Session 3

Periodic Solutions. Internal Stability. Discrete Time Systems.

Reading Assignment

Rugh Ch 5, 6, [7, 8], 20, 21
Some reference material on operator theory was handed out: Ch6 of "Luenberger, D.G., Optimization by Vector Space Methods, Wiley 1969".

Exercise 3.1 Let S be the shift operator on ℓ_2 as in Example of Lecture 3. Find $\mathcal{N}(\lambda I - S^*)$ and $\mathcal{R}(\lambda I - S^*)$ for all (complex) λ. For what λ the latter set is closed (in ℓ_2). (Hint: Use discrete Laplace transformation.)

Exercise 3.2 = Rugh 5.8 (5.6) (Does Floquet decomposition apply?)

Exercise 3.3 = Rugh 5.19 (5.16)

Exercise 3.4 = Rugh 5.24 (5.19)

Exercise 3.5 Find the minimal energy $\int_0^1 |u(t)|^2 dt$ of an input signal u such that $\dot{x}(t) = -x(t) + u(t)$, $x(0) = 0$, $x(1) = 1$.

Exercise 3.6 = Rugh 20.12 (Draw block diagram.)

Exercise 3.7 = Rugh 21.8

Exercise 3.8 = Rugh 6.18 (6.13)

Hand in problems

Solutions to the following problems should be handed in.

Exercise 3.9 = Rugh 6.13 (6.8)

Exercise 3.10 *Inverted pendulum with oscillating suspension point:*

Can the topmost, usually unstable, equilibrium position of a pendulum become stable if the point of suspension oscillates in the vertical direction?

Let the length of the pendulum be l, the amplitude of the oscillation of the point of suspension be $a << l$, the period of oscillation of the point of suspension 2τ, and, moreover, in the course of every half-period let the acceleration of the point of suspension be *constant* and equal to $\pm c$ (then $c = 8a/\tau^2$).

The equation of motion can be written in the form $\ddot{x} = (\omega^2 \pm d^2)x$ (the sign changes after time τ), where $\omega^2 = g/l$ and $d^2 = c/l$. If the oscillation of the suspension is fast enough, $c > g$, then $d^2 = 8a/(l\tau^2) > \omega^2$.

a. Show that $\Phi(2\tau, 0) = A_2 A_1$

$$A_1 = \begin{bmatrix} \cosh k\tau & 1/k \sinh k\tau \\ k \sinh k\tau & \cosh k\tau \end{bmatrix} \quad A_2 = \begin{bmatrix} \cos \Omega\tau & 1/\Omega \sin \Omega\tau \\ -\Omega \sin \Omega\tau & \cos \Omega\tau \end{bmatrix}$$

where $k^2 = d^2 + \omega^2$ and $\Omega^2 = d^2 - \omega^2$.

b. Show that $\det(\Phi(t, 0)) = 1$, $t = 2\tau, 4\tau, 6\tau, \ldots$ and that the criterion for stability is $|\mathrm{tr}(\Phi(2\tau, 0))| < 2$

c. Suppose that $a << l$. Show that the system is stable for sufficiently large $c >> g$. (The stability condition is approximately $c/g > (3/2)\, l/a$.)