STATE-OF-THE-ART FOR THE MARGINALIZED PARTICLE FILTER

Fredrik Gustafsson, Thomas B. Schön, Rickard Karlsson and Per-Johan Nordlund

Division of Automatic Control
Department of Electrical Engineering
Linköping University, Sweden
SE–581 83 Linköping, Sweden
{fredrik, schon, rickard, perno}@isy.liu.se

ABSTRACT

The marginalized particle filter is a powerful combination of the particle filter and the Kalman filter, which can be used when the underlying model contains a linear substructure subject to Gaussian noise. This paper surveys state of the art for theory and practice.

1. INTRODUCTION

Consider the problem of state estimation using the following model with a mixture of linear and nonlinear dynamics

$$x^n_{t+1} = f^n_t(x^n_t) + A^n_t(x^n_t)x^l_t + G^n_t(x^n_t)w^n_t, \qquad (1a)$$

$$x^l_{t+1} = f^l_t(x^n_t) + A^l_t(x^n_t)x^l_t + G^l_t(x^n_t)w^l_t, \qquad (1b)$$

$$y_t = h_t(x^n_t) + C_t(x^n_t)x^l_t + e_t, \qquad (1c)$$

with the following statistical assumptions

$$w_t = \begin{pmatrix} w^l_t \\ w^n_t \end{pmatrix} \sim \mathcal{N}(0, Q_t),\ Q_t = \begin{pmatrix} Q^l_t & Q^{ln}_t \\ (Q^{ln}_t)^T & Q^n_t \end{pmatrix}, \qquad (1d)$$

$$e_t \sim \mathcal{N}(0, R_t), \qquad (1e)$$

$$x^l_0 \sim \mathcal{N}(\bar{x}_0, \bar{P}_0). \qquad (1f)$$

The most principal approaches are to use the *extended Kalman Filter* (EKF) [13] that linearizes the nonlinear dynamics or the *particle filter* (PF) [8, 12, 24] which applies to general nonlinear models, and does not utilize the linear dynamics in (1).

The *marginalized particle filter* (MPF), or Rao- Blackwellized particle filter, [2, 3, 5, 9, 23, 27] combines the good features of the *Kalman filter* (KF) and the PF. The posterior distribution of the state vector x^l appearing linearly in (1a) are represented by its mean vector and covariance matrix computed by the Kalman filter. The PF computes the posterior of x^n using a set of samples, where each sample has one associated KF. Figure 1 illustrates this as an waterfall view of the posterior distribution.

The paper is based on [28] and it discusses state-of-the-art of MPF theory and some applications.

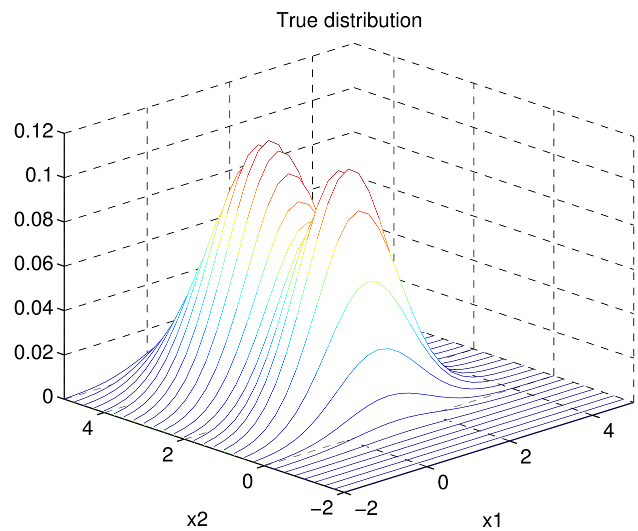

Fig. 1. Representation of posterior distribution in the MPF seen as a waterfall view. The nonlinear state, here x_2, is represented by a set of discrete samples, each sample associated with a Gaussian distribution for x_1.

