Magnetic-Field Based Odometry

Gustaf Hendeby, gustaf.hendeby@liu.se
Manon Kok, M.Kok-1@tudelft.nl
Isaac Skog, isaac.skog@liu.se

LINKÖPING UNIVERSITY

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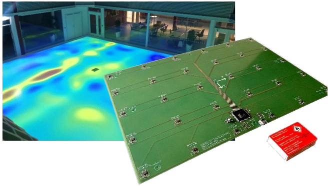
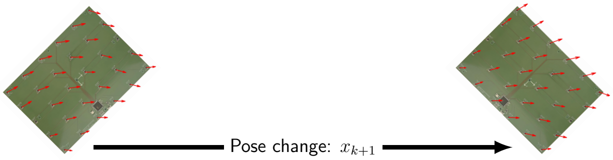
Magnetic Field Based Odometry: Basic idea

By measuring how the shape of the local magnetic field varies the pose change of the array can be estimated.





Two approaches two estimate the pose change:

- Differential equation based magnetic field odometry
- Model based magnetic field odometry

Differential Equation Based Magnetic Field Odometry

Diff. Eq. Based Magnetic Field Odometry: Dorveaux *et al.*

- The change experienced magnetic field is given by a differential equation:

$$\frac{dm}{dt} = m \times \omega + \frac{dm}{dr} v$$

- Quantities that can be derived from the array:
  - ω: measured with a gyroscope.
  - m: directly available from the array.
  - $\frac{dm}{dr}$: approximated using numerical derivatives.
  - $\frac{dm}{dt}$: approximated using numerical derivative.
- The speed v can be solved for.

Curtesy: Eric Dorveaux

Diff. Eq. Based Magnetic Odometry: Properties

- Requires gyroscope measurements.
- The numerical derivatives does not take the measurement noise into consideration. $\Rightarrow$ Sensitive to measurement noise in magnetometers and gyroscope.
- Resulting v is in body frame!
- Practical results are promising.

References

E. Dorveaux. *Magneto-inertial navigation: principles and application to an indoor pedometer.* PhD thesis, Paris Institute of Technology, 2011.

E. Dorveaux, T. Boudot, M. Hillion, and N. Petit. Combining inertial measurements and distributed magnetometry for motion estimation. In *Proc. of American Control Conf.*, San Francisco, CA, June 2011.

E. Dorveaux and N. Petit. Presentation of a magneto-inertial positioning system: navigating through magnetic disturbances. In *Int. Conf. on Indoor Positioning and Indoor Navigation (IPIN)*, Guimaraes, Portugal, Sept. 2011.

INS Integration: Zmitri *et al.*

- Continues Dorveaux's work.
- An intricate *extended Kalman filter* (EKF) and machine learning to solve the magnetic differential equation.
- INS drift theoretically reduced from cubic to linear in time.

References

M. Zmitri, H. Fourati, and C. Prieur. Improving inertial velocity estimation through magnetic field gradient-based extended Kalman filter. In *Int. Conf. on Indoor Positioning and Indoor Navigation (IPIN)*, Pisa, Italy, Oct. 2019.

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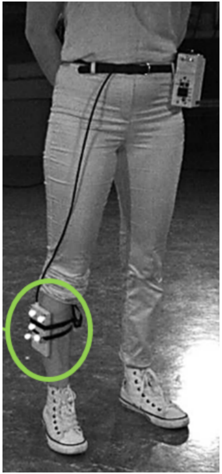
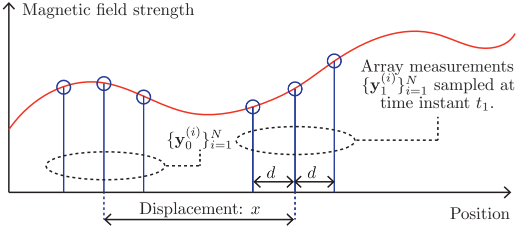


