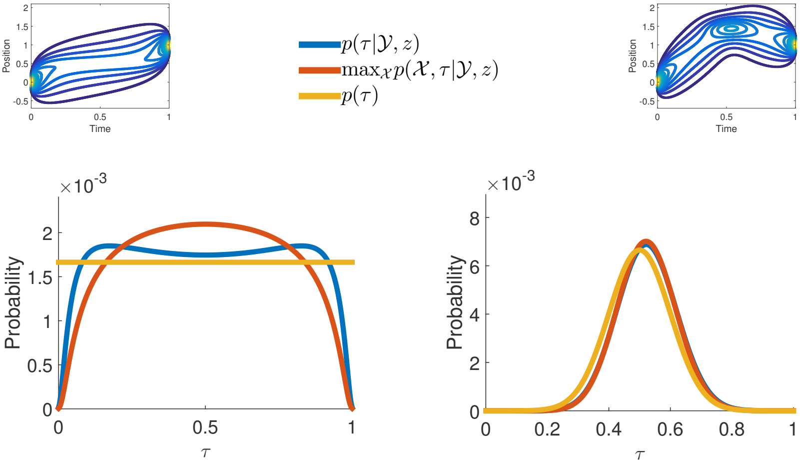
$$= \sum_{\tau \in \mathcal{K}} w_\tau \cdot \mathcal{N}(\mathcal{X}|\hat{\mathcal{X}}^\tau, \mathbf{P}^\tau).$$

The marginal posterior distribution is

$$p(\mathbf{x}_k|\mathcal{Y}, \mathbf{z}) = \sum_{\tau \in \mathcal{K}} w_\tau \cdot \mathcal{N}(\mathbf{x}_k|\hat{\mathbf{x}}_k^\tau, \mathbf{P}_k^\tau).$$

Posterior Distributions - Time and States



Estimators - Minimum Mean Squared Error

The MMSE estimator of $p(\mathcal{X}|\mathcal{Y}, \mathbf{z})$ is

$$\hat{\mathcal{X}}^{MMSE} = \sum_{\tau \in \mathcal{K}} w_\tau \hat{\mathcal{X}}^\tau,$$

which is equivalent for $p(\mathbf{x}_k|\mathcal{Y}, \mathbf{z})$.

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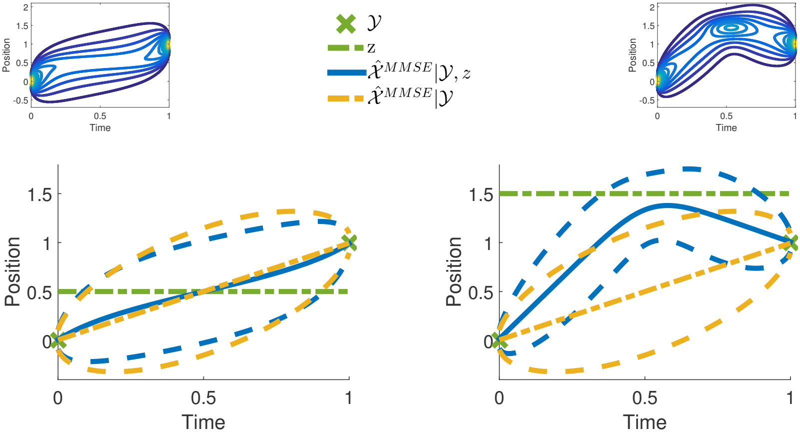
which is equivalent for $p(\mathbf{x}_k|\mathcal{Y},\mathbf{z})$. The MSE is

$$\mathbf{P}^{MMSE} = \sum_{\tau\in\mathcal{K}} w_\tau \Big(\mathbf{P}^\tau + (\hat{\mathcal{X}}^\tau - \hat{\mathcal{X}}^{MMSE})(\hat{\mathcal{X}}^\tau - \hat{\mathcal{X}}^{MMSE})^T\Big).$$

Estimators - Minimum Mean Squared Error



Estimators - Maximum A Posteriori

The MAP estimate of the full posterior distribution is

$$\hat{\mathcal{X}}^{MAP} = \arg\max_{\mathcal{X}} p(\mathcal{X}|\mathcal{Y}, \mathbf{z}).$$

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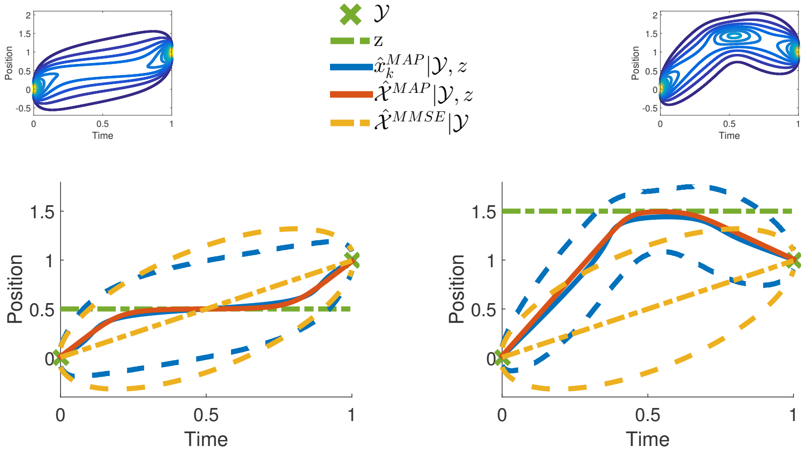
The MAP estimate of the marginal posterior distribution is

$$\hat{\mathbf{x}}_k^{MAP} = \arg\max_{\mathbf{x}_k} p(\mathbf{x}_k|\mathcal{Y}, \mathbf{z}).$$

Estimators - Maximum A Posteriori



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Summary

- Observation with uncertain timestamp
- Relevant distributions and estimators have been derived
- A simple example has been analysed
- Computationally demanding even for simple problems

Future Work

- Multiple observations with uncertain timestamp
- Corresponding filtering problem
- Nonlinear models

Thank you for listening!

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