# PERSPECTIVES
on the
# PROCESS
of
# IDENTIFICATION

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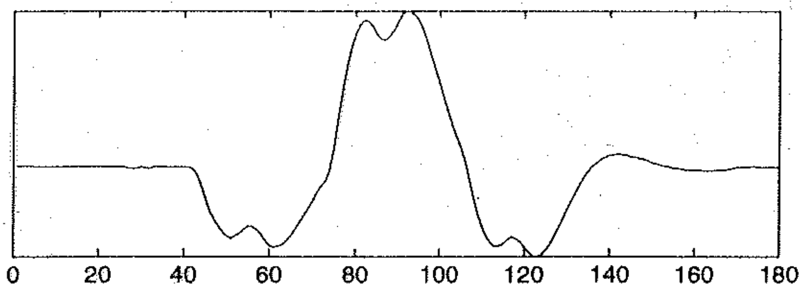
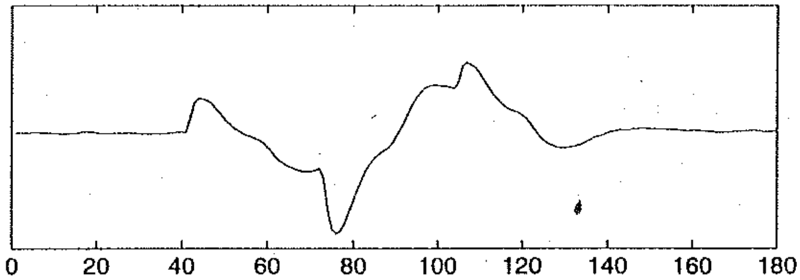
Linköping