2. THEORY AND ANALYSIS

The theory and analysis are here split in the following areas:

- *Background theory.* The basic algorithms are found in [2, 5, 9, 27]. The different twists that occur when certain terms in (1a) disappear are thoroughly discussed in [27]. The most important term is $C_t(x^n_t)x^l_t$: without that term the Ricatti equation becomes the same for all KF's, leading to substantial savings in computations.

- *Variance reduction.* One important advantage of MPF is the variance reduction that follows from the relation

$$\mathsf{Var}\left(g(U,V)\right) = \\ \mathsf{Var}\left(\mathsf{E}\left(g(U,V)|V\right)\right) + \mathsf{E}\left(\mathsf{Var}\left(g(U,V)|V\right)\right),$$

  In the MPF setup, U and V are represented by the linear and nonlinear states, respectively. This is sometimes referred to as Rao-Blackwellization, see e.g., [25]. The last term disappear in the MPF, which leads to the variance reduction.

- *Complexity analysis.* The complexity for the two cases mentioned above (with same or different Ricatti equations for the KF's) is analyzed in [18].

There are certainly more topics that fit within the MPF framework, for instance quantization, [16], data association, and *simultaneous localization and mapping* (SLAM) aspects, [1].

3. APPLICATIONS

Positioning applications:

- *Underwater terrain-aided positioning* [14, 15]
- *Aircraft terrain-aided positioning* [27]
- *Automotive map-aided positioning* [29]
- *GPS navigation* [11]
- *SLAM* [21, 22]

Target tracking applications:

- *Automotive target tracking* [10]
- *Bearings-only target tracking* [17]
- *Radar target tracking* [28]

Other applications:

- *Communication applications* [6, 30]
- *System identification* [7, 19, 20, 26]
- *Audio applications* [4].

Acknowledgment

This work was supported by VINNOVA's Center of Excellence ISIS (Information Systems for Industrial Control and Supervision), and by the Swedish Research Council (VR).

4. REFERENCES

[1] Lecture notes from SLAM summer school. H. Christensen (editor). KTH, Stochholm, Sweden, Aug. 2002.

[2] C. Andrieu and A. Doucet. Particle filtering for partially observed Gaussian state space models. *Journal of the Royal Statistical Society*, 64(4):827–836, 2002.

[3] C. Andrieu, A. Doucet, and E. Punskaya. *Sequential Monte Carlo Methods in Practice*, chapter Sequential Monte Carlo Methods for Optimal Filtering. Statistics for Engineering and Information Science. Springer Verlag, 2001.

[4] C. Andrieu and S. Godsill. A particle filter for model based audio source separation. In *Proceedings of the International Workshop on Independent Component Analysis and Blind Signal Separation (ICA 2000)*, Helsinki, Finland, June 2000.

[5] R. Chen and J. Liu. Mixture Kalman filters. *Journal of the Royal Statistical Society*, 62(3):493–508, 2000.

[6] R. Chen, X. Wang, and J. Liu. Adaptive joint detection in flat-fading channels via mixture Kalman filtering. *IEEE Transactions on Information Theory*, 46(6):2079–2094, 2000.

[7] M. J. Daly, J. P. Reilly, and M. R. Morelande. Rao-Blackwellised particle filtering for blind system identification. In *Proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing*, Philadelphia, PA, USA, Mar. 2005.

[8] A. Doucet, N. de Freitas, and N. Gordon, editors. *Sequential Monte Carlo Methods in Practice*. Springer Verlag, 2001.

[9] A. Doucet, S. Godsill, and C. Andrieu. On sequential Monte Carlo sampling methods for Bayesian filtering. *Statistics and Computing*, 10(3):197–208, 2000.

[10] A. Eidehall, T. B. Schön, and F. Gustafsson. The marginalized particle filter for automotive tracking applications. In *Proceedings of the IEEE Intelligent Vehicle Symposium*, pages 369–374, Las Vegas, USA, June 2005.

[11] A. Giremus and J. Tourneret. Joint detection/estimation of multipath effects for the global positioning system. In *Proceedings of IEEE International Conference on Acoustics, Speech, and Signal Processing*, volume 4, pages 17–20, Philadelphia, PA, USA, Mar. 2005.