Curtesy: Makia Zmitri

Model Based Magnetic-Field Odometry

An Optical Flow Point of View

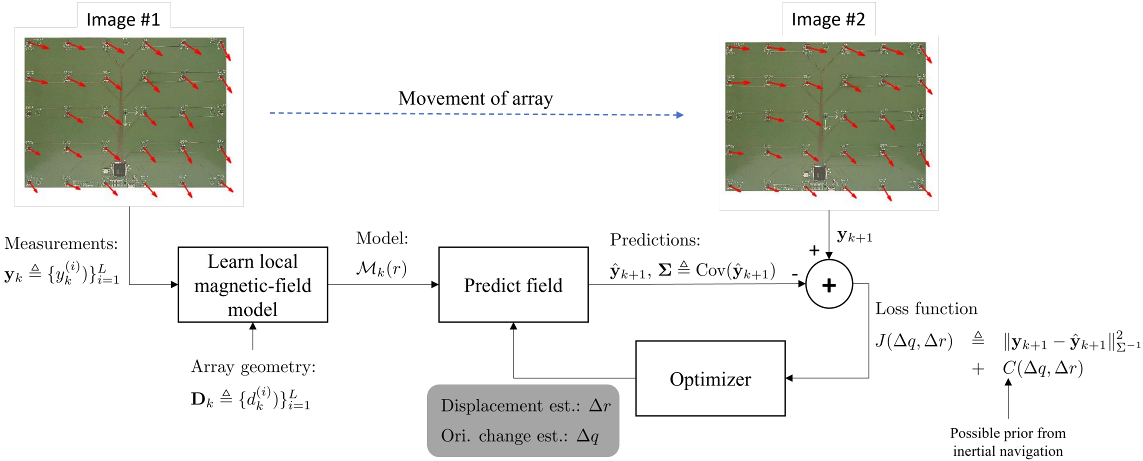
Optical Flow *Follow points between two visual images.*



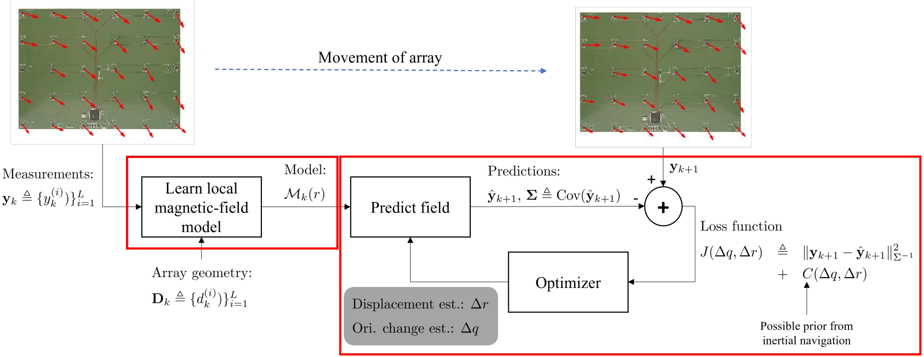
"Magnetic Flow"

Follow magnetic changes between two magnetic-field "images" and use a magnetic-field model to interpolate between the sparse measurement points.

Model Based Approach: General Idea



Model Based Approach: Key Questions



Key questions:

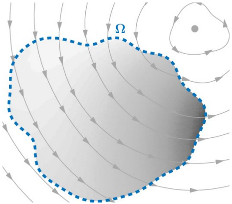
- How to choose the model $\mathcal{M}_k(r;\theta)$ and train it?
- How to use the predicted field to estimate the pose change?

Magnetic field properties and modeling

Maxwell's equations (in vacuum, no charges or currents):

$$\nabla \cdot E = 0 \qquad \nabla \times E = -\frac{\partial M}{\partial t}$$

$$\nabla \cdot M = 0 \qquad \nabla \times M = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$



Curtesy: Arno Solin

Observations:

- The magnetic field is divergence free: $\nabla \cdot M = 0$
- In a static electric field, the magnetic field is curl free: $\nabla \times M = 0$

$\Longrightarrow$ A model $\mathcal{M}(r;\theta)$ ($\theta =$ model parameters) of the magnetic-field should (preferably) fulfill this property. Examples of models that can be designed to have these properties are: sum of dipoles, **polynomial**, **Gaussian processes**, and neural networks.

Polynomial Model: General Case

- Use 3 independent polynomials to describe the magnetic field in the 3 directions.
- Number of parameters needed for a degree ℓ field model: $\dim(\theta) = 3 \cdot \frac{(\ell+3)(\ell+2)(\ell+1)}{6}$.