y
φ

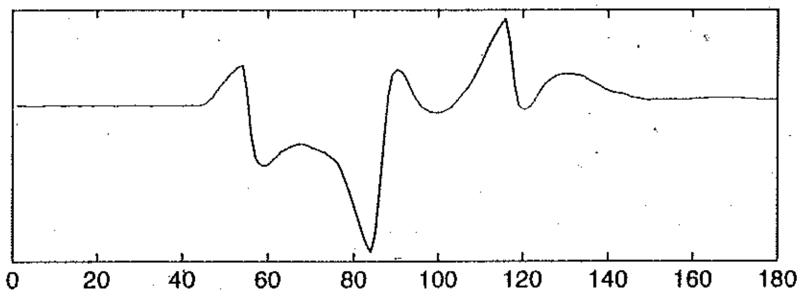
*Pitch rate*



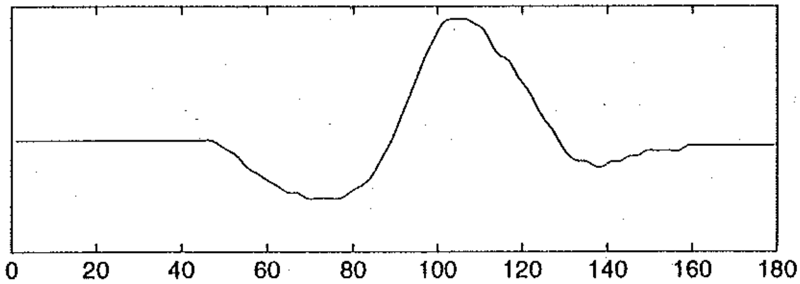

*Elevator Angle*




*Canard Angle*

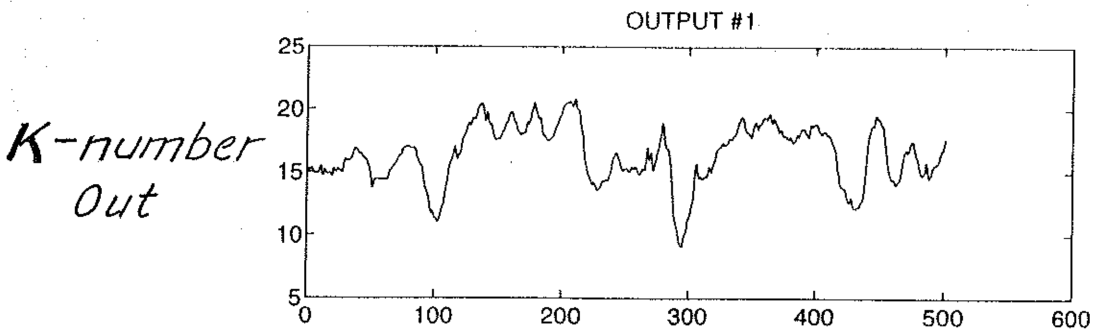


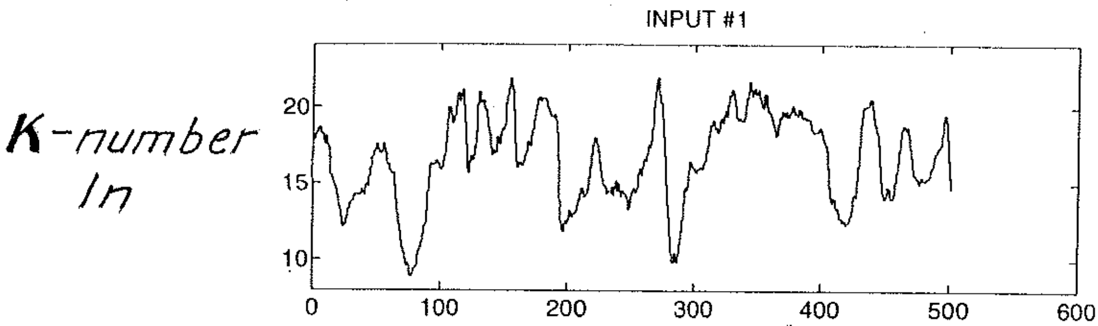

*Leading Egde Flap*

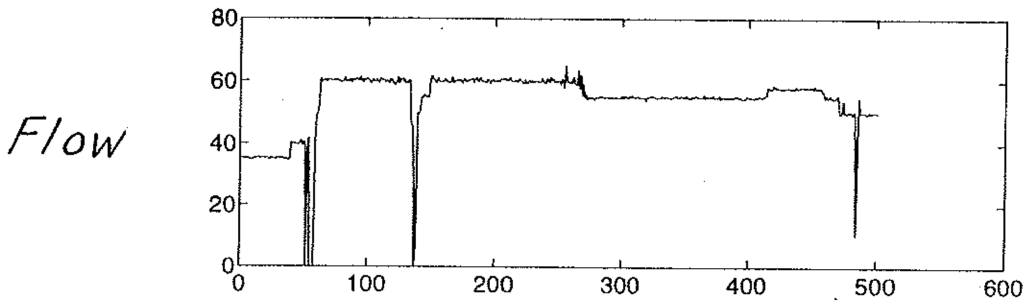


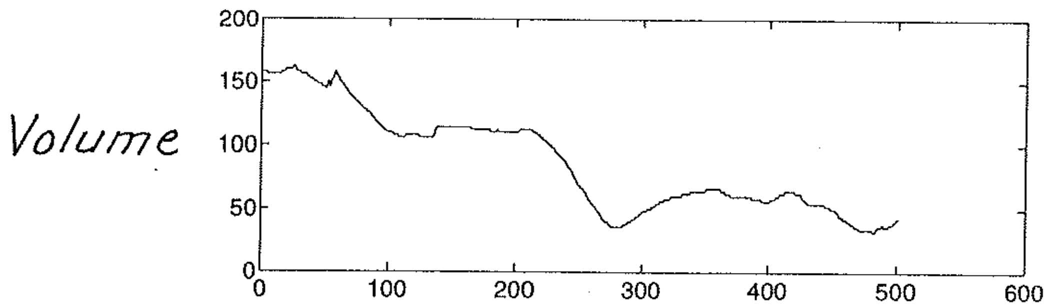
# Questions

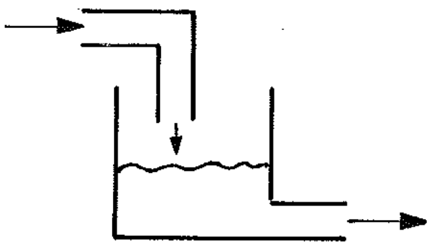
- How do the rudder angles affect the pitch rate?
- Aerodynamical derivatives?
- How to use the information in flight data?


OUTPUT #1
K-number Out


INPUT #1
K-number In


Flow


Volume



# Questions

- Time mark the pulp as it passes through the different vessels!
- What about the residence time in the vessels?
- How to use the information in the observed data?

# The Engineer's Perspective

- How to use the information in the observed data to build a model?

- How to know if the model is any good?

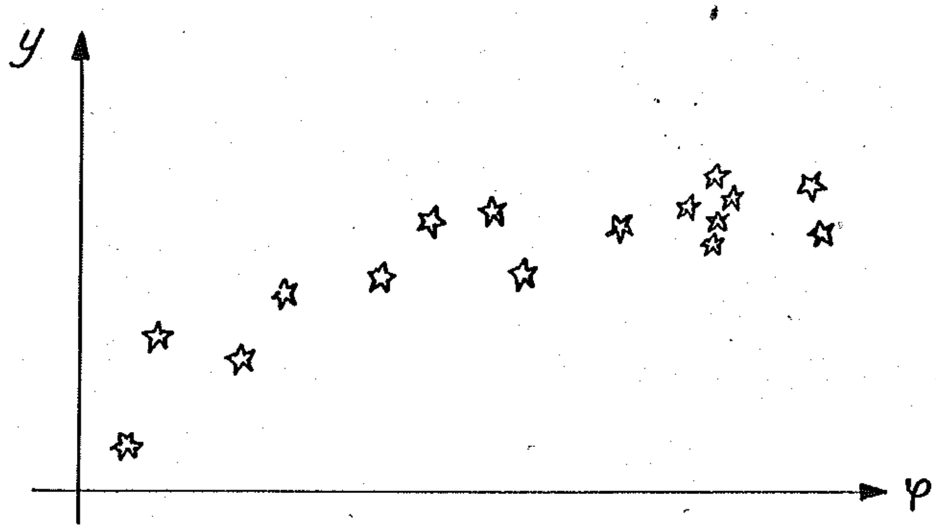
- What kind of software is available for the tasks?

