Session 2

Existence and Uniqueness. Grönwall's Inequality. Transition matrix. Change of Variables. Adjoint Equation.

Reading Assignment

Rugh pp. Ch 3, 4.

Exercises

Exercise 2.1 = Rugh 3.1

Exercise 2.2 = Rugh 3.3

Exercise 2.3 = Rugh 3.7 (3.4 in old edition)

Exercise 2.4 = Rugh 3.15 (3.12)

Exercise 2.5 = Rugh 4.2 (4.1)

Exercise 2.6 = Rugh 4.3 (4.2)

Exercise 2.7 = Rugh 4.10 (4.9)

Exercise 2.8 Given $v \in \mathbf{R}^m$ and $M \in \mathbf{R}^{n\times m}$ with MM^T invertible, prove that the minimal $|u|$ subject to the constraint $Mu = v$ is given by

$$\hat{u} \;=\; M^T(MM^T)^{-1}v$$

(Hint: Prove first the alignment property mentioned in the lecture.)
Does your proof generalize to operators on $\mathbf{L}_2^m[0,\infty)$?

Hand in problems

Solutions to the following problems should be handed in at the exercise session.

Exercise 2.9 = Rugh 4.17 (4.15)

Exercise 2.10 = Rugh 3.5-6 using Maple

Exercise 2.11 Consider two well-mixed tanks in series with timevarying flow $q(t)$ and constant volumes V_1 and V_2 (see example in Lecture 1). Regard the inlet concentration as input and the tank concentrations as state variables. Determine the transition matrix as a function of q, V_1, and V_2. Express the relationship between inlet and outlet concentrations by an integral equation.