Example:

Field model consisting of three quadratic polynomials

$$\mathcal{M}(r;\theta) = \begin{pmatrix} r^T C_{\mathsf{x}}^2 r + C_{\mathsf{x}}^1 r + C_{\mathsf{x}}^0 \\ r^T C_{\mathsf{y}}^2 r + C_{\mathsf{y}}^1 r + C_{\mathsf{y}}^0 \\ r^T C_{\mathsf{z}}^2 r + C_{\mathsf{z}}^1 r + C_{\mathsf{z}}^0 \end{pmatrix}$$

- $\theta = \{C_{\mathsf{x}}^2, C_{\mathsf{x}}^1, C_{\mathsf{x}}^0, C_{\mathsf{y}}^2, C_{\mathsf{y}}^1, C_{\mathsf{y}}^0, C_{\mathsf{z}}^2, C_{\mathsf{z}}^1, C_{\mathsf{z}}^0\}$
- $\dim(\theta) = 30$

Polynomial Model: Curl-free Case

- Fields generated from potentials are always curl free.
- Model the magnetic field in terms of its magnetic potential $\varphi(r;\theta):\ \mathbb{R}^3 \mapsto \mathbb{R}$.
- Number of parameters needed for a degree ℓ field model (degree $(\ell+1)$ potential): $\dim(\theta) = \frac{(\ell+4)(\ell+3)(\ell+2)}{6} - 1$.

Example :
The underlying potential is a cubic polynomial defined by $h(r)\theta$.

$$\mathcal{M}(r;\theta) = \nabla_r \underbrace{\varphi(r;\theta)}_{\text{potential}} = \nabla_r \underbrace{h(r)\theta}_{\text{poly.}} = A(r)\theta$$

- Number of parameters: $\dim(\theta) = 19$

Polynomial Model: Divergence- & Curl-free Case

- Divergence-free implies that $\nabla_r \cdot \mathcal{M}(r;\theta) = 0$.
- For a polynomial model this is a linear constrain in θ:

$$\nabla_r \cdot \mathcal{M}(r;\theta) = \nabla_r \cdot A(r)\theta = B\theta = 0$$

- The number of linear constrains are $K = \frac{(\ell+2)(\ell+1)\ell}{6}$.
- θ must reside in the null space of B. $\Rightarrow$ New par. $\theta = B^{\perp}\theta_l$, $B^{\perp} = \text{null}(B)$.
- $\dim(\theta_l) = \dim(\theta) - K = \ell^2 + 4\ell + 3$.

Example :
The underlying potential is a cubic polynomial in r.

$$\mathcal{M}(r;\theta_l) = \Phi(r)\theta_l \qquad \Phi(r) \triangleq \nabla_r h(r)\,\text{null}\big(\nabla_r \cdot (\nabla_r h(r))\big) \qquad \dim(\theta_l) = 15$$

Polynomial Model Parameter Estimation (Training/Learning)

- Estimation (learning) of the model parameters is given by

$$\hat{\theta}_l = \left(\sum_{i=1}^{L} \Phi^\top(d^{(i)}) \Phi(d^{(i)}) \right)^{-1} \sum_{i=1}^{L} \Phi^\top(d^{(i)}) y_k^{(i)}.$$

- The cross-covariance of the two estimates $\hat{\mathcal{M}}_k(r^{(i)})$ and $\hat{\mathcal{M}}_k(r^{(j)})$ is given by

$$\Sigma_{\hat{\mathcal{M}}}^{(i,j)} = \hat{\sigma}_e^2 \, \Phi(r^{(i)}) \left(\sum_{i=1}^{L} \Phi^\top(d^{(i)}) \Phi(d^{(i)}) \right)^{-1} \Phi^\top(r^{(j)})$$

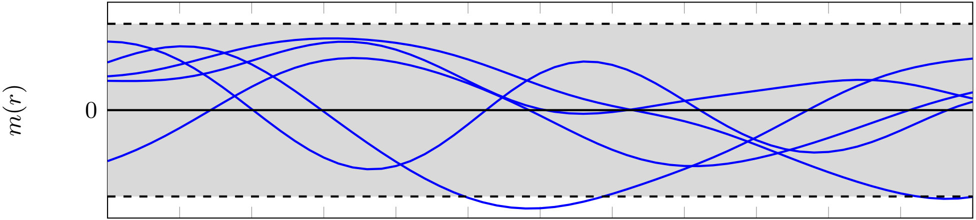
where

$$\hat{\sigma}_e^2 = \frac{1}{3L} \sum_{i=1}^{L} \| y_k^{(i)} - \Phi(d^{(i)}) \hat{\theta}_l \|^2.$$