# The Essence of the Problem

- See $z(t) = [y(t), \varphi(t)]$ for $t = 1, 2, \ldots, N$
- $\varphi(t)$: "Available Information, Past Data"
- Now see $\varphi(N+1)$!
- Say something about $y(N+1)$!
- $y(t)$ and $\varphi(t)$ could take values in any kind of sets.

# Patterns

What we have really is a number of points in $R^d, d = \text{dim}y + \text{dim}\varphi$




See the pattern!

# Two Basic Problems:

- Cannot have all possible $\varphi(t)$ in the observed data set.
  - Interpolation, extrapolation

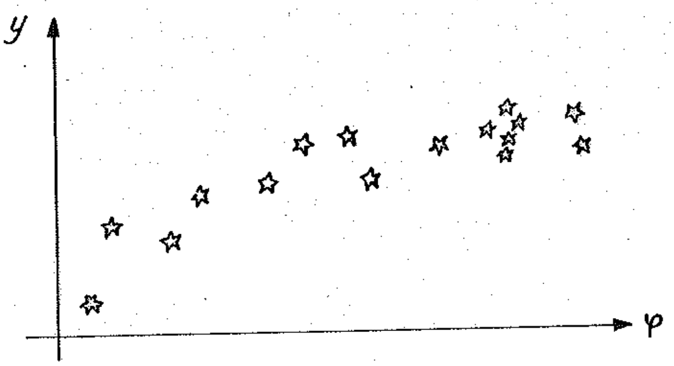
- No Exact Reproducibility
  - "noise", disturbance assumptions

# Perspectives

- Statistical
  - model for non-reproducibility
- Pattern Recognition
  - $y$ discrete-valued
- Projection methods (in statistics)
  - subspaces
  - linear/nonlinear regressions

- Learning theory
  - how many data points are required to distinguish patterns?
- Machine learning, Knowledge Acquisition
  - build up rules from examples
  - trees

Bottom line:




- Parameterize the "data cluster areas"!
  - $h(y(t), \varphi(t), \theta) \approx 0$
  - The function $h$ provides for the extra- and interpolations
  - Adjust $\theta$ using the "examples" of $\{y(t), \varphi(t)\}$
  - Non-reproducibility $\Longleftrightarrow$ "$\approx$"

# The Control Scientist's Perspective: System Identification

The two basic problems:

- Interpolations and extrapolations over the data-space is the task of the *Model Structure*
- Non-reproducibility is blamed on the *Unmeasured Input* $v(t)$ $\Rightarrow$ Average out by redundancy in a selection criterion.

**How to cope with the unmeasured input ("disturbances, noise")?**

**How to pick a "selection rule"?**

- Constrain the set of possible $v$:s

$$|v(t)| \leq C \quad \forall t$$

- Assign probabilities to the different possible $v$:s:

$$v \text{ has pdf } p_v(\cdot, \theta)$$

# Approaches

- Non-probabilistic $v(t) \in V$
  - Unknown-but-bounded
  - Set membership

- Probabilistic
  - The pdf for $v$ gives a pdf for $z$
  - Maximum likelihood

- Pragmatic
  - $\hat{y}(t|\theta) = g_t(\theta, \varphi(t))$ *The Model Structure*
  - $y(t) = \hat{y}(t|\theta) + e(t)$
  - min $V(\theta) = \sum ||y(t) - \hat{y}(t|\theta)||$
    - Contains ML and set membership

# The Crux: The Model Structure How to extra-/interpolate over the data-space

$$\widehat{y}(t|\theta) = g_t(\theta, \varphi(t))$$

- Black-Box
- Physical Modeling
- Semi-Physical Modeling

# The Crux: The Model Structure How to extra-/interpolate over the data-space

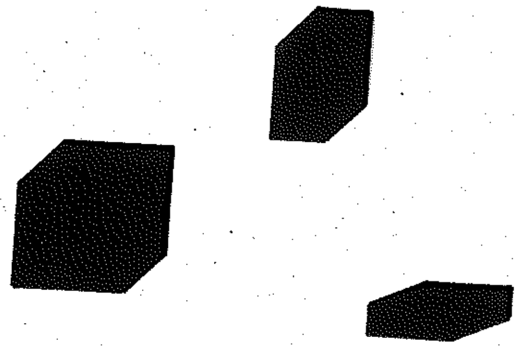
Guessed output "past data"

