Session 4

Reachability and Controllability. Observability and Reconstructability. Controller and Observer Forms. Balanced Realizations.

Reading Assignment

Rugh, chapters 9,13 and 25.

Exercises

Exercise 4.1 = Rugh 9.3 (9.2)

Exercise 4.2 = Rugh 9.4

Exercise 4.3

a. Show that $\{A, B\}$ is controllable iff $\{PAP^{-1}, PB\}$ is controllable for some P with $\det P \neq 0$ iff $\{PAP^{-1}, PB\}$ is controllable for all P with $\det P \neq 0$.

b. Prove that $\{A, B\}$ is controllable iff $\{A - BL, B\}$ is controllable for some L iff $\{A - BL, B\}$ is controllable for all L.

c. Prove that $\{A, B\}$ is controllable iff $\{A, BB^T\}$ is controllable.

Exercise 4.4 Use the PBH test to show that the controller form realization of $b(s)/a(s)$ is observable iff $a(s), b(s)$ are coprime.

Exercise 4.5 What are the controllability indices of $\dot{x} = Ax + Bu$ when

$$A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$$

How is the rank of B reflected in the indices?

Exercise 4.6 Show that the set of controllability indices is invariant under change of coordinates.

Exercise 4.7 Formulate observability and reconstructability using operators, i.e. the possibility to find $x(t_0)$ and $x(t_1)$ given the output and the input during the interval $[t_0, t_1]$. The nullspaces of the corresponding operators are now the important spaces. (Hint: Use

$$\begin{aligned} y(t) &= (L_1 x_0 + L_2 u_{[t_0,t_1]})(t) \\ &= C(t)\Phi(t,t_0)x_0 + C(t)\int_{t_0}^{t} \Phi(t,s)B(s)u(s)\,ds \end{aligned}$$

and discuss the solvability of the equation $L_1 x_0 = b$. Describe L_1^* and use $\mathcal{N}(L_1) = \mathcal{N}(L_1^* L_1)$.)

Exercise 4.8 Rugh 25.10

Hand in problems

Solutions to the following problems should be handed in at the exercise session.

Exercise 4.9 *Interpretation of Reachability Gramian*

Consider the double integrator

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= u\end{aligned}$$

Determine for different t, which state that can be reached from $x(0) = 0$, with input signal energy less than 1, i.e.

$$\|u\|^2 = \int_0^t u^2(t)\,dt \leq 1$$

Plot the regions using e.g. Matlab.

Exercise 4.10 Find controllability indices and controller form for the "double effect evaporator" model $\dot{x} = Ax + Bu$, where

$$A = \begin{bmatrix} 0 & -0.0016 & -0.0711 & 0 & 0\\ 0 & -0.1419 & 0.0711 & 0 & 0\\ 0 & -0.0088 & -1.1020 & 0 & 0\\ 0 & -0.0013 & -0.1489 & 0 & -0.0013\\ 0 & 0.0605 & 0.1489 & 0 & -0.0591 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & -0.1430 & 0\\ 0 & 0 & 0\\ 0.3920 & 0 & 0\\ 0 & 0.1080 & -0.0592\\ 0 & -0.0486 & 0 \end{bmatrix}$$

A Matlab-file is available: `~fulinsys/linsys93/matlab/s12b.m`