Gaussian Processes (GPs)

- Stochastic process, such that every finite collection of those random variables has a multivariate normal distribution.
- Commonly used for non-parametric function approximation, i.e., $m(r) \sim \mathcal{GP}(\mu(r), \kappa(r, r'))$.
- The kernel function $\kappa(r, r')$ describes the correlation between input r and r'; the mean is described by $\mu(r)$.

Example:



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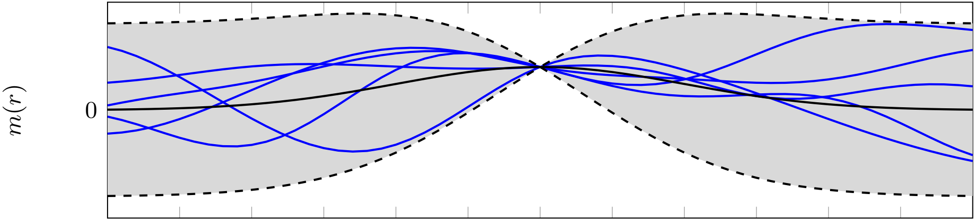

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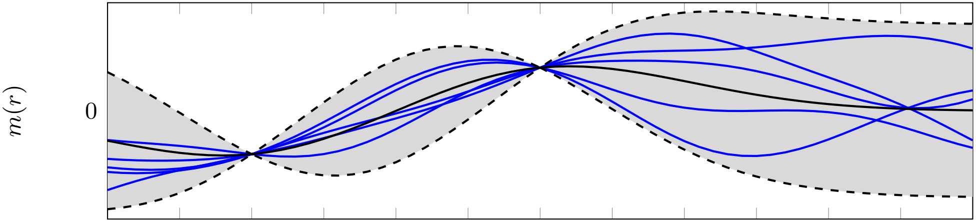


GP Model: Curl-free Case

- The magnetic field can be modeled as a GP, in its basic form

$$\mathcal{M}(r;\theta) = \mathcal{GP}(0, \kappa_B(r, r')) \qquad \text{(Mean has been marginalized)}$$

- A curlfree field can be enforced with an appropriate kernel choice, *e.g.*:

$$\kappa_B(r, r') = \sigma^2_{\text{lin}} I_3 + \sigma_f^2 e^{-\frac{\|r-r'\|^2}{2\ell^2}} \cdot \left(\frac{(r-r')(r-r')^\top}{\ell^2} + \left(2 - \frac{\|r-r'\|^2}{\ell^2}\right) I_3 \right)$$

  As with polynomial model case, model the potential as a GP to get a curl-free field.
- The hyper parameters represent freedom to vary (σ^2_{lin} and σ^2_f) and the length scale of the variations (ℓ).
  - Can be optimize in an outer loop.
  - Can be picked based on physical insight.

GP Model: Divergence- & Curl-free Case

- Add magnetization $\eta(r)$, which is known to be 0 in air, to the GP model.
- The resulting GP:

$$\begin{pmatrix} \mathcal{M}(r;\theta) \\ \eta(r) \end{pmatrix} = \mathcal{GP}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \kappa_B(r,r') & \kappa_B(r,r') \\ \kappa_B(r,r') & \kappa_B(r,r') + \kappa_H(r,r') \end{pmatrix}\right)$$

$$\kappa_H(r,r') = \sigma_{\text{lin}}^2 I_3 + \sigma_f^2 e^{-\frac{\|r-r'\|^2}{2\ell^2}} \cdot \left(I_3 - \frac{(r-r')(r-r')^\top}{\ell^2} \right).$$

- Make virtual measurements to enforce $\eta(r) = 0$.
- Additional complexity seems to pay of in practice.

Modelling the magnetic field

Gaussian process models: Estimate a function $f(x)$ from observations and prior knowledge about the shape of the function.