$$\hat{y}(t|\theta) = g_t(\theta, \varphi(t))$$

parameters to adjust

- Black-Box
- Physical Modeling
- Semi-Physical Modeling

# Black Boxes



Idea: Interpolate between the $\varphi$:s by smooth standard functions

$$\widehat{y}(t|\theta) = \sum_{k=1}^{d} \theta_k h_k(\varphi(t))$$

$$\varphi(t) = [y(t-1), \ldots, y(t-n), u(t-1), \ldots, u(t-m)]$$

$h_k(\varphi)$ are basis functions that are mappings from the $\varphi$-space to the $y$-space. They may depend on $\theta$:

$$h_k(\varphi, \theta)$$

# Black-Box Basis Functions

Basic property:

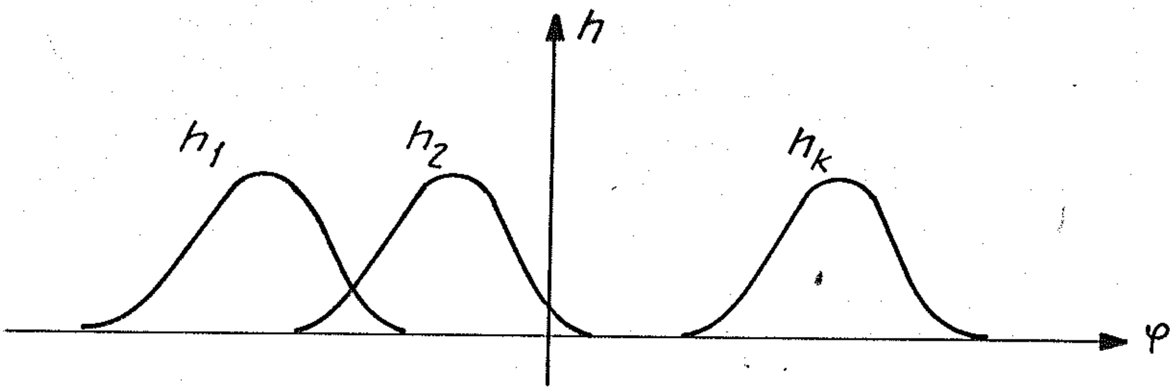
- $h_k(\varphi), k = 1, \ldots$ form a basis for all (reasonable) functions from the $\varphi$-space to the $y$-space.

- $d = d(N) \to \infty$ as $N \to \infty$: Non-parametric (regression) methods.

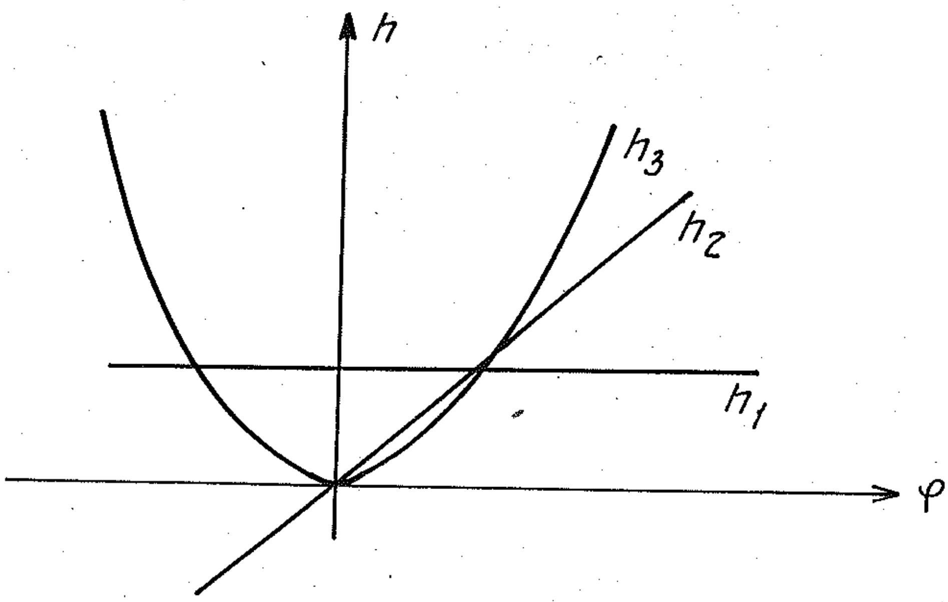
- Hope to "do well" with just a few of them

