Session 5

Realizations from weighting patterns, impulse responses, and Markov parameters. Minimal Realizations

Reading Assignment

Rugh, chapter 10, 11 and 26. Skip some of the proofs.

Exercises

Exercise 5.1 = Rugh 10.7 (10.4)

Exercise 5.2 = Rugh 10.9 (10.6)

Exercise 5.3 = Rugh 10.12 (10.11)

Exercise 5.4 = Rugh 11.12 (11.11)

Exercise 5.5 Consider the discrete time system

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix}$$

Determine the reachable subspace and the unobservable subspace. Determine Kalman's decomposition (See CCS). Determine a minimal realisation. Determine the least k such that $x(k) = 0$ for any $x(0)$.

Exercise 5.6 Perform the calculations in Rugh Example 26.21 (p497) for $\alpha = -2, 0, 1$.

Hand in problems

Exercise 5.7 The following system is given

$$\begin{aligned} \dot{x}_1 &= \sin(t)u(t) \\ \dot{x}_2 &= \cos(t)u(t) \\ y(t) &= \sin(t)x_1(t) + \cos(t)x_2(t) \end{aligned}$$

Calculate the weighting pattern, and show that it is stationary. Then give a time invariant realisation.

Exercise 5.8 Find a minimal realization of

$$R(s) = \begin{bmatrix} \frac{s+1}{s^2+2s+1} & \frac{s}{s^2+1} \\ \frac{1}{s+2} & \frac{2}{s^2+3s+2} \end{bmatrix}$$