How To Estimate The Pose Change?

- Measurement model with prefect magnetic field model

$$y_{k+1} = h(x_{k+1}; \mathcal{M}_k(r)) + e_{k+1}$$

where x_{k+1} = pose change, $\mathcal{M}_k(r)$ = mag. field model, and e_{k+1} = meas. error with covariance $\text{Cov}(e_k) = \Sigma_{e_k}$.

- If $\hat{\mathcal{M}}_k(r)$ is unbiased and the model error small, then

$$y_{k+1} \approx h(x_{k+1}; \hat{\mathcal{M}}_k(r)) + \varepsilon_{k+1}$$

where

$$\Sigma_{\varepsilon_k} \triangleq \text{Cov}(\varepsilon_k) = \underbrace{\Sigma_{\hat{\mathcal{M}}_{k-1}}(x_k)}_{\text{additional uncertainty due to model errors}} + \underbrace{\Sigma_{e_k}}_{\text{uncertainty due to measurement errors}}$$

- Least squares pose change estimate

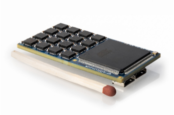
$$\hat{x}_{k+1} = \arg\min_x V(x) \qquad V(x) = \|y_{k+1} - h(x; \hat{\mathcal{M}}_k(r))\|^2_{\Sigma^{-1}_{\varepsilon_{k+1}}(x)}.$$

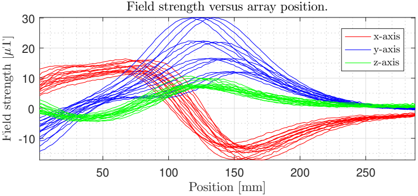
Results

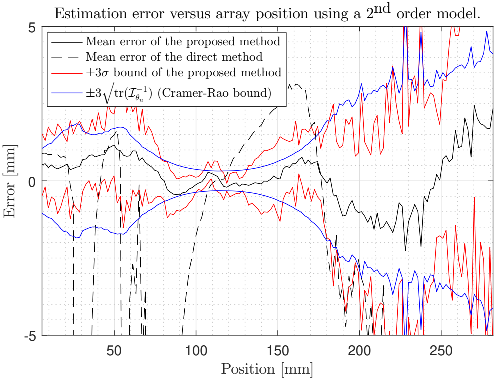
Scaled Experiments

Proof-of-concept experiment

- Controlled linear translation
- Scaled down, realistic field
- Proposed polynomial based method works well



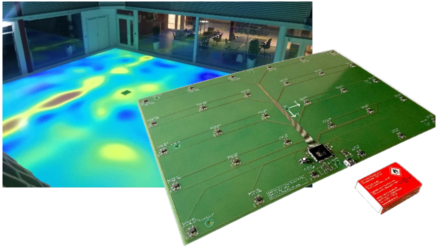


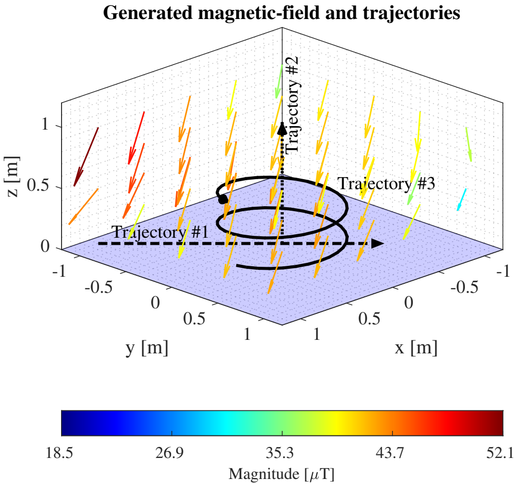


Reference

I. Skog, G. Hendeby, and F. Gustafsson. Magnetic odometry — a model-based approach using a sensor array. In *Proceedings of 21th IEEE International Conference on Information Fusion*, Cambridge, UK, July 10–13 2018

Results: Full Scale Experiment (1/3)





Reference

I. Skog, G. Hendeby, and F. Trulsson. Magnetic-field based odometry — an optical flow inspired approach. In *Proceedings of Eleventh International Conference on Indoor Positioning and Indoor Navigation*, Lloret de Mar, Spain, Nov. 29–Dec. 2 2021