# Character of the basis functions:

- Local



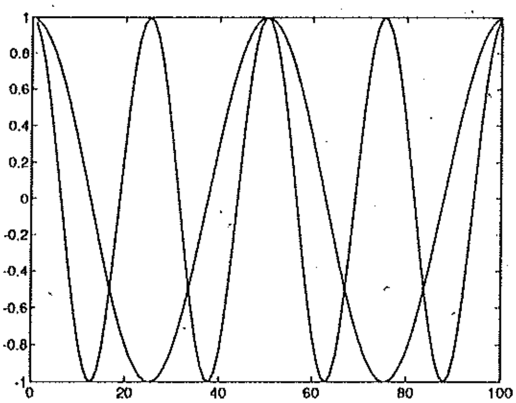

- Global



# Common Choices of Basis Functio

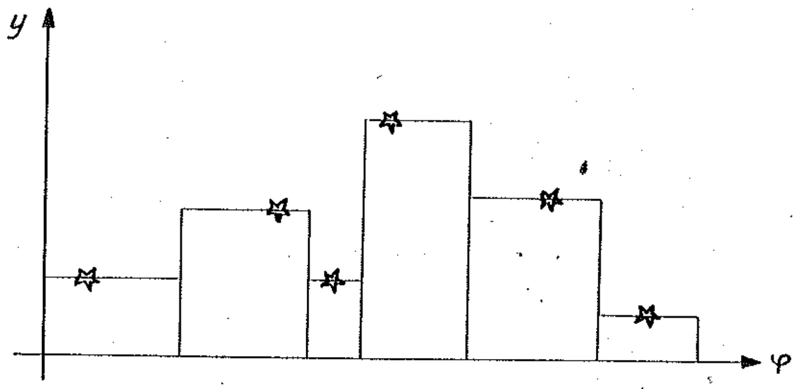
- "Classic System Identification"
  - Linear $\varphi$-spaces: $h_k(\varphi(t)) = u(t-k)$ or $y(t-k)$ (or $\hat{y}(t-k|\theta)$): The black-box difference equation family. (ARX, ARMAX, etc)

    Can also be viewed as bases in the space of frequency functions:



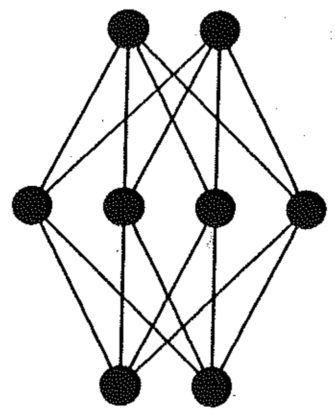

  - Volterra and other non-linear counterparts

- "Classic non-parametric regression"
  - Nearest Neighbor: $h_k(\varphi)$ indicator function for smallest possible data box

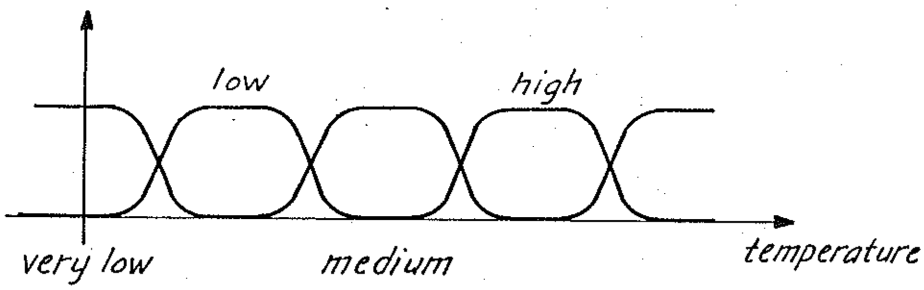



  - Average boxes (Radial basis Neural Networks): (Smooth) indicator function for somewhat bigger boxes.
  - Trees

- Neural Networks
  - Explicit equations for $h_k$ complicated, but easy recursions

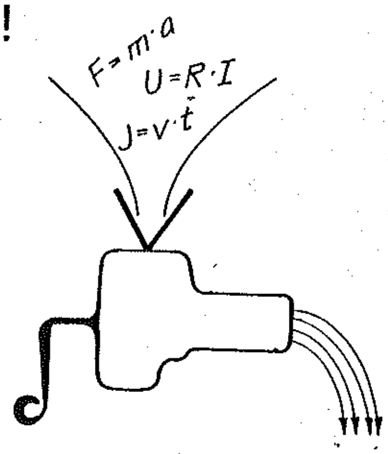


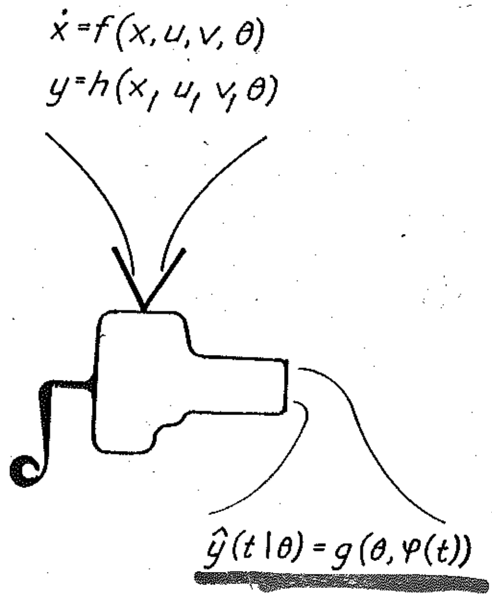
- Fuzzy Models
  - Membership functions – interpolation functions – $h_k$



# Physical Model Structures

Basic Guideline: Don't Estimate What You Already Know!