[12] N. J. Gordon, D. J. Salmond, and A. F. M. Smith. Novel approach to nonlinear/non-Gaussian Bayesian state estimation. In *IEE Proceedings on Radar and Signal Processing*, volume 140, pages 107–113, 1993.

[13] T. Kailath, A. Sayed, and B. Hassibi. *Linear Estimation*. Information and System Sciences Series. Prentice Hall, Upper Saddle River, New Jersey, 2000.

[14] R. Karlsson and F. Gustafsson. Surface and underwater navigation using particle filters. *To appear IEEE Transactions on Signal Processing*.

[15] R. Karlsson and F. Gustafsson. Particle filter for underwater navigation. In *Proceedings of the Statistical Signal Processing Workshop*, pages 509–512, St. Louis, USA, Sept. 2003.

[16] R. Karlsson and F. Gustafsson. Particle filtering for quantized sensor information. In *Proceedings of the 13th European Signal Processing Conference*, Antalya, Turkey, Sept. 2005.

[17] R. Karlsson and F. Gustafsson. Recursive Bayesian estimation – bearings-only applications. *IEE Proceedings on Radar, Sonar, and Navigation. Special issue on target tracking: Algorithms and Applications*, 152(5):305–313, Oct. 2005.

[18] R. Karlsson, T. Schön, and F. Gustafsson. Complexity analysis of the marginalized particle filter. *IEEE Transactions on Signal Processing*, 53(11):4408–4411, Nov. 2005.

[19] P. Li, R. Goodall, and V. Kadirkamanathan. Parameter estimation of railway vehicle dynamic model using Rao-Blackwellised particle filter. In *Proceedings of the European Control Conference*, Cambridge, UK, Sept. 2003.

[20] P. Li, R. Goodall, and V. Kadirkamanathan. Estimation of parameters in a linear state space model using rao-blackwellise particle filter. *IEEE Proceedings of Control Theory Applications*, 151(6):727–738, Nov. 2004.

[21] M. Montemerlo, S. Thrun, D. Koller, and B. Wegbreit. Fastslam: A factored solution to the simultaneous localization and mapping problem. In *Proceedings of the AAAI National Conference on Artificial Intelligence*, 2002.

[22] K. Murphy. Bayesian map learning in dynamic environments. In *Advances in Neural Information Processing Systems*, pages 1015–1021, 2000.

[23] K. Murphy and S. Russel. *Sequential Monte Carlo Methods in Practice*, chapter Rao-Blackwellised particle Filtering for Dynamics Bayesian Networks. Statistics for Engineering and Information Science. Springer Verlag, 2001.

[24] B. Ristic, S. Arulampalam, and N. Gordon. *Beyond the Kalman Filter: particle filters for tracking applications*. Artech House, London, UK, 2004.

[25] C. P. Robert and G. Casella. *Monte Carlo Statistical Methods*. Springer texts in statistics. Springer, New York, 1999.

[26] T. Schön and F. Gustafsson. Particle filters for system identification of state-space models linear in either parameters or states. In *Proceedings of the 13th IFAC Symposium on System Identification*, pages 1287–1292, Rotterdam, The Netherlands, Sept. 2003. Invited paper.

[27] T. Schön, F. Gustafsson, and P.-J. Nordlund. Marginalized particle filters for mixed linear/nonlinear state-space models. *IEEE Transactions on Signal Processing*, 53(7):2279–2289, July 2005.

[28] T. Schön, R. Karlsson, and F. Gustafsson. The marginalized particle filter in practice. In *Proceedings 2006 IEEE Aerospace Conference*, Big Sky, USA, Mar 2006.

[29] N. Svenzén. Real time implementation of map aided positioning using a Bayesian approach. Master's Thesis No LiTH-ISY-EX-3297-2002, Department of Electrical Engineering, Linköpings universitet, Sweden, Dec. 2002.

[30] X. Wang, R. Chen, and D. Guo. Delayed-pilot sampling for mixture Kalman filter with application in fading channels. *IEEE Transactions on Signal Processing*, 50(2):241–254, Feb. 2002.