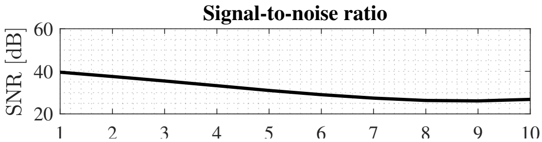
Results: Full Scale Experiment (2/3)

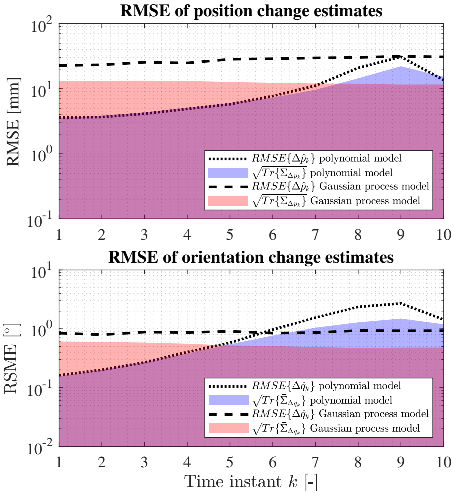
Aims

- Judge the feasibility of the proposed method under realistic conditions
- Compare polynomial and GP model assumption

Trajectory 2

- Motion out of the plane
- Displacement: 100 mm

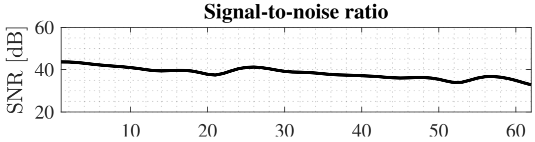




Results: Full Scale Experiment (3/3)

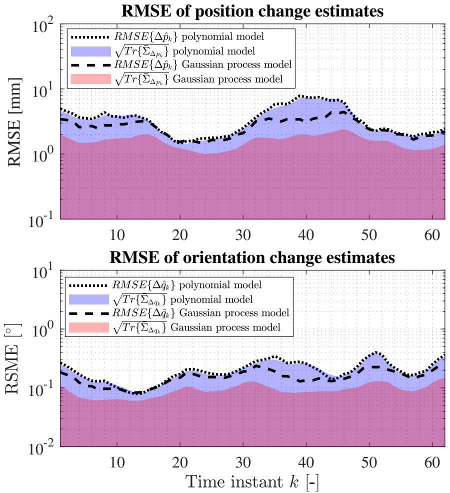
Trajectory 3

- Motion out of the plane + rotations
- Displacement: 100 mm
- Rotation: 3 °



Conclusions

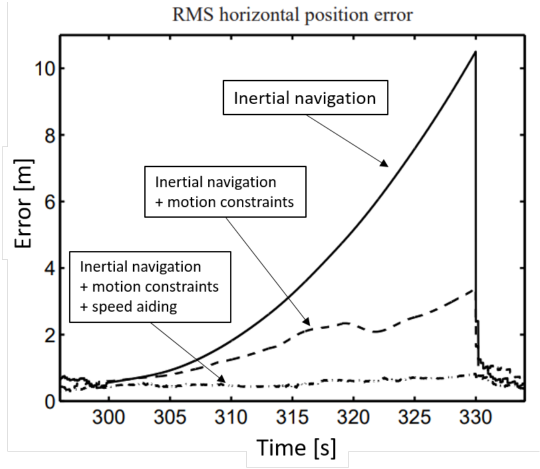
- Good est. accuracy at high, but realistic, SNR values.
- Poly. model simpler and more accurate.



Magnetic Odometry Aided INS

Magnetic Odometry Aided INS

- Inertial navigation systems (INSs) inherently drift over time.
- Speed/displacement information can limit the drift.
- The proposed estimated pose change comes with uncertainty information, making it perfect for INS integration.
- Use a filter (*e.g.*, extended Kalman filter (EKF)), compensate the predicted displacement based on the magnetic measurements.



Loose Versus Tight Integration

Loose INS integration

- Pose change estimates used as measurements for the filter in the INS aiding.
- Can keep existing INS aiding filter structures.
- Typically makes the system *less* robust.