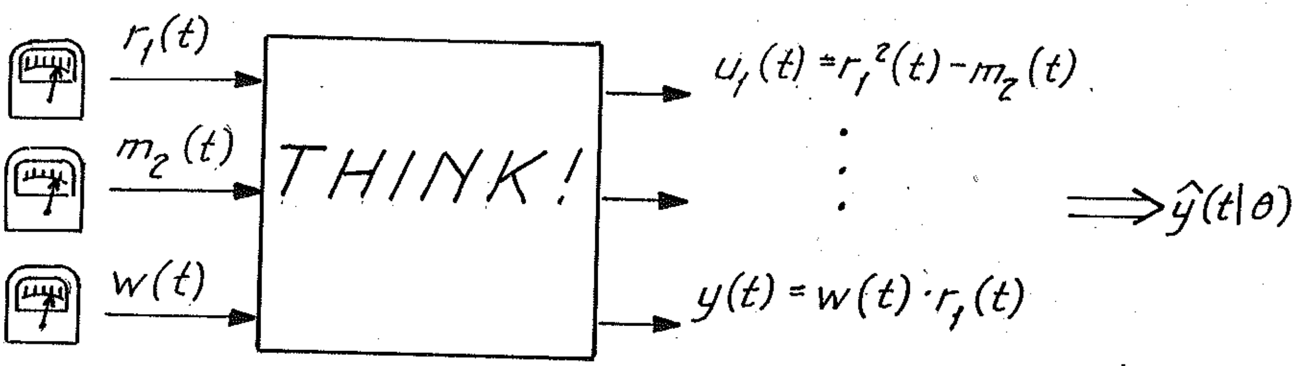




The Physics is used to interpolate and EXTRAPOLATE in the $\varphi$-space

# Semi-Physical Model Structures

Introduce essential non-linearities "by hand"

Again: Don't estimate what you already know



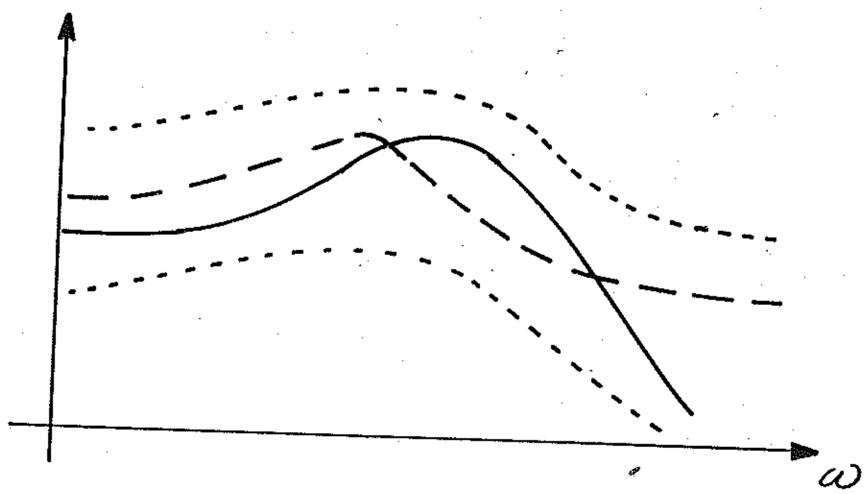
# The Heart of the Matter: Model Validation

The basic process of identification can be seen as a way to provide candidate models to be subjected to validation:

- How far away might it be from a correct description?
  - Next page!
- Are my model structure assumptions consistent with the observed data?
  - (Classical) residual analysis
- Is it good enough?
  - Subjective!

# "Model Error Modeling"

- Again the two basic problems:
  - Not the right interpolation rules: *Bias Error*
  - Getting fooled by the "noise": *Random Error*



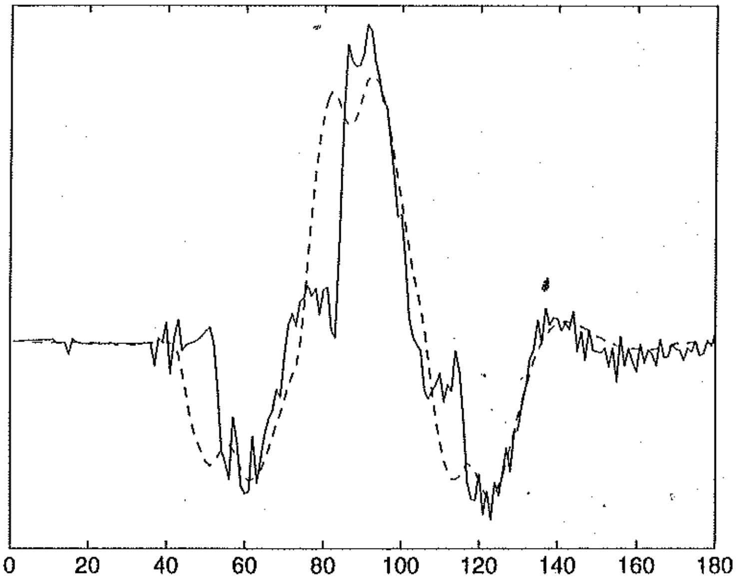
- Basic Advice:
  - Determine a model that passes the validation tests.
  - $\Rightarrow$ Bias error $\leq$ random error
  - Reduce model if necessary – with respect to its purpose

