Session 6

Linear Feedback, Eigenvalue Assignment, State Observation, Youla Parameterization

Reading Assignment

Rugh, chapter 14, 15 and 28, 29.

Exercises

Exercise 6.1 = Rugh 14.8

Exercise 6.2 To further discuss the freedom in choice of feedback matrix for MIMO systems, consider the open loop system ([Moore, AC Oct 1976, pp. 691])

$$A = \begin{bmatrix} -1.25 & 0.75 & -0.75 \\ 1 & -1.5 & -0.75 \\ 1 & -1 & -1.25 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$
$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

which has controllable eigenvalues -1.25, -2.25 and an uncontrollable eigenvalue at -0.5. Assume that the control objective is to shift the eigenvalues to -5,-6,-0.5 and to obtain a reasonable response for the initial condition $[0\ 0\ 1]^T$, which represents a disturbance in $x_3(t)$.

a. Check that

$$L = \begin{bmatrix} 4.5344 & -4.5344 & 6.6915 \\ 1.0784 & -1.0784 & 4.0440 \end{bmatrix}$$

gives the correct poles, and simulate the system (hint: matlab).

b. The design above is not satisfactory since the slow uncontrollable mode is present in y_1. Find a state feedback so that the eigenvector corresponding to the uncontrollable mode is $X_3 = [0\ 1\ 0]^T$, and hence is unobservable in the output.

Exercise 6.3 = Rugh 14.4

Exercise 6.4 = Rugh 14.11

Exercise 6.5 = Rugh 15.5 (15.4)

Exercise 6.6 = Rugh 15.6

Exercise 6.7 Reduced order observer as a fast full state observer Consider Examples 15.6 (p276). What H is needed to place one pole at -5 and one at $-1/T$? Calculate the transfer functions from y to u and to $\hat{x}$. Show that the limits for small T are equal to the corresponding transfer functions in Example 15.8 (p280).

Hand in problems

Exercise 6.8 Rugh 29.4

Exercise 6.9 Consider the following model of a flexible servo with two tachometers and two motors:

$$\dot{x} = \begin{bmatrix} 0 & 0 & -a_1 & a_1 \\ 0 & 0 & a_2 & -a_2 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} b_1 & 0 \\ 0 & b_2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x$$

a. Calculate the controllability indices.

b. Choose state feedback $u = r - Kx$ to obtain the charactersistic polynomial $[s^2+2\zeta\omega s+\omega^2]^2$. (If symbolic computations are problematic use numerical values $a_i = b_i = 1, \zeta = 0.7, \omega = 1$)

c. Calculate a reduced order observer and place the two observer poles in p_1 and p_2 . ($p_1 = -2$ and $p_2 = -3$).