Tight INS integration

- Raw magnetometer data used as measurements for the filter in the INS aiding.
- New INS aiding filter structures must be developed.
- Typically makes the system *more* robust.

Tight Integration Using A Polynomial Model: Basic Idea (1/2)

1. Note that the parameters change due to the shift of the center of a polynomial model

$$g(r;\theta,r_0)=\sum_{i=0}^{p}\theta_i(r-r_0)^i \quad \Leftrightarrow \quad g(r;\theta',r_0+\Delta r)\sum_{i=0}^{p}\theta'_i(r-r_0-\Delta r)^i$$

   can be described by a linear transformation of the form $\theta' = A(\Delta r)\theta$.

2. Describe the local magnetic-field center at the origin of the array using a polynomial model and add the coefficients to the navigation state-vector x_k.

$$x_{k+1}^{\text{ext}} \triangleq \begin{bmatrix} x_k \\ \theta_{k+1} \end{bmatrix} = f^{\text{ext}}(x_k^{\text{ext}}, u_k, w_k^{\text{ext}}) = \begin{bmatrix} \underbrace{f(x_k,u_k,w_k)}_{\text{INS nav. eq}} \\ \underbrace{A(x_k,u_k)\theta_k + w_k^{\theta}}_{\text{poly. coeff. update}} \end{bmatrix}$$

Tight Integration Using A Polynomial Model: Basic Idea (2/2)

3. Create measurement equation using the polynomial model equation

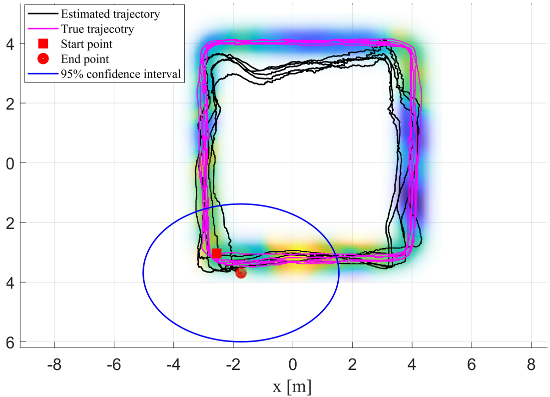
$$y_k^{\text{mag}} = \begin{bmatrix} 0 & \dots & \Phi(d^{(1)}) \\ 0 & \dots & \vdots \\ 0 & \dots & \Phi(d^{(M)}) \end{bmatrix} x_k^{\text{ext}}$$

4. Estimate the state x_k^{ext} with the your favourite filter...

References

- C. Huang, G. Hendeby, and I. Skog. A tightly-integrated magnetic-field aided inertial navigation system. In *IEEE Int. Conf. on Information Fusion*, Linköping, Sweden, July 2022

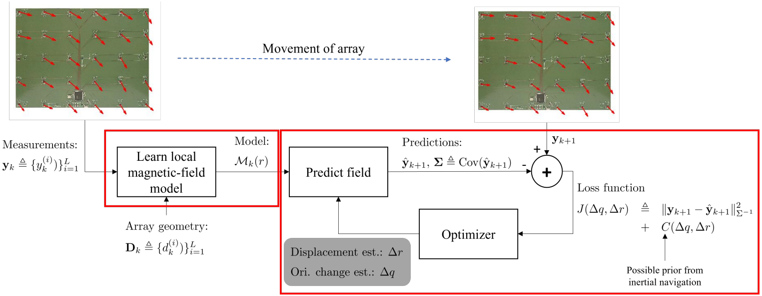
Preliminary Results: Tightly Integrated Magnetic-Field Aided INS



- Verified on experimental data from (repeated) indoor trajectories.
- Sub-meter accuracy observed after $3+$ min.
- Drastically reduced drift compared to pure dead reckoning

Summary

Summary



- Odometrics can be estimated from measuring the natural magnetic field.
- The field variations limits the performance.
- The odometric measurements are suitable as supporting measurements in INS systems

- Challenges:
  - The size of the array.
  - Quality and calibration of magnetometers.
  - How to best integrate this into a SLAM solution?

Gustaf Hendeby, gustaf.hendeby@liu.se
Manon Kok, M.Kok-1@tudelft.nl
Isaac Skog, isaac.skog@liu.se

www.liu.se

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