# The Engineer's Perspective II
# Solving the Problem

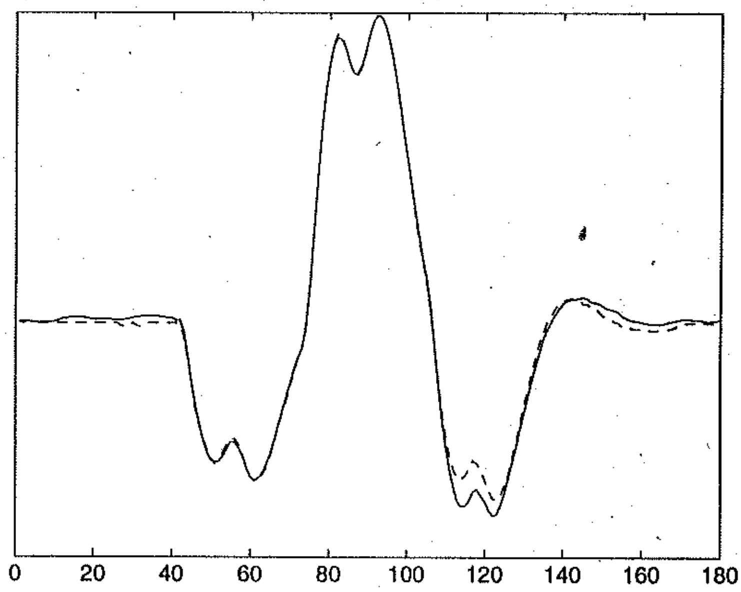
A recipe for dynamical systems:

1. `compare(z,arx(z(1:200,:),[4 4 1]))`
2. Does it look good?
   - Yes: Congratulations!
   - NO:
     - Higher order
     - More inputs
     - Apply semi-physical modelling
     - Give up!

# Aircraft Dynamics

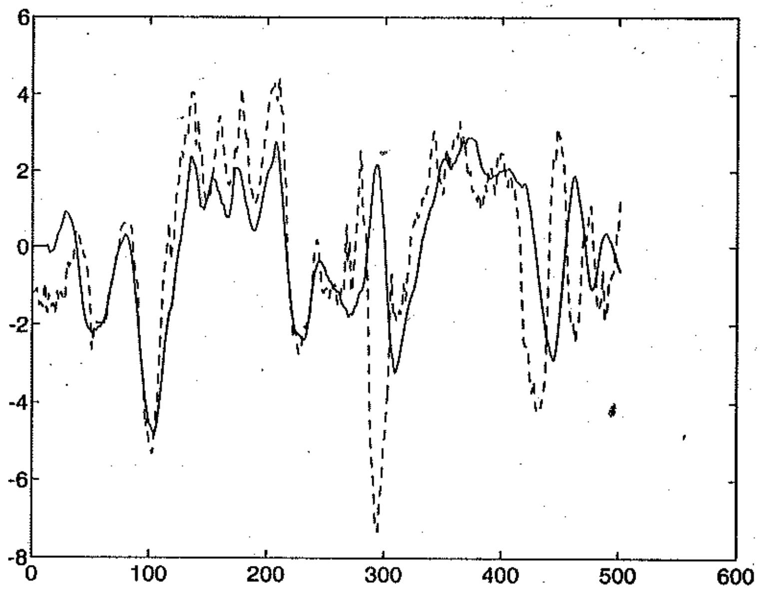



Dashed line: Actual Pitch rate. Solid line: 10 step ahead predicted pitch rate, based on the fourth order model from canard angle only.




As above but using all three inputs.

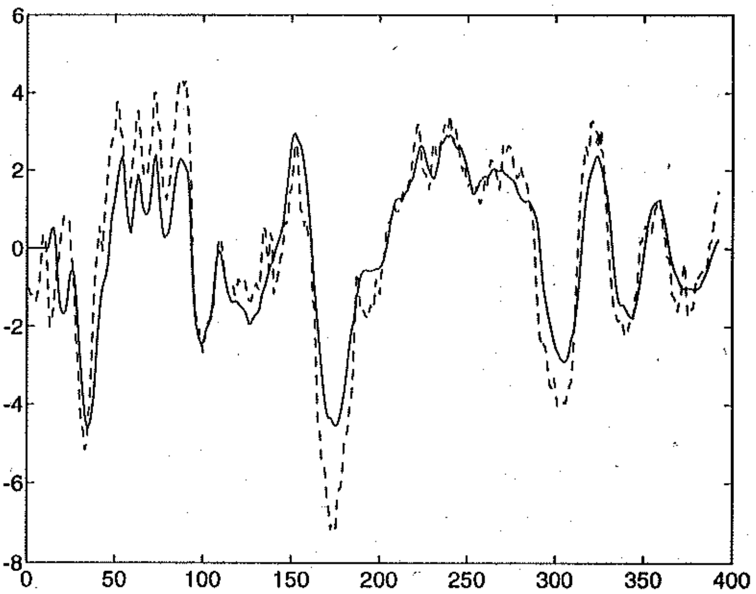
# Buffer Vessel Dynamics




Dashed line: $\kappa$-number after the vessel, actual measurements. Solid line: Simulated $\kappa$-number using the input only and a fourth order linear model with delay 12, estimated using the first 200 data points.

Think: ....

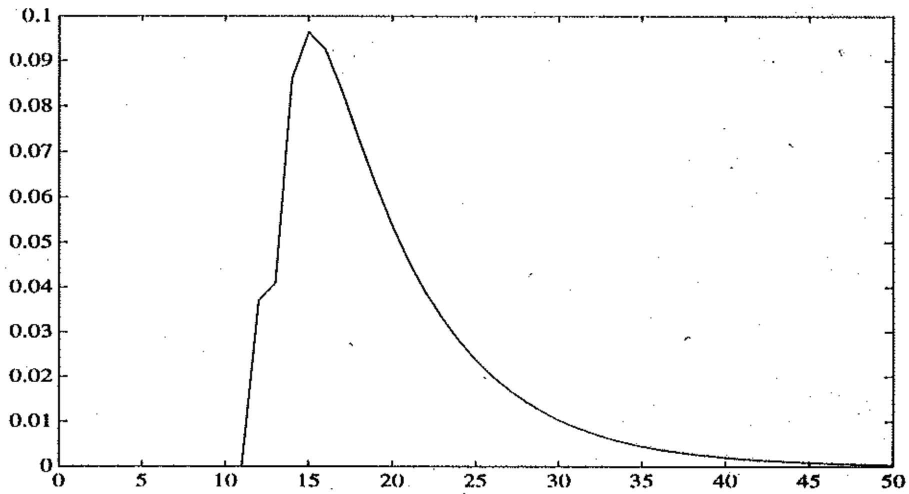
```
z = [y,u]; pf = flow./level;
t = 1:length(z)
newt = table1([cumsum(pf),t],[pf(1):sum(pf)]');
newz = table1([t,z], newt);
```




Same as previous figure but applied to resampled data

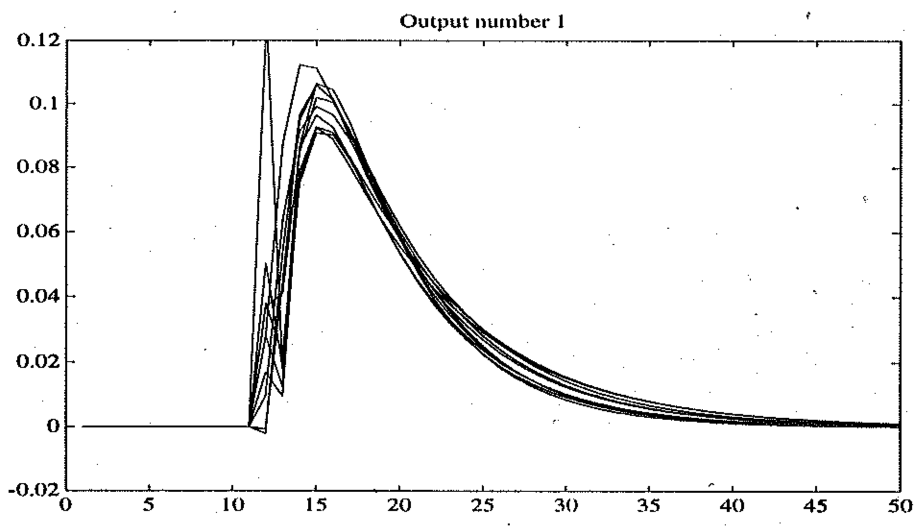
# What's the impulse response of our model?

```
m=arx(ze,nn);
impres=idsim([1;zeros(49,1)],m);
plot(impres)
```



# What's the uncertainty?

```
idsimsd([1;zeros(49,1)],m)
```



# Conclusions

- Process identification is meeting place for practical problems and fairly advanced theory
- The pragmatic approach ("Curve fitting") has many theoretical interpretations
- Important to see the links between "hot" new approaches and classic theory
- Good software support
- The area starts and ends with real data

Bottom line:

See the pattern in observed data!